Chapter 1

Compensator Design

1.1 Design Procedure

1.1.1 Introduction

In this chapter we will demonstrate a procedure for designing frequency compensators for the standard feedback configuration shown in Figure 1.1. $G(s)$, as before, represents the plant transfer function; $H(s)$ represents the feedback gain and is used to set the closed loop gain and $G_c(s)$ represents the compensator.

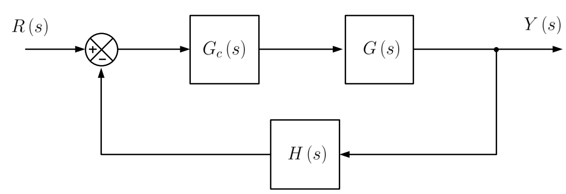

Figure 1.1: Feedback System Block Diagram

To demonstrate the design procedure, in the sequel we will use a plant and feedback gain with the following transfer functions:

$$G(s) = \frac{G_o}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\left(1 + \frac{s}{\omega_3}\right)} \tag{1.1}$$

$$H(s) = k \tag{1.2}$$

where $G_o = 500$, $\omega_1 = 2\pi(10)$, $\omega_2 = 2\pi(100)$, $\omega_3 = 2\pi(300)$, and $k = 0.5$.

The compensators considered in the sequel are the following:

1) Proportional (P) compensator:

$$G_c(s) = k_p \tag{1.3}$$

2) Dominant pole (I, integrator) compensator:

$$G_c(s) = \frac{\omega_I}{s} \tag{1.4}$$

3) Dominant pole with zero (PI, proportional plus integrator) compensator:

$$G_c(s) = \frac{\omega_I}{s}\left(1 + \frac{s}{\omega_z}\right) \tag{1.5}$$

4) Lead compensator:

$$G_c(s) = G_{c_o}\frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}, \qquad \omega_z < \omega_p \tag{1.6}$$

5) Lead with integrator and zero compensator

$$G_c(s) = \frac{\omega_I\left(1 + \frac{s}{\omega_{z_1}}\right)\left(1 + \frac{s}{\omega_{z_2}}\right)}{s\left(1 + \frac{s}{\omega_p}\right)} \tag{1.7}$$

The first three compensators may be considered to be members of the three term controller, PID (proportional, integral, derivative), family of compensators. We will see that as the complexity of the compensator increases the performance also improves. The performance measures used are the rise time, settling time and percentage overshoot of the step response. For the simpler compensators, i.e. proportional and dominant pole compensators, only one design parameter is needed to be found. For the most involved compensator, the lead with integrator and zero compensator, there are a total of four design parameters to be determined.

1.1.2 Uncompensated System

We start our evaluation with the uncompensated loop gain $T(s) = kG_c(s)\,G(s)$, where $G_c(s) = 1$. The loop gain is given as

$$T(s) = \frac{T_o}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\left(1 + \frac{s}{\omega_3}\right)} \tag{1.8}$$

where

$$T_o = G_o k = 500 \cdot 0.5 = 250$$

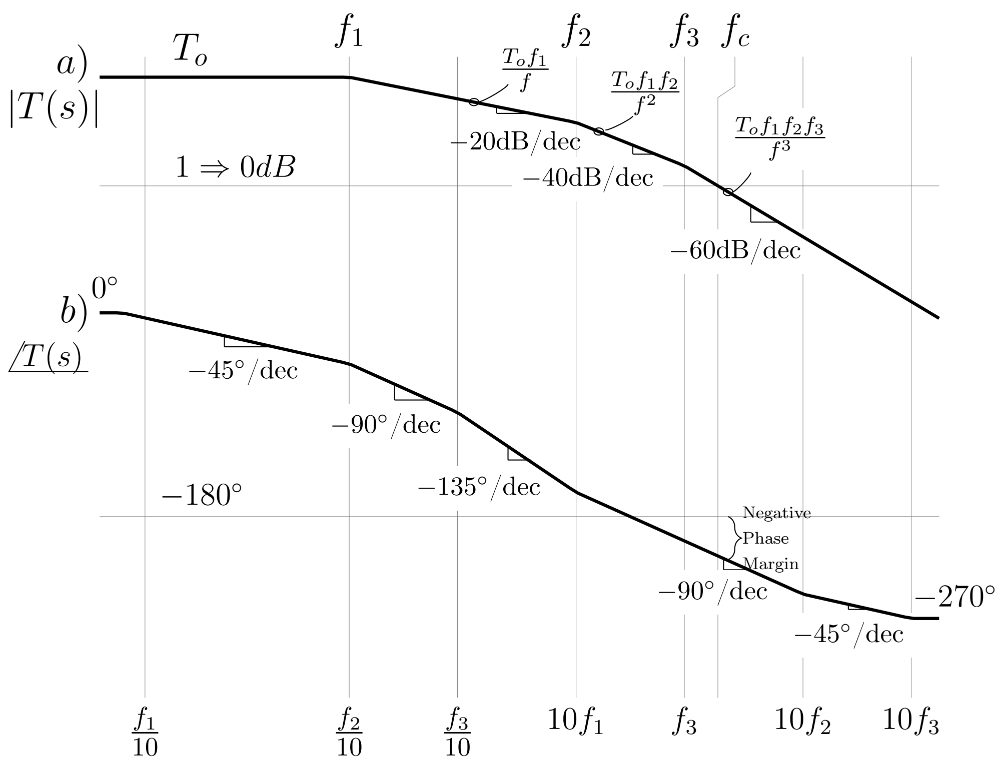

Figure 1.2: Bode Plot: Uncompensated System

$$\omega_1 = 2\pi\,(10)\,, \omega_2 = 2\pi\,(100)\,, \omega_3 = 2\pi\,(300)$$

We construct the Bode plot of the loop gain, shown in Figure 1.2. Using this constructed plot we can easily determine simplified (approximate) expressions for the f_c, the unity gain crossover frequency, and PM, the phase margin:

$$\frac{T_o f_1 f_2 f_3}{f_c^3} = 1 \implies f_c = \sqrt[3]{T_o f_1 f_2 f_3} \tag{1.9}$$

$$PM = 180 - \arctan\left(\frac{f_c}{f_1}\right) - \arctan\left(\frac{f_c}{f_2}\right) - \arctan\left(\frac{f_c}{f_3}\right) \tag{1.10}$$

Equations (1.9) and (1.10) result in $f_c = 422$ Hz and $PM = -40^\circ$.

In a similar fashion we can also determine f_{GM}, the frequency at which the phase reaches -180°, and subsequently the gain margin:

$$-180 = -\arctan\left(\frac{f_{GM}}{f_1}\right) - \arctan\left(\frac{f_{GM}}{f_2}\right) - \arctan\left(\frac{f_{GM}}{f_3}\right) \tag{1.11}$$

$$GM = -20\log\left(\frac{T_o f_1 f_2}{f_{GM}^2}\right) \tag{1.12}$$

Evaluating 1.11 and 1.12 results in $f_{GM} = 184$ Hz and $GM = -17.3$ dB, respectively.

Figure 1.3 is a Matlab Bode plot of the uncompensated loop gain produced using the 'margin' command. Matlab uses the unapproximated transfer function models and so is able to accurately determine the margins and associated frequencies: $f_c = 385$ Hz, $PM = -36.1°$, $f_{GM} = 184$ Hz and $GM = -14.8$ dB. A side by side comparison of these results with those from the asymptotic line analysis is shown in Table 1.1.

Table 1.1: Uncompensated System margins

	PM (°)	f_C **(Hz)**	**GM (dB)**	f_{GM}**(Hz)**
Asymptotes	-40	422	-17.3	184
Matlab	-36.1	385	-14.8	184

The phase margin test indicates that the uncompensated closed loop system is unstable. A Matlab time domain simulation of the step response of the uncompensated system is shown in Figure 1.4. The output quickly becomes unbounded for the unit step input indicative of an unstable system. So compensation is needed to make the system stable and further to improve the performance.

1.1.3 Proportional Compensated System

In our first compensator design we will assess the efficacy of using a proportional compensator:

$$G_c(s) = k_p \tag{1.13}$$

k_p simply represents a constant gain. Note that the effect of varying the value of k_p is to raise and lower the magnitude Bode plot while keeping the phase plot unaffected. So the value of k_p can be set to obtain a unity gain crossover frequency (f_c) which results in an acceptable phase margin.

Generally the design procedure would require that asymptotic Bode plots for the now compensated loop gain be constructed, however, in the case of a proportional compensator, since the shape of the plots (magnitude and phase) are unchanged we need simply to replace any occurrences of the To with k_pTo in Figure 1.2 and proceed accordingly.

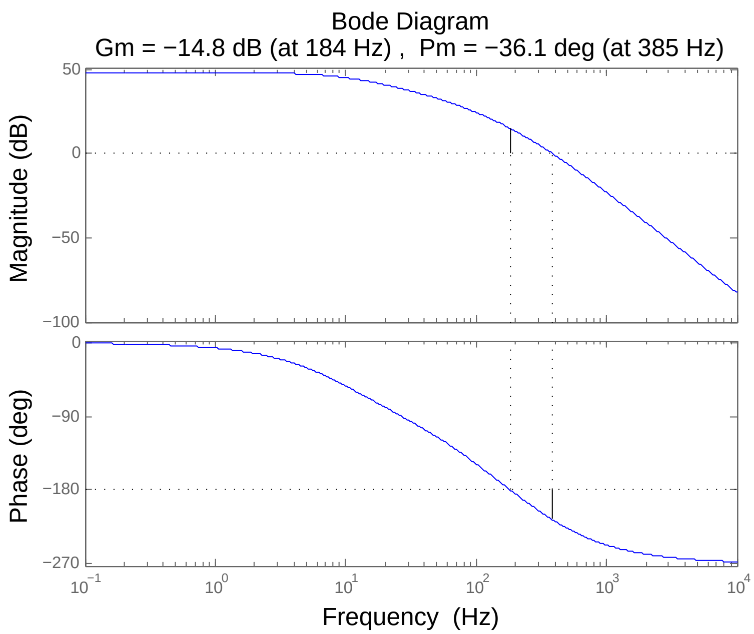

Figure 1.3: Matlab Analysis of Uncompensated System

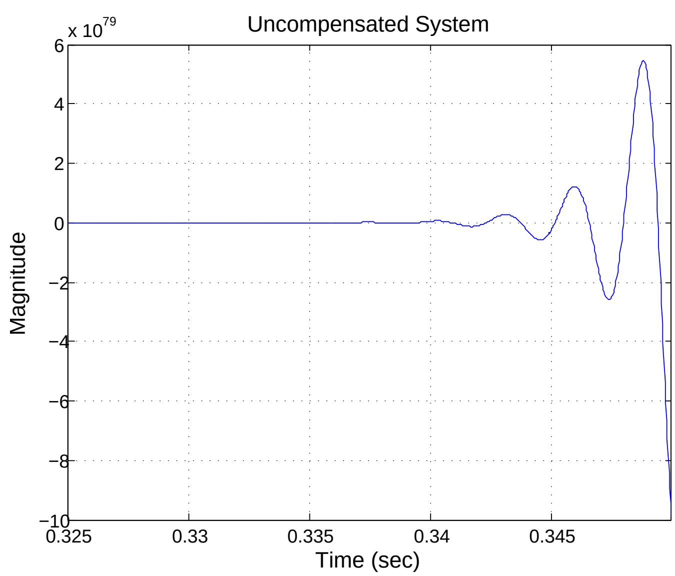

Figure 1.4: Matlab Analysis of Uncompensated System

As a general rule of thumb, to obtain an acceptable phase margin (generally $45° \leq PM \leq 60°$) one usually sets the unity gain crossover frequency (f_c) to occur in the segment of the asymptotic magnitude plot that has a slope of -20dB/dec. From the constructed magnitude plot we find

$$\frac{k_p T_o f_1}{f_c} = 1 \implies f_c = k_p T_o f_1 \tag{1.14}$$

We can substitute the above equation for f_c into the phase margin equation shown next to determine the required value of k_p.

