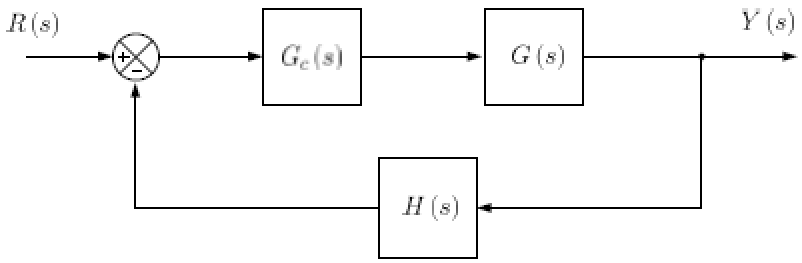
SOLUTION

ECE317

Problem:

Consider a feedback system



where,

$$G(s) = \frac{G_o}{1 + \frac{s}{\omega_0}}$$

and $G_o = 5,\ \omega_0 = 1000\, rds/s\ ,\ H(s) = 1$.

A: Uncompensated system: $\boldsymbol{G_c(s) = 1}$

Determine the loop gain of the uncompensated system $T_{uncomp}(s)$, and for this loop gain using asymptotic approximations only,

i. Sketch the Bode magnitude and phase plots in standard Bode plot form where the phase response is shown below the magnitude response. Label all break frequencies, slopes of sloping lines, gains of sloping lines and phase levels on zero slope lines.
ii. Determine the phase margin and the associated crossover frequency.
iii. Determine whether the closed loop system is stable.
iv. Determine the steady-state error to a unit step.

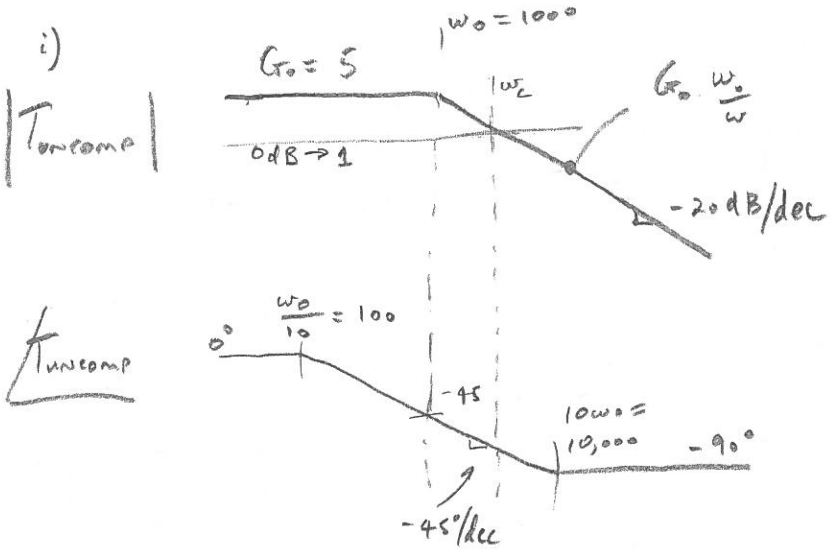
B: Compensated system, Integral Control: $\boldsymbol{G_c(s) = K/s}$

Determine the loop gain of the compensated system $T_{comp}(s)$, and for this loop gain using asymptotic approximations only,

i. Sketch the Bode magnitude and phase plots in standard Bode plot form where the phase response is shown below the magnitude response. Label all break frequencies, slopes of sloping lines, gains of sloping lines and phase levels on zero slope lines.
ii. Determine a value for *K* which achieves a phase margin of 60°.
iii. For your value of *K* what is the unity gain crossover frequency.
iv. Determine the steady-state error to a unit step.

A: UNCOMPENSATED SYSTEM: $T_{uncomp}(s) = \dfrac{G_o}{1+\frac{s}{\omega_o}}$ $\quad G_o = 5$, $\omega_o = 1000$

i)



ii) $G_o \dfrac{\omega_o}{\omega_c} = 1 \Rightarrow \omega_c = G_o \omega_o$

$\Rightarrow \omega_c = 5 \times 1000 = 5{,}000$

phase at ω_c $\quad \phi_c = -\operatorname{atan}\left(\dfrac{\omega_c}{\omega_o}\right)$

$= -\operatorname{atan}\left(\dfrac{5000}{1000}\right)$

$= -\operatorname{atan}(5)$

$= -78.7°$

$\Rightarrow$ phase margin (PM) $= 180 - 78.7 = 101.3°$

ALTERNATE APPROX. METHOD:

