# Solution

## ECE317
## HW #5

# Problem 1:

Utilizing the Routh-Hurwitz criterion, determine the stability of the following polynomials. Determine the number of roots, if any, in the RHP. If it is adjustable, determine the range of *K* that results in a stable system.

a) $s^3 + 4s^2 + 8s + 4$

b) $s^3 + 2s^2 - 6s + 20$

c) $s^5 + s^4 + 2s^3 + s + 6$

d) $s^4 + s^3 + 3s^2 + 2s + K$

e) $s^5 + s^4 + 2s^3 + s^2 + s + K$

## a) Solution:

Given

$$s^3 + 4s^2 + 8s + 4 \ ,$$

we have the Routh array

| | | |
|---|---|---|
| $s^3$ | 1 | 8 |
| $s^2$ | 4 | 4 |
| $s^1$ | 7 | 0 |
| $s^o$ | 4 | |

Each element in the first column is positive, thus the system is stable.

## b) Solution:

Given

$$s^3 + 2s^2 - 6s + 20 \; ,$$

we determine by inspection that the system is unstable, since it is necessary that all coefficients have the same sign. There are two roots in the right half-plane.

## c) Solution:

Given

$$s^5 + s^4 + 2s^3 + s + 6 \; ,$$

we know the system is unstable since the coefficient of the $s^2$ term is missing. There are two roots in the right half-plane.

## d) Solution:

Given

$$s^4 + s^3 + 3s^2 + 2s + K \; ,$$

we have the Routh array

$$\begin{array}{c|ccc} s^4 & 1 & 3 & K \\ s^3 & 1 & 2 & 0 \\ s^2 & 1 & K & \\ s^1 & 2-K & 0 & \\ s^o & K & & \end{array}$$

Examining the first column, we determine that the system is stable for $0 < K < 2$.

## e) Solution:

Given

$$s^5 + s^4 + 2s^3 + s^2 + s + K \ ,$$

we have the Routh array

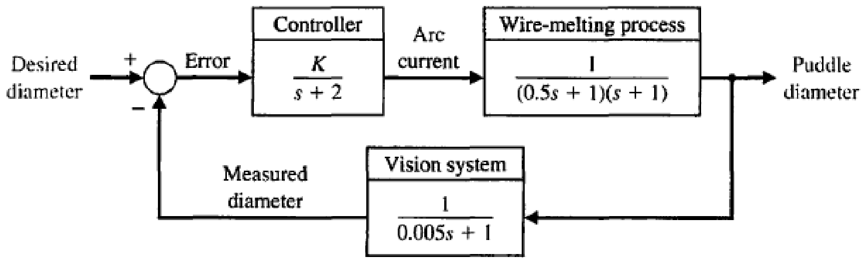
$$\begin{array}{c|ccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 1 & K \\ s^3 & 1 & 1-K & \\ s^2 & K & K & \\ s^1 & -K & 0 & \\ s^o & K & & \end{array}$$

Examining the first column, we determine that for stability we need $K > 0$ and $K < 0$. Therefore the system is unstable for all $K$.

## Problem 2:

Arc welding is one of the most important areas of application of industrial robots. In most manufacturing welding situations, uncertainties in dimensions of the part, geometry of the joint, and the welding process itself require the use sensors for maintaining weld quality. Several systems use a vision system to measure the geometry of the puddle of melted metal, as shown below. This system uses a constant rate of feeding the wire to be melted.

Calculate the maximum value for *K* of the system that will result in a stable system.




## Solution:

Given

$$G(s) = \frac{K}{(s+1)(s+2)(0.5s+1)} ,$$

and

$$H(s) = \frac{1}{0.005s+1} ,$$

the closed-loop transfer function is

$$T(s) = \frac{K(0.005s+1)}{0.0025s^4 + 0.5125s^3 + 2.52s^2 + 4.01s + 2 + K} .$$

Therefore, the characteristic equation is

$$0.0025s^4 + 0.5125s^3 + 2.52s^2 + 4.01s + (2+K) = 0 .$$

The Routh array is given by

| | | | |
|---|---|---|---|
| $s^4$ | 0.0025 | 2.52 | $2+K$ |
| $s^3$ | 0.5125 | 4.01 | 0 |
| $s^2$ | 2.50 | $2+K$ | |
| $s^1$ | $3.6 - 0.205K$ | 0 | |
| $s^o$ | $2+K$ | | |

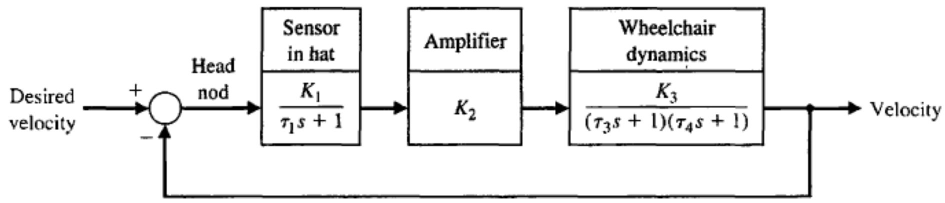
Examining the first column, we determine that for stability we require

$$-2 < K < 17.6 .$$

## Problem 3:

A very interesting and useful velocity control system has been designed for a wheelchair control system. We want to enable people paralyzed from the neck down to drive themselves in motorized wheelchairs. A proposed system utilizing velocity sensors mounted in a headgear in shown below. The headgear sensors provides an output proportional to the magnitude of the head movement. There is a sensor mounted at $90^\circ$ intervals so that forward, left, right, or reverse can be commanded. Typical values for the time constants are $\tau_1 = 0.5$ s, $\tau_3 = 1$ s, and $\tau_4 = \frac{1}{4}$ s.

Determine the limiting gain $K = K_1 K_2 K_3$ for a stable systems.




## Solution:

The closed-loop characteristic equation is

$$1 + \frac{K}{(0.5s+1)(s+1)(\frac{1}{4}s+1)} = 0 ,$$

or

$$s^3 + 7s^2 + 14s + 8(1+K) = 0 .$$

The Routh array is

| | | |
|---|---|---|
| $s^3$ | 1 | 14 |
| $s^2$ | 7 | $8(1+K)$ |
| $s^1$ | $b$ | |
| $s^o$ | $8(1+K)$ | |

where

$$b = \frac{7(14) - 8(1+K)}{7} .$$

For stability, we require $b > 0$ and $8(1+K) > 0$. Therefore, the range of $K$ for stability is

$$-1 < K < 11.25 .$$