$$\begin{aligned} PM &= 180 - \arctan\left(\frac{f_c}{f_1}\right) - \arctan\left(\frac{f_c}{f_2}\right) - \arctan\left(\frac{f_c}{f_3}\right) \\ &= 180 - \arctan\left(k_p T_o\right) - \arctan\left(\frac{k_p T_o f_1}{f_2}\right) - \arctan\left(\frac{k_p T_o f_1}{f_3}\right) \end{aligned} \tag{1.15}$$

With a desired value of phase margin of $PM = 45°$ equation (1.15) evaluates to $k_p = 0.0311$ and $f_c = 77.65$.

Using the obtained value of k_p an evaluation of the compensated loop gain with the unapproximated transfer functions was performed by Matlab and is shown in Figure 1.5 where we see that the obtained phase margin is 55°. This value, due to the approximate nature of our design equations, turns out to more than required, but nonetheless acceptable. Had it not been so, one could simply iterate.

A step response simulation of the proportional compensated closed-loop system is shown in Figure 1.6. A summary of all the performance results (as furnished by the Matlab *stepinfo* function) are given in the table 1.2. There we see in particular that the overshoot, rise-time, settling-time and steady-state error values are 20%, 2.9 ms, 15.4 ms and −11%, respectively.

Table 1.2: Proportionally Compensated System Performance Features

Proportional Compensation	
Characteristics	Value
Overshoot	20 %
Rise time	2.9 ms
Settling time	15.4 ms
Steady-state error	−11 %
Bandwidth	63.4 Hz
Phase margin	54.7°
Gain margin	15.4 dB

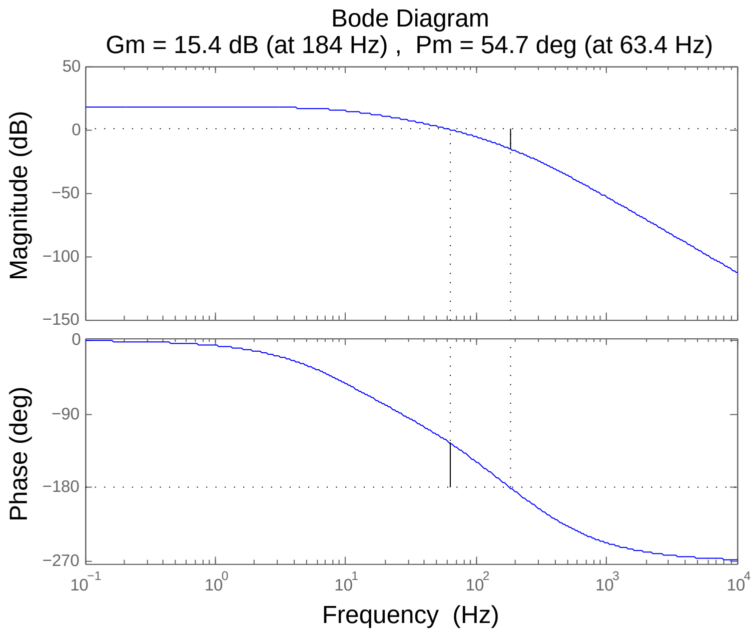

Figure 1.5: Matlab Proportional Compensated Loop Gain Bode Plot

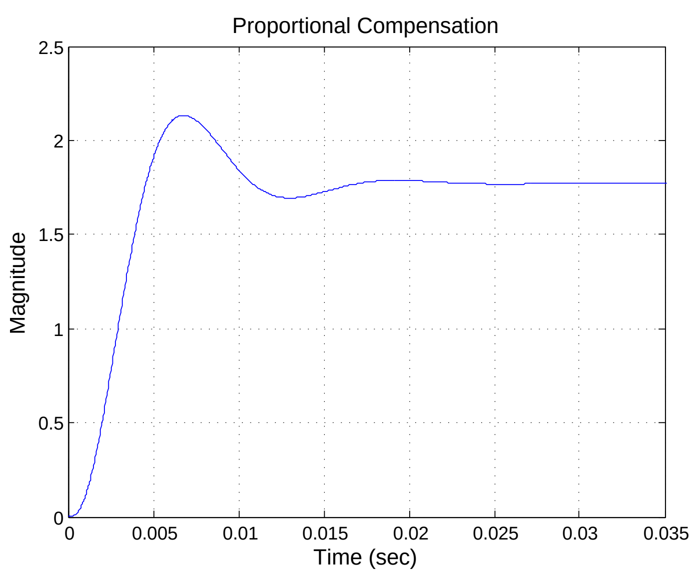

Figure 1.6: Step Response of Proportional Compensated Closed-Loop System

1.1.4 Dominant Pole Compensated System

The next form of frequency compensation to be discussed is the dominant pole compensation. As the name suggests a pole is inserted in the loop gain which dominates the dynamics of the loop. That is to say that the frequency response of the loop gain up to the unity gain crossover frequency is mainly determined by the dominant pole. To achieve this the pole needs to be placed at a frequency much lower (usually a decade or so) than the lowest pole or zero of the uncompensated loop gain. This requirement unfortunately reduces the loop bandwidth and subsequently the speed of response. However, compensator design is simplified and good stability margins may be easily obtained. If the pole is placed at zero frequency then this represents an integrator which will result in obtaining a zero steady state error characteristic.

In the design that follows, an integrator is employed so that the compensator transfer function is given by:

$$G_c(s) = \frac{\omega_I}{s}$$

where $\omega_I = 2\pi \cdot f_I$ is an appropriately chosen design constant. It represents the frequency at which the compensator gain is at unity. Figure 1.7 shows the Bode plot asymptotes for the magnitude and phase of this compensator.

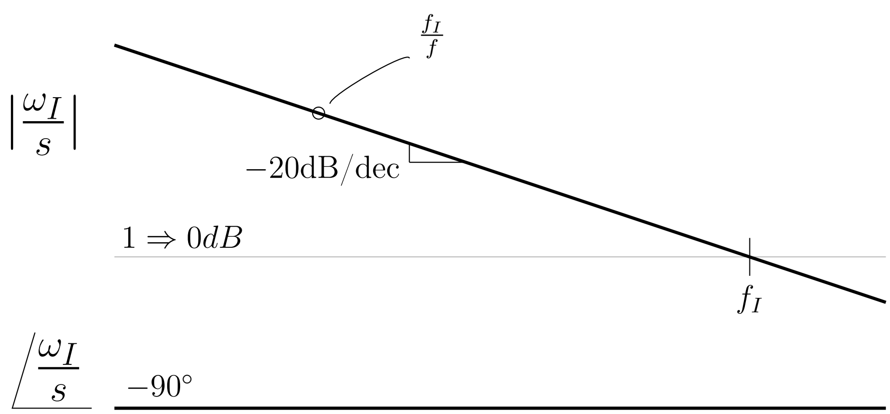

Figure 1.7: Bode Plot: Dominant Pole Compensator

Design of the compensator now consists of selecting an appropriate compensator parameter, f_I.

Figure 1.8 shows the graphical construction of the phase asymptotes for the loop gain with the compensator. Note that because the plant's transfer function is third order (with no zeros), it features a phase shift of $-270°$ at high frequencies. Also, since the next higher frequency $f_2 \geq 10 \cdot f_1$ the phase shift at f_1 is $-45°$. Furthermore, the compensator contributes its own $-90°$ phase shift and does so for all frequencies. Consequently, the total phase shift of the compensated open loop transfer function at f_1 is $-135°$. Consequently, in this design the frequency f_1 is chosen to be the unity gain crossover frequency of the overall system, so that we get a $+45°$ phase margin.

Figure 1.9 shows how the plant and compensator transfer functions combine to produce the magnitude response of the compensated loop. To achieve a phase margin that is $+45°$, we require the magnitude at f_1 to equal 1 (0dB), therefore $f_c = f_1$.

$$\frac{f_I T_o}{f_1} = 1$$

$$f_I = \frac{f_1}{T_o} = \frac{10}{250} = 0.04$$

The dominant pole compensator in this case is:

$$G_c(s) = \frac{\omega_I}{s} = \frac{2\pi \cdot 0.04}{s}$$

Figure 1.10 shows resulting gain and phase asymptotic construction of the Bode plot. We next run the full, unapproximated, compensated loop transfer function through the Matlab 'margin' command to verify the design results. Figure 1.11 shows the results obtained. In particular, a phase margin of $PM = 45.9°$ with a unity gain crossover frequency $f_c = 7.84$ Hz was obtained, which compares favorably with the design values of $PM = 45°$ and $f_c = f_1 = 10$ Hz.

Figure 1.12 shows the unit step response of the dominant pole compensated closed loop system. It is clearly seen that zero steady state response has been attained but not without going through significant overshoot first. The features of the closed loop system are summarized in the Table 1.3.

Table 1.3: Dominant Pole Compensated System Features

Dominant Pole Compensation	
Characteristics	Value
Peak amplitude	22.2% overshoot
Rise time	24.5 ms
Settling time	136.5 ms
Steady-state error	0 %
Bandwidth	7.84 Hz
Phase margin	45.9°
Gain margin	18.2 dB

1.1.5 Dominant Pole Compensated System with zero

The dominant pole compensator of the previous section, while stable and featuring zero steady state error performance, exhibits several undesirable characteristics which include a large overshoot and long settling time. In our effort to achieve stability, we have sacrificed the bandwidth of the system. Unfortunately, the lower the bandwidth, the slower the response. Thus, we need to increase the bandwidth to speed up the response. This can be done by adding a zero to the compensator that will cancel the lowest frequency pole of the plant. By doing this, the canceled pole no longer affects the phase lag seen at the gain crossover frequency. For our system, the 10 Hz pole is canceled so that the remaining poles at 100 and 300 Hz are now the poles which limit the bandwidth. Accordingly we will be able to extend the bandwidth by a decade.