$\phi_c = -45 \log_{10}\left(\dfrac{10\omega_c}{\omega_o}\right)$

$= -45 \log_{10}(50)$

$= -76.5 \Rightarrow PM = 180 - 76.5$

$= 103.5°$

iii) $PM > 0 \Rightarrow$ STABLE

iv) SYSTEM TYPE # = 0

$\Rightarrow$ ERROR TO A STEP $= \dfrac{1}{1+K_o}$

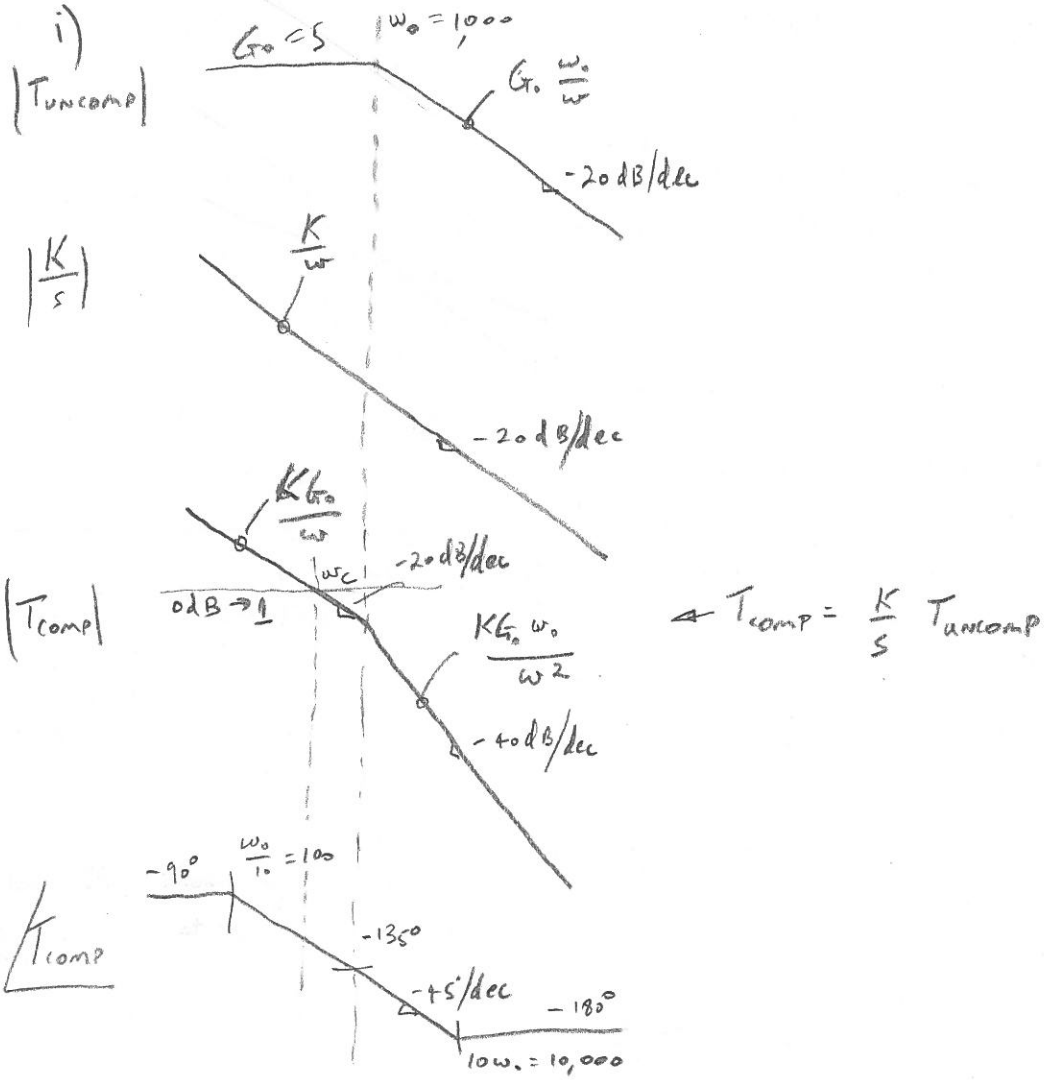
WHERE ERROR COEFFICIENT $K_o = \lim_{s \to 0} \dfrac{G_o}{1+\frac{s}{\omega_o}}$

$= G_o$

$\Rightarrow$ ERROR $= \dfrac{1}{1+G_o} = \dfrac{1}{1+5} = \dfrac{1}{6}$

B: COMPENSATED SYSTEM:

i)



ii) + iii) phase margin 60° $\Rightarrow$ PHASE $= -120°$

PHASE $\phi = -120 = -90 - \mathrm{atan}\left(\frac{\omega_c}{\omega_0}\right)$ @ ω_c

$\Rightarrow \quad \omega_c = 1000 \tan(30°)$

$= 577$ rad/s

$\therefore$ at $\omega_c \quad \frac{K G_0}{\omega_c} = 1$

$\Rightarrow \quad K = \frac{\omega_c}{G_0} = \frac{577}{5} = 115.5$

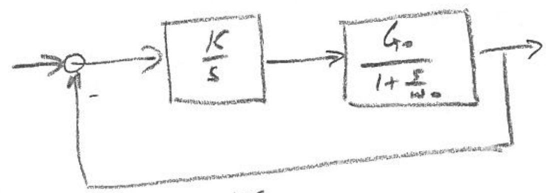
ALTERNATIVE APPROX. METHOD:

PHASE $\phi = -120 = -90 - 45 \log_{10}\left(\frac{10\omega_c}{\omega_0}\right)$ @ ω_c

$\Rightarrow \quad \omega_c = 1000 \times 10^{\frac{30}{45}} = 464$

$\Rightarrow \quad K = \frac{\omega_c}{G_0} = \frac{464}{5} = 92.8$

iv) The number of poles at zero in the loop gain gives what is known as the system type number. With the addition of the integrator the system type number has increased from 0 to 1. A system type number of 1 or more assures that there is zero error to a step input.



$T_{comp} = \frac{K G_0}{s\left(1+\frac{s}{\omega_0}\right)}$ ONE POLE @ $s = 0$.

$\Rightarrow$ SYSTEM TYPE # = 1

$\Rightarrow$ ERROR TO A STEP = 0