The transfer function for the new compensator is given by

$$G_c\left(s\right) = \frac{\omega_I}{s}\left(1 + \frac{s}{\omega_z}\right)$$

where

$$\omega_z = \omega_1$$

The compensated loop gain $T(s) = kG_c(s)G(s)$ with this compensator now becomes:

$$T(s) = \frac{T_o\omega_I}{s\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}$$

The construction of the compensated magnitude plot is shown in Figure 1.13 where the uncompensated plant and compensator magnitude asymptotes are combined. The final magnitude and phase plots using the asymptotic construction procedure is shown in Figure 1.14. Using the low frequency magnitude asymptote we see that at the unity gain crossover frequency f_c

$$\frac{T_o f_I}{f_c} = 1$$

$$\Longrightarrow \; f_c = T_o f_I \tag{1.16}$$

The general expression for ϕ_f, the phase response at an arbitrary frequency f is given by:

$$\phi_f = -90 - \arctan\left(\frac{f}{f_2}\right) - \arctan\left(\frac{f}{f_3}\right), \quad \forall f \tag{1.17}$$

Consequently the phase margin is given by:

$$
\begin{aligned}
PM &= 180 + \phi_{f_c} \\
&= 90 - \arctan\left(\frac{f_c}{f_2}\right) - \arctan\left(\frac{f_c}{f_3}\right) \\
&= 90 - \arctan\left(\frac{T_o f_I}{f_2}\right) - \arctan\left(\frac{T_o f_I}{f_3}\right)
\end{aligned} \tag{1.18}
$$

With a design value of $PM = 45°$ the required value of f_I is solved using Equation (1.18) to yield $f_I = 0.258$ so that $\omega_I = 2\pi f_I = 1.623$. This results, using (1.16), in a gain crossover frequency, $f_c = 64.58$ Hz.

To verify the design procedure the unapproximated loop gain transfer function is run through the Matlab 'margin' command which produces the plot shown in Figure 1.15. The obtained phase margin and bandwidth are seen to be $PM = 51°$ and $f_c = 56$ Hz. The bandwidth has been greatly improved.

The unit step response of the closed loop system is shown in Figure 1.16. A summary of the performance using this compensator is shown in Table 1.4.

Table 1.4: Dominant Pole with Zero Compensated System Features

Dominant Pole with Zero Compensation	
Characteristics	Value
Overshoot	17.48%
Rise time	3.4 ms
Settling time	17.5 ms
Steady-state error	0 %
Bandwidth	55.5 Hz
Phase margin	50.5 degree
Gain margin	15.8 dB

1.1.6 Dominant Pole Compensated System with zero, improved phase margin

The addition of the zero to dominant pole compensation has allowed us to increase the speed of response by extending bandwidth. This is manifested in the time domain response by a substantial decrease in the rise and settling times. However, the overshoot, while slightly improved, may be seen as overly large. Nevertheless, it too may be reduced by appreciating the fact that overshoot and phase margin are inversely related. So we will increase the phase margin to reduce the overshoot.

Using the same compensator as in the previous section, we will now redesign it to achieve a phase margin of 60°. The zero frequency is left unchanged, so that $f_z = f_1$, but a new value of f_I will be determined to achieve the desired 60° phase margin. Solving equation (1.18), now with $PM = 60$ we see the required value of $f_I = 0.1636$ so that $\omega_I = 2\pi f_I = 1.0276$.

To verify the design the Bode plot of the loop gain was produced using the 'margin' command in Matlab. This plot is shown in figure 1.17. We see that a phase margin of 62° was achieved with a bandwidth of 37.9 Hz. Of course the extension in phase margin is necessarily accompanied by a reduction in bandwidth.

The step response of the closed loop system is shown in Figure 1.18. The resulting performance characteristics are tabulated in the Table 1.5. There we see that the overshoot has been reduced to 6.1%. Recall that a phase margin of 45° had previously resulted in a 17.48% overshoot. This reduction in overshoot has been attained at the expense of an increase in rise time. However, the settling time has been slightly reduced.

Table 1.5: Dominant Pole with Zero (Improved PM) Compensated System Features

Dominant Pole with Zero Compensation (Improved Margin)	
Characteristics	Value
Overshoot	6.10 %
Rise time	5.3 ms
Settling time	15.9 ms
Steady-state error	0
Bandwidth	37.9 Hz
Phase margin	62 degree
Gain margin	19.8 dB

1.1.7 Lead Compensated System

Next we consider the case of using a lead compensator, the transfer function of which is given by:

$$G_c(s) = G_{c_o} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}, \qquad \omega_z < \omega_p \tag{1.19}$$

The design of this compensator requires the appropriate determination of the three variables, G_{c_o}, f_z and f_p, the low frequency gain, zero frequency and pole frequency, respectively. Note in particular that the zero frequency is required to be at a lower value than the pole frequency. This constraint exists so that the phase response, first starting at zero at low frequencies, becomes positive, i.e. leads, as the frequency increases before returning to zero at high frequencies. In essence, the lead compensator provides a phase boost that is adjustable based on the pole and zero frequencies. The maximum phase boost ϕ_{max} possible is $\phi_{max} = 90°$ and occurs at a frequency $f_{\phi_{max}}$ which is the geometric mean of the zero and pole frequencies of the compensator. The geometric mean of two numbers represents the midpoint between these numbers when represented on a logarithmic scale.

$$f_{\phi_{max}} = \sqrt{f_z f_p} \tag{1.20}$$

The compensator also provides a gain boost at higher frequencies so that with proper design it can both extend loop bandwidth while increasing the phase margin. Proper design of the compensator requires that the frequency of maximum boost afforded by this compensator is set to the unity gain crossover frequency, f_c. The asymptotic Bode plot of the lead compensator is shown in Figure 1.19.

When the compensator is placed in the loop, the loop gain of the system now becomes:

$$T(s) = \frac{T_o G_{co}\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_p}\right)\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}$$

The construction of the asymptotic magnitude response from the constituent parts is shown in Figure 1.20. Both the resulting magnitude and phase asymptotic responses of the loop gain are shown in Figure 1.21. As for previous compensators, the annotations on these plots are used in the design procedure. The exact expression for the phase ϕ is given by:

$$\begin{aligned}\phi_f = \arctan\left(\frac{f}{f_z}\right) - \arctan\left(\frac{f}{f_p}\right) - \arctan\left(\frac{f}{f_1}\right) \\ - \arctan\left(\frac{f}{f_2}\right) - \arctan\left(\frac{f}{f_3}\right), \quad \forall f\end{aligned} \tag{1.21}$$

Consequently the phase margin is given by:

$$\begin{aligned}PM &= 180 + \phi_{f_c} \\ &= 180 + \arctan\left(\frac{f_c}{f_z}\right) - \arctan\left(\frac{f_c}{f_p}\right) - \arctan\left(\frac{f_c}{f_1}\right) \\ &\quad - \arctan\left(\frac{f_c}{f_2}\right) - \arctan\left(\frac{f_c}{f_3}\right)\end{aligned} \tag{1.22}$$

As mentioned previously, we will set the unity gain crossover frequency f_c to the maximum phase boost frequency $f_{\phi_{max}}$ so that using equation (1.20) we have:

$$\begin{aligned} f_c &= f_{\phi_{max}} \\ &= \sqrt{f_z f_p} \end{aligned} \tag{1.23}$$

In order to minimize the effect on the phase margin of the phase lag due to the compensator pole we will set this pole frequency an order of magnitude above the crossover frequency:

$$f_p = 10 f_c \tag{1.24}$$

Equation (1.24) together with equation (1.23) results in a relationship between the zero and pole frequencies of the compensator:

$$f_p = 100 f_z \tag{1.25}$$

We can use the zero frequency of the compensator to cancel the pole frequency f_2 of the plant, so that $f_z = f_2$. This together with equations (1.25) and (1.22) may be used to determine the unity gain crossover frequency, f_c, for a given desired phase margin. For a margin of 60° we find $f_c = 187$ Hz.

From the magnitude asymptote we see that

$$\frac{T_o G_{c_o} f_1}{f_c} = 1 \tag{1.26}$$

so that for a given f_c we can solve for the required compensator low frequency gain G_{c_o}:

$$G_{c_o} = \frac{f_c}{T_o f_1} \tag{1.27}$$

For our design we obtain a value of $G_{c_o} = 0.0749$.

To verify our design we produce the plot using the Matlab 'margin' command on the unapproximated transfer functions of the compensated loop. This is shown in Figure 1.22. There we find that the obtained phase margin is $PM = 63.9°$ with $f_c = 164$ Hz, which validates our design procedure.

The unit step response of the closed loop system with this compensation is shown in Figure 1.23. The features of the step response are presented in the Table 1.6. We see, due to the extended bandwidth, that the speed of response, represented by the rise and settling times is quite good. Percentage overshoot is also relatively low due to the 60° phase margin employed. However, as there is no longer an integrator in the forward path, the steady state error is now non-zero. This will be remedied in the next and final compensator design.

Table 1.6: Lead Compensation

Lead Compensation	
Feature	Value
Overshoot	8.06 %
Rise time	1.3 ms
Settling time	3.9 ms
Steady-state error	-5.07%
Bandwidth	164 Hz
Phase margin	63.9°
Gain margin	35.1 dB

1.1.8 Lead Compensated System with integrator and zero

In this final compensator design, we will augment the lead compensator of the previous section with an integrator and zero. The integrator is added to provide the closed loop system with a zero steady state error performance. This is contrasted with the approach previously discussed where the integrator was primarily used to as the dominant pole. In the present case by canceling lower frequency poles, we're able to extend the unity gain bandwidth of the loop gain to obtain quick closed loop response. The zero is added to leave the high frequency magnitude and phase response unchanged from the lead compensator case.

The transfer function of the compensator considered here is given by:

$$G_c\left(s\right) = \frac{\omega_I\left(1 + \frac{s}{\omega_{z_1}}\right)\left(1 + \frac{s}{\omega_{z_2}}\right)}{s\left(1 + \frac{s}{\omega_p}\right)} \tag{1.28}$$

The parameters ω_{z_2} and ω_p correspond to ω_z and ω_p of the lead compensator design, which leaves ω_I and ω_{z_1} to be determined. ω_{z_1} can be simply set to ω_1. The low frequency gain of the lead compensator of the previous section was denoted G_{c_o}. This was the value of the loop magnitude at f_1 (in particular, and below this frequency, in general). To maintain this value of gain at f_1 we will adjust ω_I to achieve this. The low frequency magnitude asymptote is given by $\frac{f_I}{f}$ so that at f_1 we have

$$\frac{f_I}{f_1} = G_{c_o} \implies f_I = G_{c_o} f_1 \tag{1.29}$$

This completes the design of this compensator. To verify our design we produce the plot using the Matlab 'margin' command on the unapproximated transfer functions of the compensated loop. This is show in figure 1.24. There we find that the obtained phase margin is $PM = 60°$ with $f_c = 164$ Hz, which very closely agrees with the results obtained for the lead compensator.

Next, we exam the step response of the closed loop system which is shown in Figure 1.25. It is clear that now we have zero steady state error. Further performance features are given in the Table 1.7.

Table 1.7: Lead Compensation with integrator and zero

Lead Compensation with integrator and zero	
Feature	Value
Overshoot	8.31 %
Rise time	1.3 ms
Settling time	4.0 ms
Steady-state error	0%
Bandwidth	164 Hz
Phase margin	60.4°
Gain margin	34.8 dB

1.1.9 Summary

The following table shows the summary of all of the results.

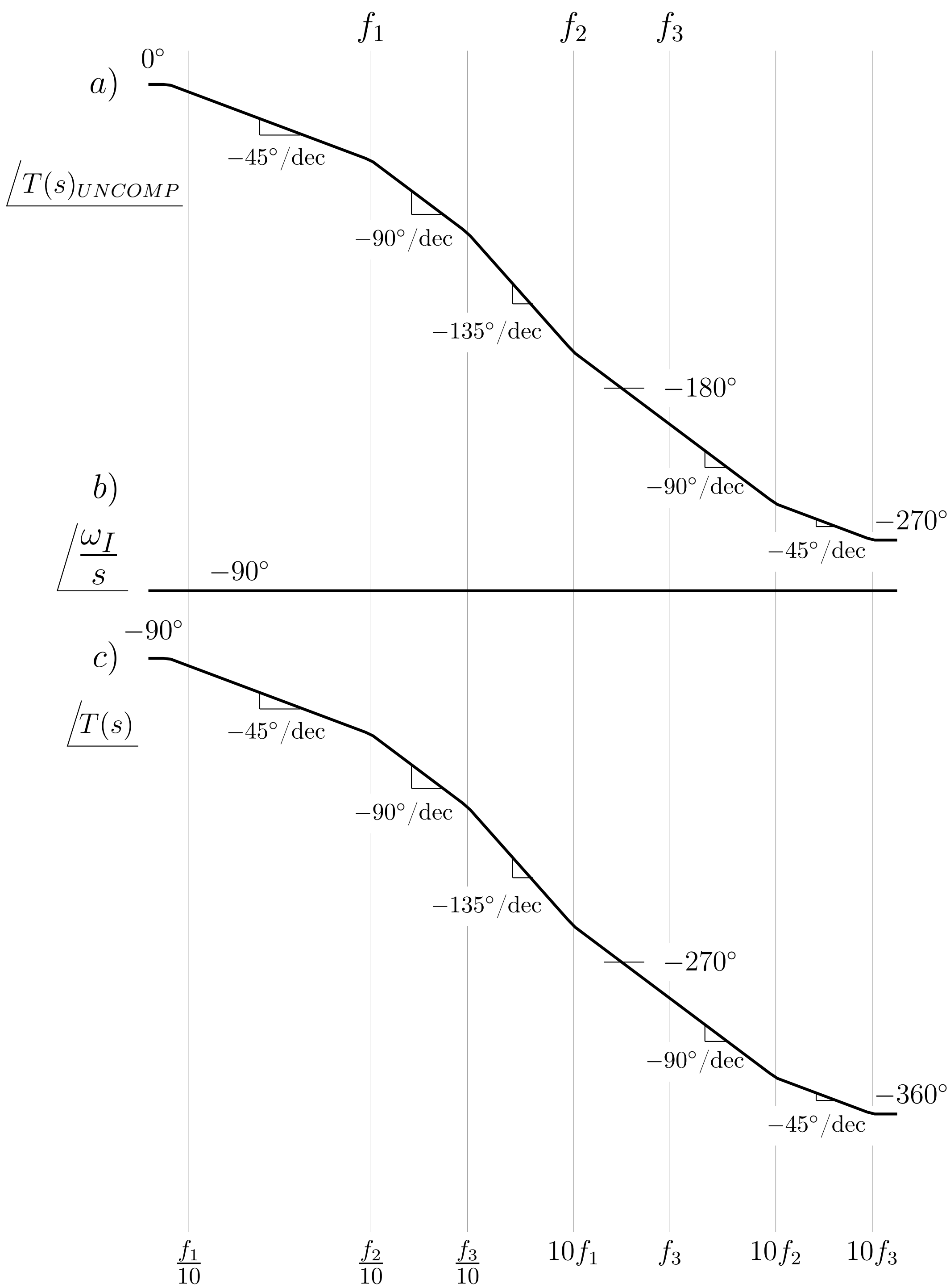

Figure 1.8: Dominant Pole: Phase Construction

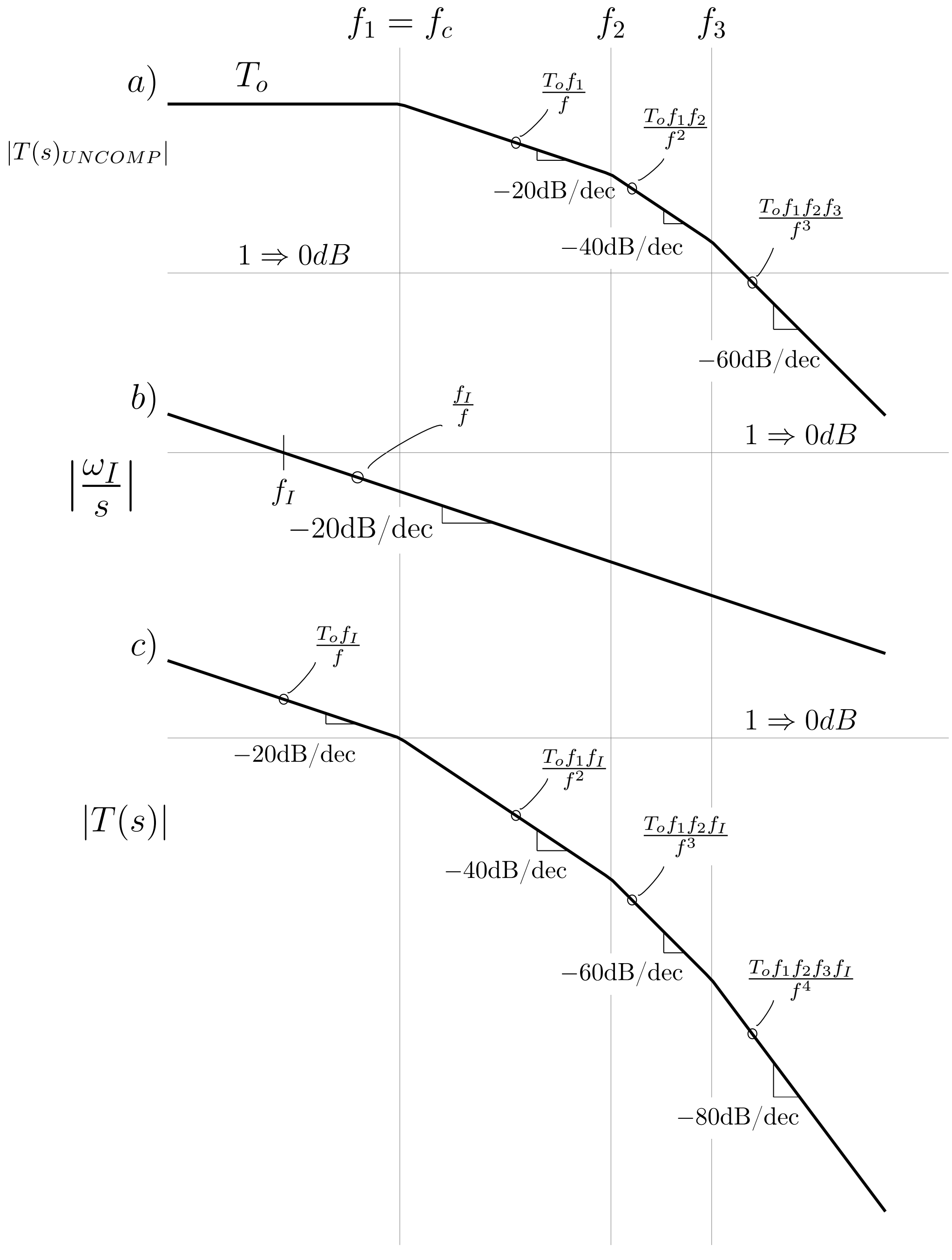

Figure 1.9: Dominant Pole: Magnitude Construction

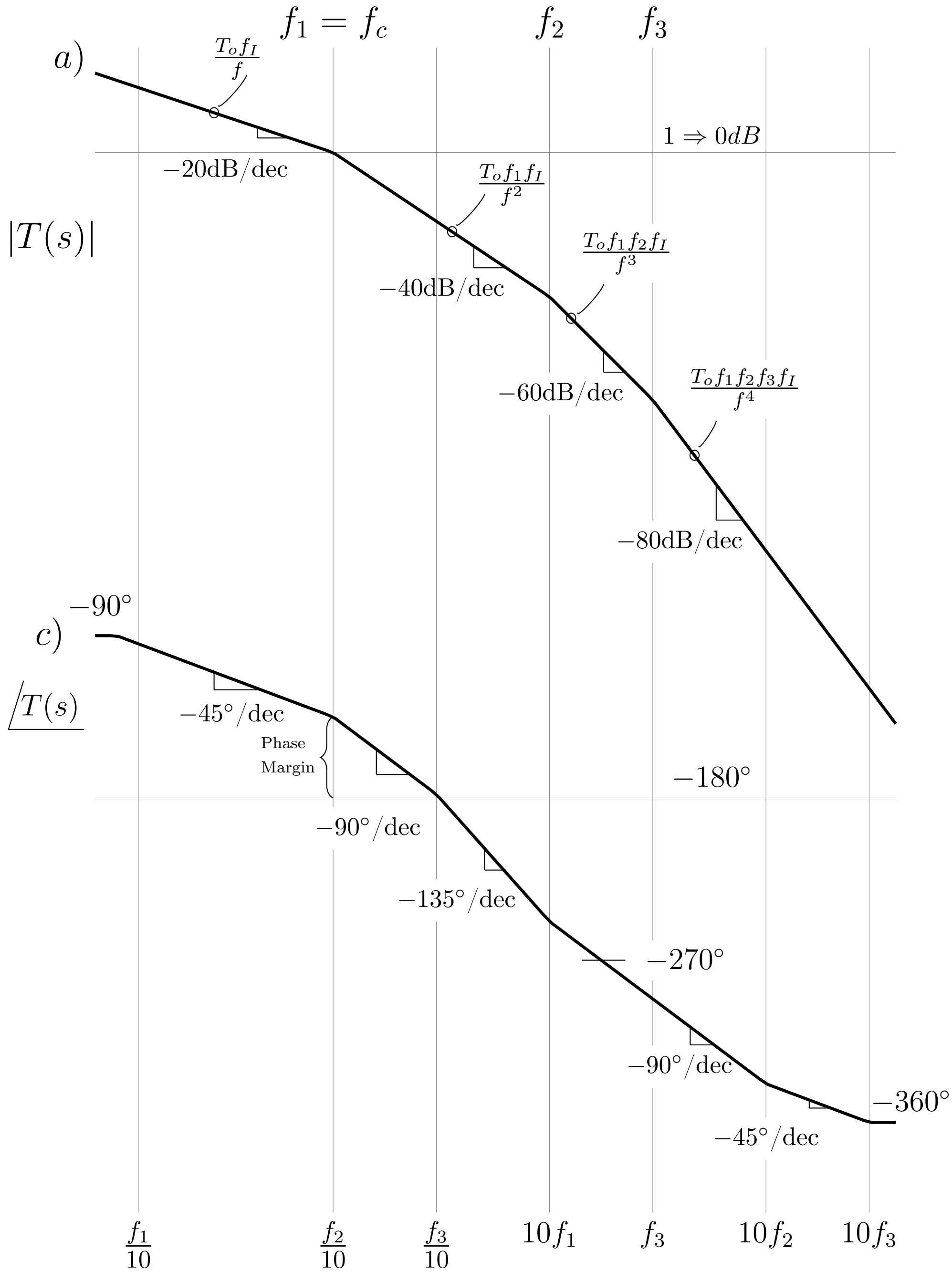

Figure 1.10: Dominant Pole Compensated System

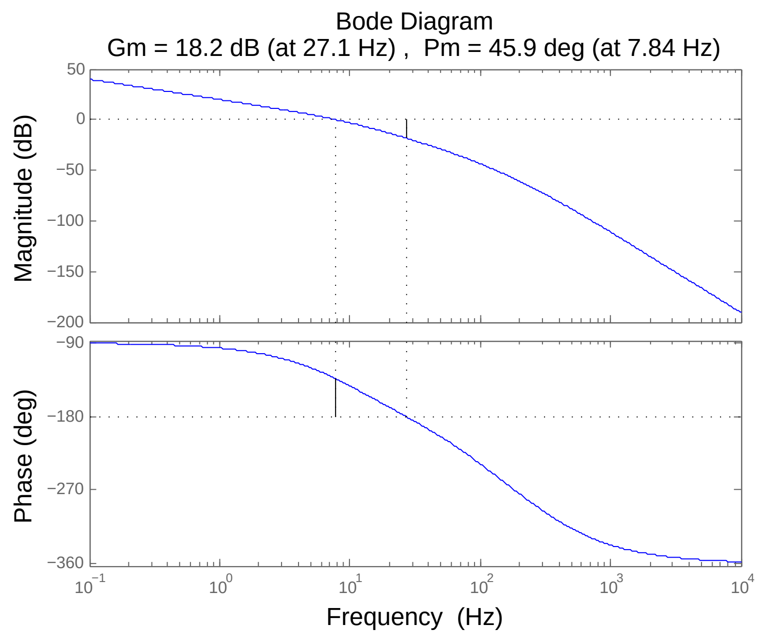

Figure 1.11: Matlab Analysis of Dominant Pole Compensated System

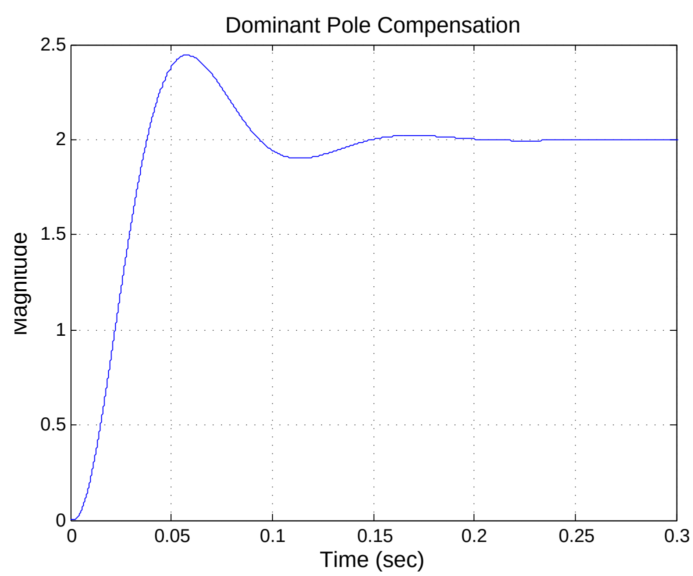

Figure 1.12: Step Response of the Dominant Pole Compensated System

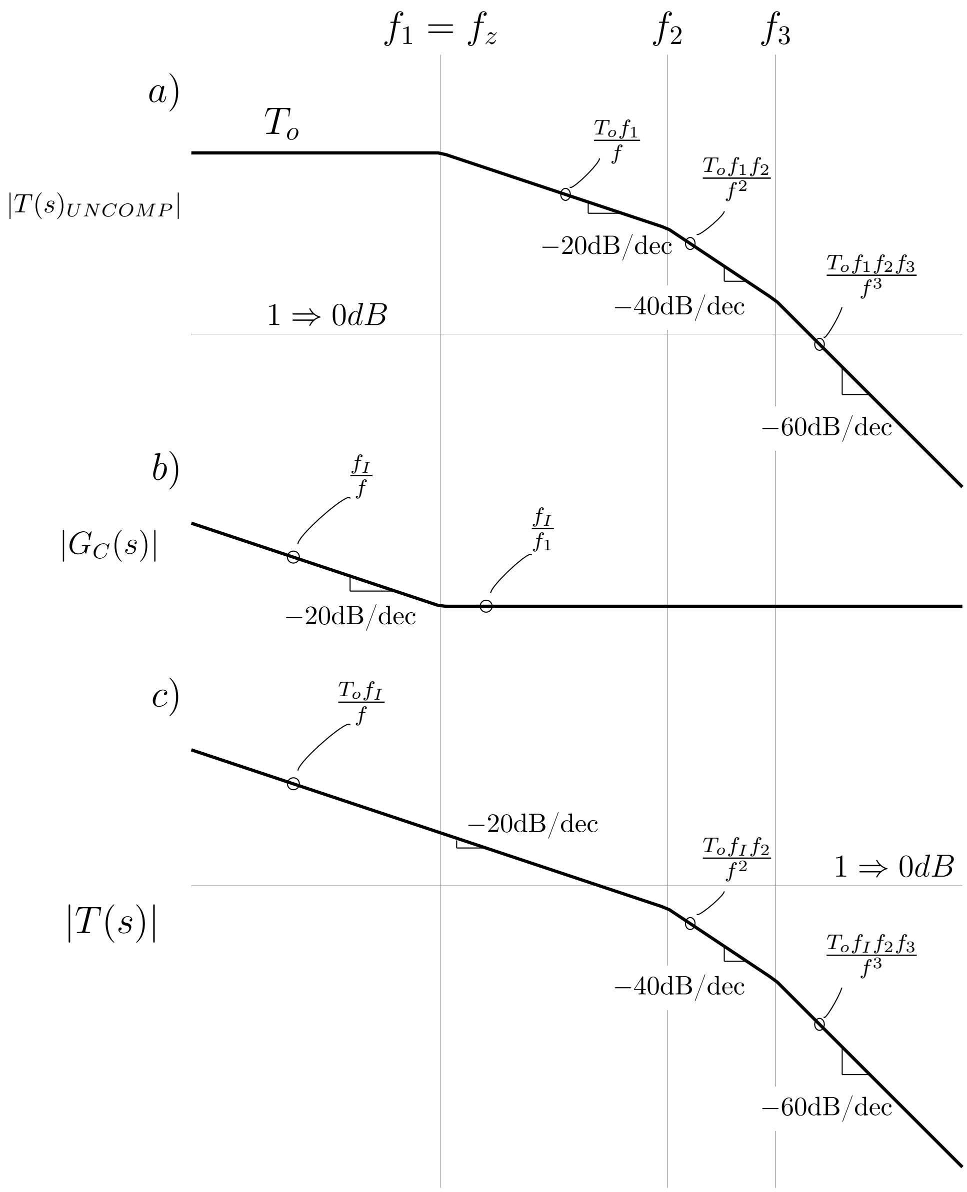

Figure 1.13: Dominant Pole with Zero Magnitude Construction

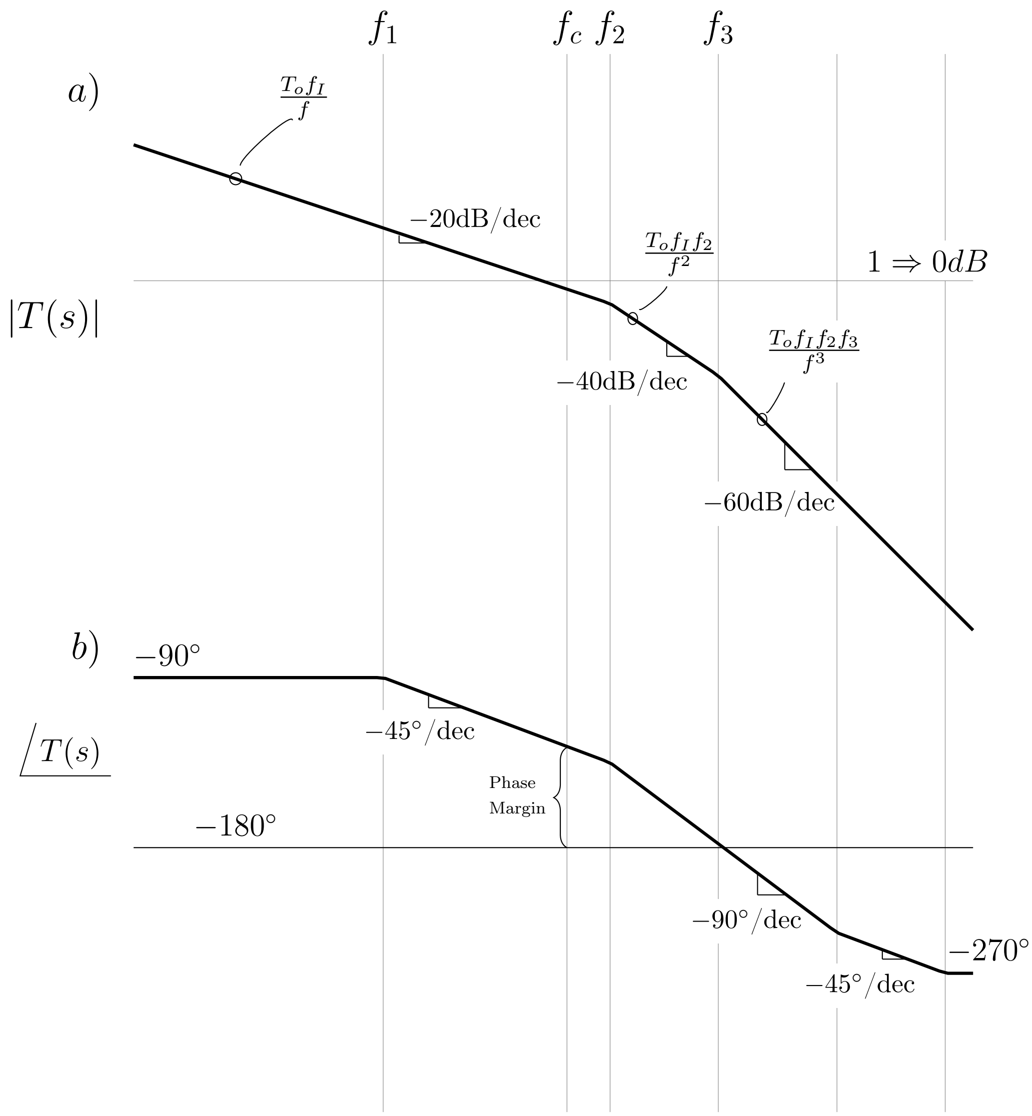

Figure 1.14: Dominant Pole with Zero Compensated System

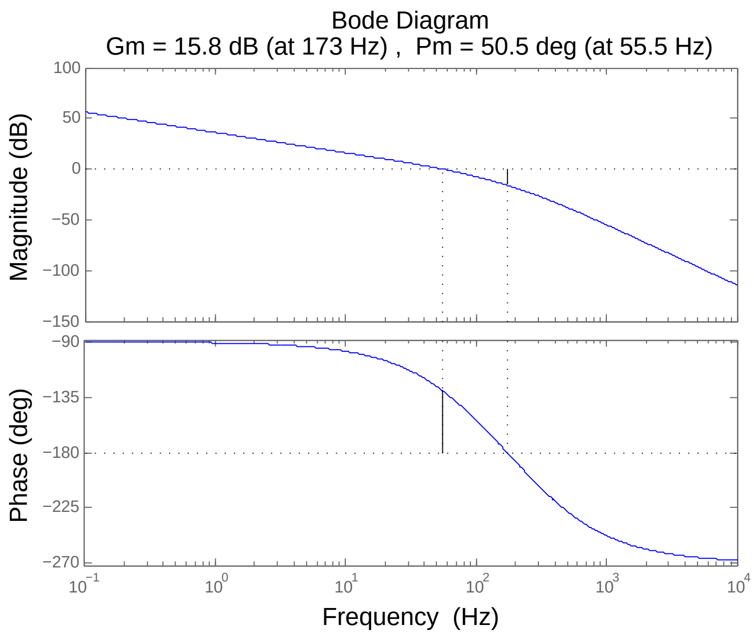

Figure 1.15: Loop Gain and Phase Response of the Dominant Pole Compensated System with Zero

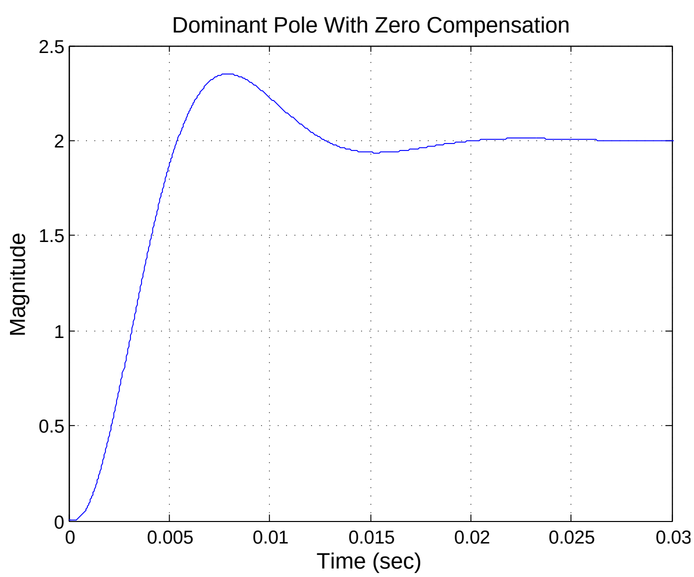

Figure 1.16: Step Response of the Dominant Pole with Zero Compensated System

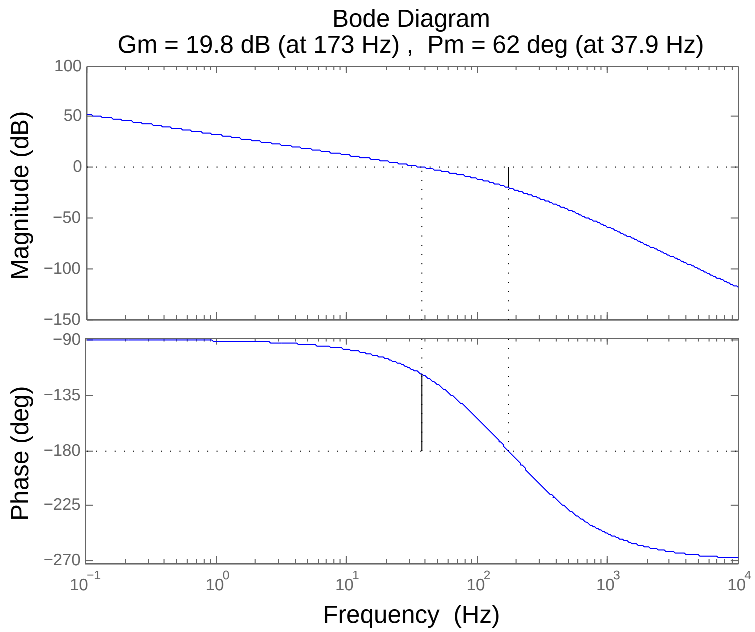

Figure 1.17: Matlab Analysis of Dominant Pole Compensated System with Zero (Improved Margin)

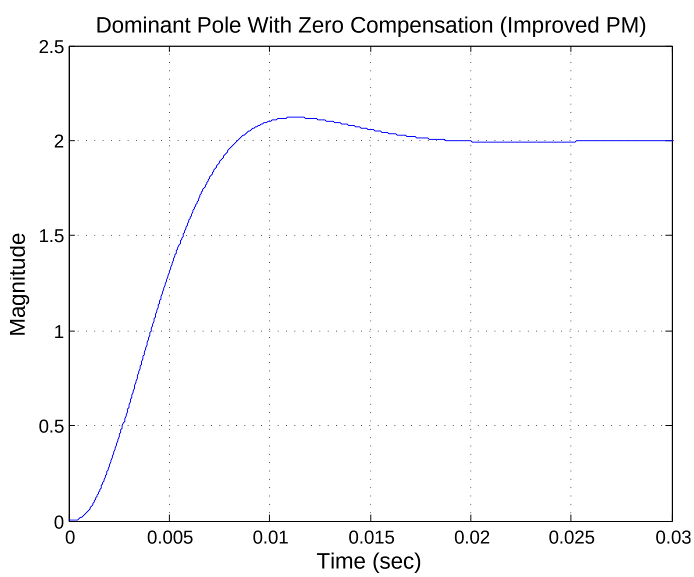

Figure 1.18: Step Response of the Dominant Pole with Zero Compensated System (Improved Margin)

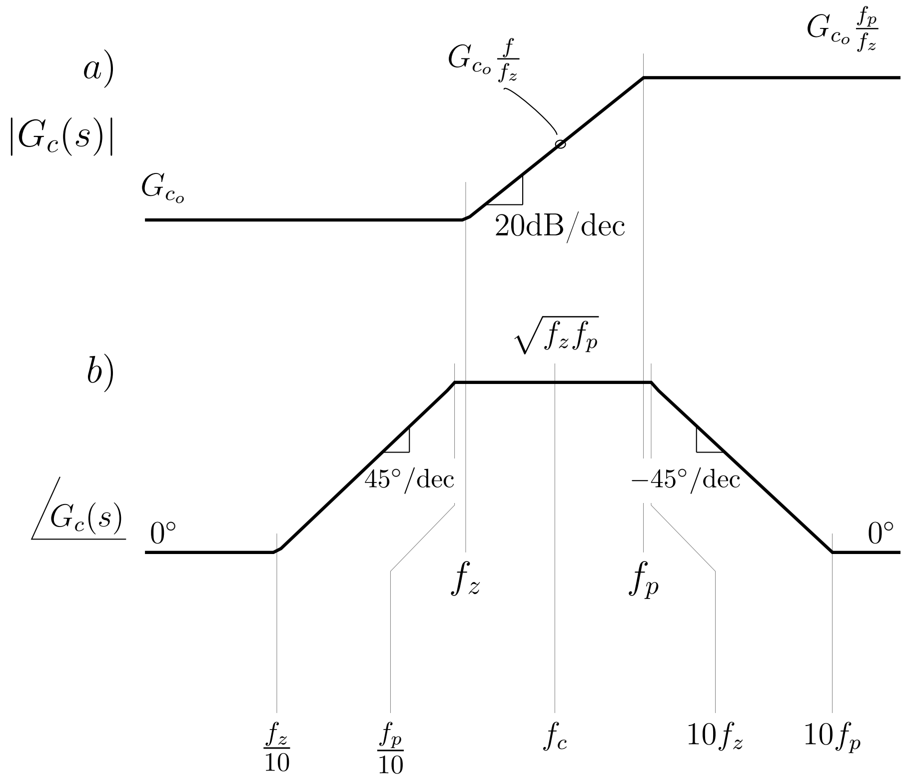

Figure 1.19: Bode Diagram: Lead Compensator

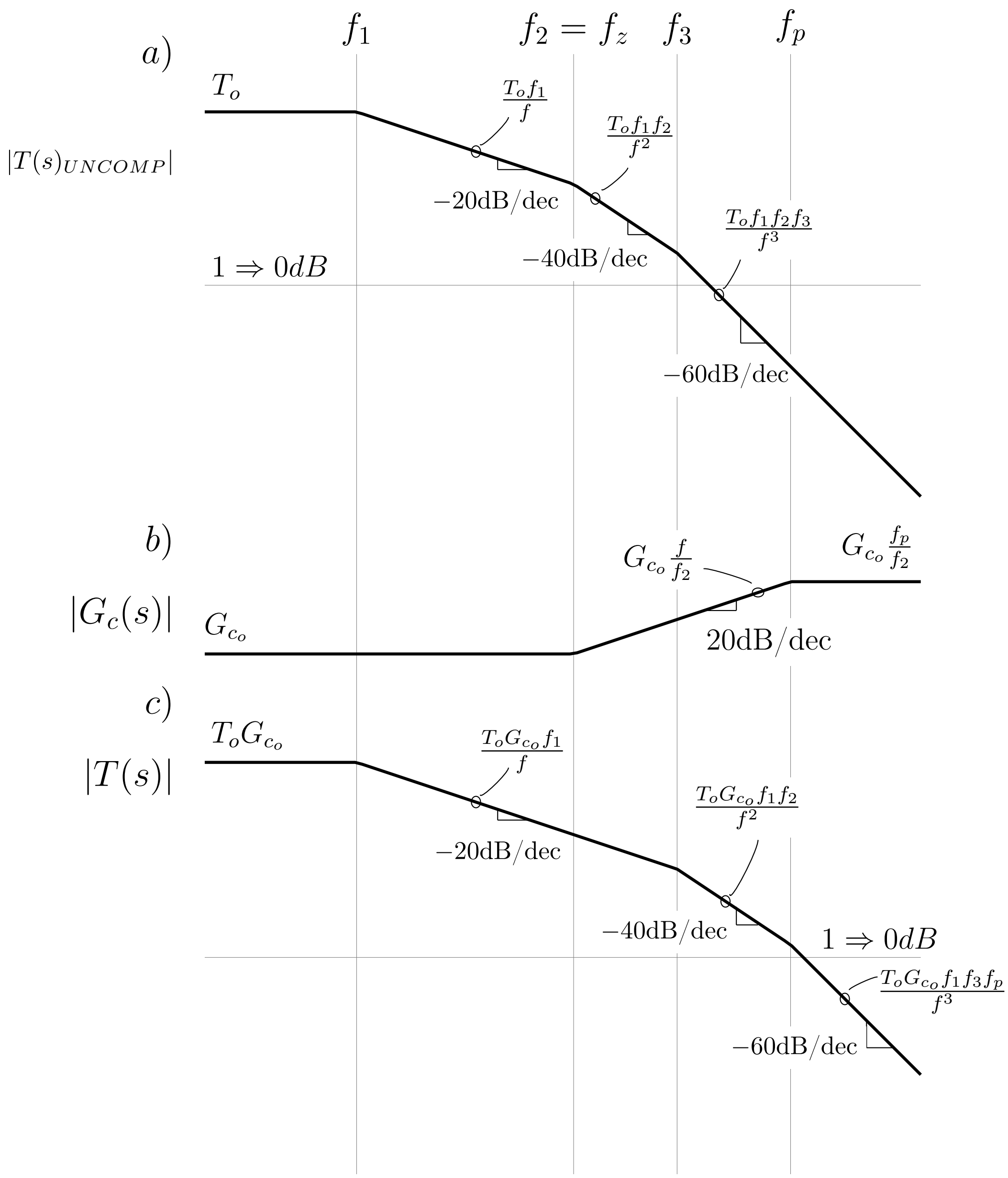

Figure 1.20: Lead Compensation Magnitude Construction

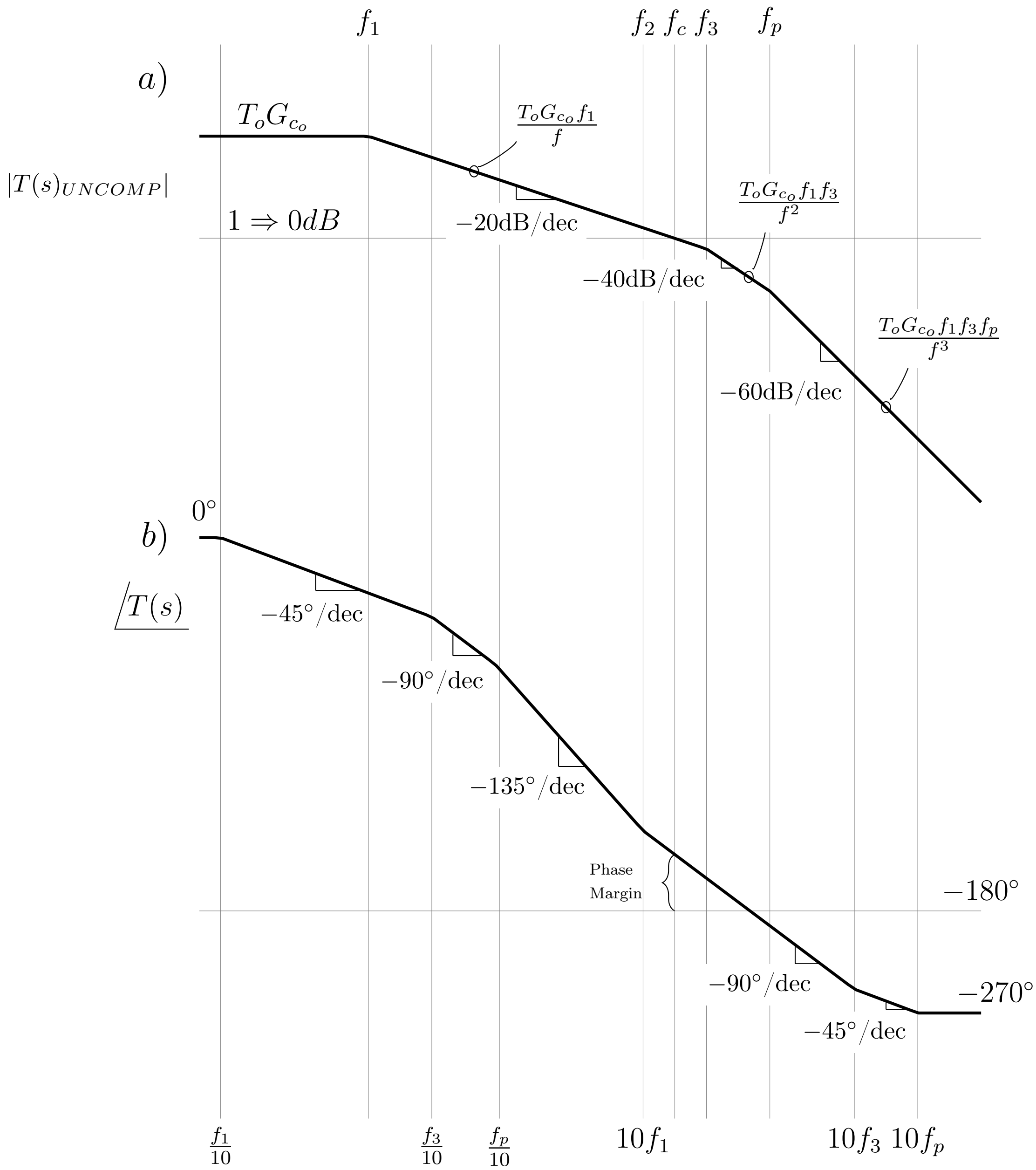

Figure 1.21: Lead Compensated System

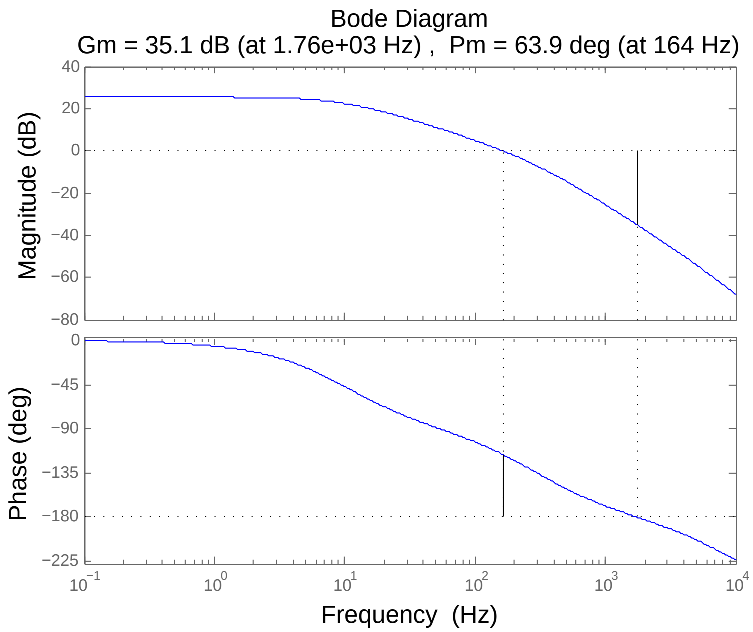

Figure 1.22: Matlab Analysis of Lead Compensated System

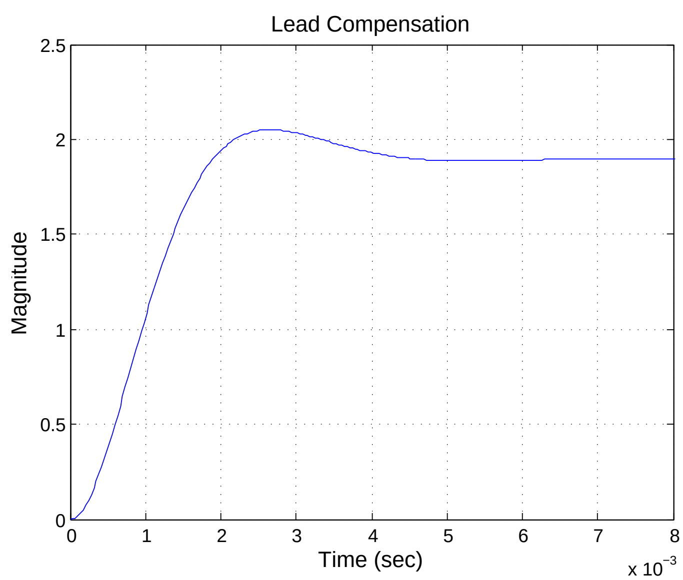

Figure 1.23: Step Response of the Lead Compensated System

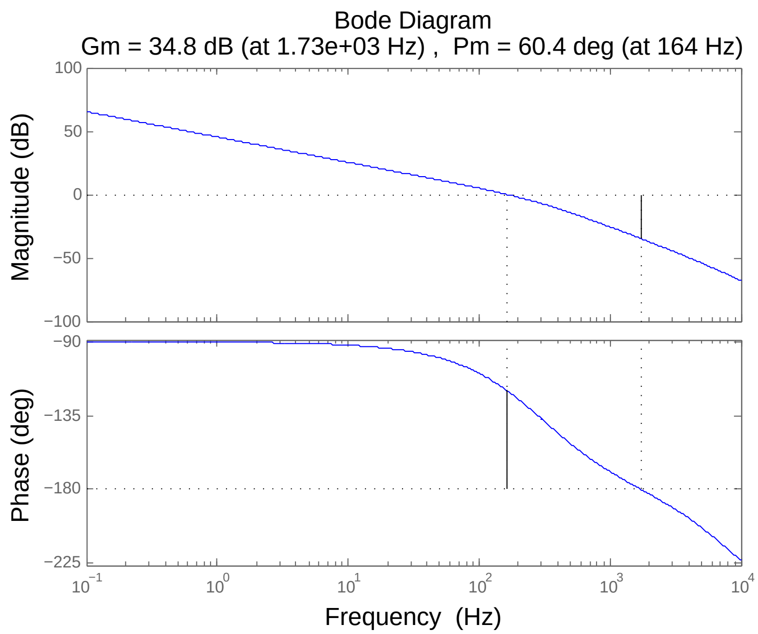

Figure 1.24: Matlab Analysis of Lead Compensated System with integrator and zero

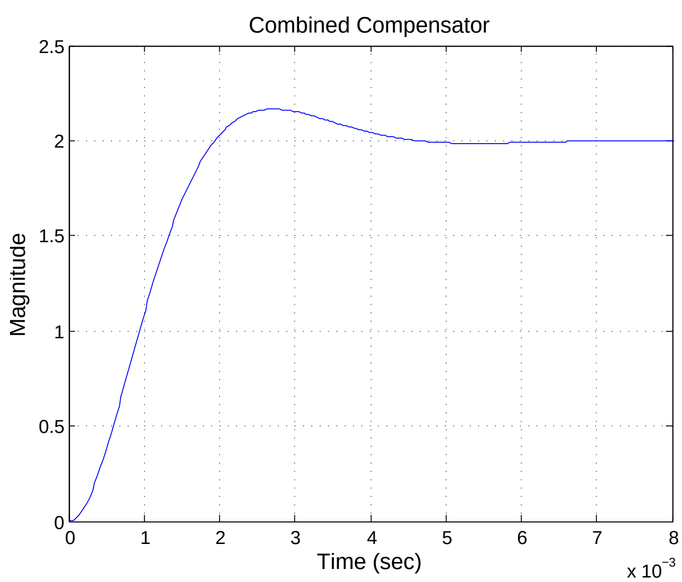

Figure 1.25: Step Response of the Lead Compensated System with integrator and zero

Table 1.8: Summary of Compensators

$$G(s) = \frac{G_o}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}$$

$$H(s) = k$$

where $G_o = 500$, $\omega_1 = 2\pi(10)$, $\omega_2 = 2\pi(100)$, $\omega_3 = 2\pi(300)$, and $k = 0.5$.

$G_c(s)$	$G_c(s)$ parameters	f_c (Hz)	ϕ_{PM} (deg)	GM (dB)	OS (%)	t_r (ms)	t_s (ms)	ERR (%)
1 (uncompensated)	none	385	-36	-15	na	na	na	∞
k_p	$k_p = 0.03$	63	55	15	20	2.9	15.4	-11
$\frac{\omega_I}{s}$	$\omega_I = 0.25$	7.8	46	18	22	25	137	0
$\frac{\omega_I}{s}\left(1+\frac{s}{\omega_z}\right)$	$\omega_I = 1.62$ $\omega_z = 2\pi(10)$	56	51	16	17	3.4	18	0
$\frac{\omega_I}{s}\left(1+\frac{s}{\omega_z}\right)$	$\omega_I = 1.03$ $\omega_z = 2\pi(10)$	38	62	20	6	5.3	16	0
$G_{c_o}\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$	$G_{c_o} = 0.075$ $\omega_z = 2\pi(100)$ $\omega_p = 2\pi(10,000)$	164	64	35	8.1	1.3	3.9	-5
$\frac{\omega_I\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{s\left(1+\frac{s}{\omega_p}\right)}$	$\omega_I = 4.71$ $\omega_{z_1} = 2\pi(10)$ $\omega_{z_2} = 2\pi(100)$ $\omega_p = 2\pi(10,000)$	164	60	35	8.3	1.3	4	0

1.1.10 MATLAB Code

```
function compensators

clear all;
close all;

t = linspace(0, 0.35, 10000);
f = logspace(-1, 4, 1000);
w = 2*pi*f;
s = tf('s');

Go = 500;
f1 = 10;
f2 = 100;
f3 = 300;

w1 = 2*pi*f1;
w2 = 2*pi*f2;
w3 = 2*pi*f3;
k = 0.5;
To = Go*k;
yf = 2;

%====================================================================
%====================================================================
% UNCOMPENSATED
%====================================================================

G = Go/((1+s/w1)*(1+s/w2)*(1+s/w3));

titl = 'Uncompensated System';
Gc = 1;

xlmt = [0.325 0.35];
disp(titl)
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0031
%     SettlingTime: 0.3500
%      SettlingMin: -9.9007e+79
%      SettlingMax: -8.9587e+79
%        Overshoot: 0
%       Undershoot: 55.0478
%             Peak: 9.9007e+79
%         PeakTime: 0.3500

%====================================================================
%====================================================================
% Proportional Compensation
%====================================================================

fn = @(kp) 135 - atand(To*f1*kp/f1) - atand(To*f1*kp/f2) ...
               - atand(To*f1*kp/f3);

kp = fzero(fn, 0)    % kp = 0.0311
```

```
fc = To*f1*kp        % fc = 77.6459

titl = 'Proportional Compensation';
Gc = kp;

xlmt = [0 0.035];
disp(titl)
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0029
%     SettlingTime: 0.0154
%      SettlingMin: 1.6011
%      SettlingMax: 2.1341
%        Overshoot: 20.4502
%       Undershoot: 0
%             Peak: 2.1341
%         PeakTime: 0.0068

%====================================================================
%====================================================================
% Dominant Pole Compensation
%====================================================================

titl = 'Dominant Pole Compensation';
wI = w1/To      %  0.2513
Gc = wI/s;

xlmt = [0 0.3];
disp(titl)
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0245
%     SettlingTime: 0.1365
%      SettlingMin: 1.8000
%      SettlingMax: 2.4440
%        Overshoot: 22.2103
%       Undershoot: 0
%             Peak: 2.4440
%         PeakTime: 0.0580

%====================================================================
%====================================================================
% Dominant Pole With Zero Compensation
%====================================================================

titl = 'Dominant Pole With Zero Compensation';

fn = @(fI) 45 - atand(To*fI/f2) - atand(To*fI/f3);
fI = fzero(fn, 0)

fc = To*fI      %   64.58
wI = 2*pi*fI    %   1.623
Gc = wI/s*(1+s/w1);


xlmt = [0 0.030];
disp(titl)
```

```
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0034
%     SettlingTime: 0.0175
%      SettlingMin: 1.8012
%      SettlingMax: 2.3496
%        Overshoot: 17.4798
%       Undershoot: 0
%             Peak: 2.3496
%         PeakTime: 0.0079

%==================================================================
%==================================================================
% Dominant Pole With Zero Compensation (Improved Phase Margin)
%==================================================================

titl = 'Dominant Pole With Zero Compensation (Improved PM)';

fn = @(fI) 30 - atand(To*fI/f2) - atand(To*fI/f3);
fI = fzero(fn, 0)

fc = To*fI              %  40.88
wI = 2*pi*fI            %  1.0276
Gc = wI/s*(1+s/w1);

xlmt = [0 0.03];
disp(titl)
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0053
%     SettlingTime: 0.0159
%      SettlingMin: 1.8014
%      SettlingMax: 2.1219
%        Overshoot: 6.0951
%       Undershoot: 0
%             Peak: 2.1219
%         PeakTime: 0.0113

%==================================================================
%==================================================================
% Lead Compensation
%==================================================================

fz = 100      %  = f2
fp = 100*fz
fn = @(fc) 120 + atand(fc/fz) - atand(fc/f1) ...
               - atand(fc/f2) - atand(fc/f3) - atand(fc/fp);
fc = fzero(fn, 0)       % 187
Gco = fz*fc/(To*f1*f2)  % 0.0749
wz = 2*pi*fz;
wp = 2*pi*fp;

titl = 'Lead Compensation';
Gc = Gco*(1+s/wz)/(1+s/wp);

xlmt = [0 0.008];
disp(titl)
```

```
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0013
%     SettlingTime: 0.0039
%      SettlingMin: 1.7192
%      SettlingMax: 2.0517
%        Overshoot: 8.0633
%       Undershoot: 0
%             Peak: 2.0517
%         PeakTime: 0.0027

%======================================================================
%======================================================================
% Combined Compensation
%======================================================================

titl = 'Combined Compensator';
% fc, unity gain xover frequency: same value as for lead compensator
wI = 2*pi*fc/To   % 4.7
wz1 = w1;
wz2 = w2;
% wp:  use the same value as for lead compensator
Gc = wI*(1+s/wz1)*(1+s/wz2)/(s*(1+s/wp));

xlmt = [0 0.008];
disp(titl)
analysis(Gc, G, k, w, titl, t, xlmt, yf)

%         RiseTime: 0.0013
%     SettlingTime: 0.0040
%      SettlingMin: 1.8121
%      SettlingMax: 2.1662
%        Overshoot: 8.3110
%       Undershoot: 0
%             Peak: 2.1662
%         PeakTime: 0.0027
end


function s = analysis(Gc, G, k, w, titl, t, xlmt, yf)

% loop gain
L = Gc*G*k;

figure
[mag, phase] = bode(L,w);
margin(mag, phase, w);

h = gcr;
h.AxesGrid.Xunits = 'Hz';
h.AxesGrid.TitleStyle.FontSize = 12;
h.AxesGrid.XLabelStyle.FontSize = 12;
h.AxesGrid.YLabelStyle.FontSize = 12;

%======================================

% closed loop gain
```



```
Gs = 1/k * L/(1+L);

% Plot of Step response results
figure
yout = step(Gs, t);
plot(t, yout);
grid on;
title(titl,'FontSize',12);
xlabel('Time (sec)','FontSize',12);
ylabel('Magnitude','FontSize',12);
xlim(xlmt);

% Time Domain Analysis Parameters
s = stepinfo(yout,t);
ss_error = (yout(end)-yf)/yf * 100
end
```