Table 4.1 *Basic Root Locus Principles*

1. The branches of the locus are continuous curves that start at each of the n poles of GH, for $K = 0$. As K approaches $+\infty$, the locus branches approach the m zeros of GH. Locus branches for excess poles extend infinitely far from the origin; for excess zeros, locus segments extend from infinity.
2. The locus includes all points along the real axis to the left of an odd number of poles plus zeros of GH.
3. As K approaches $+\infty$, the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180° + i360°}{n - m}$$

for $i = 0, \pm1, \pm2, \ldots,$ until all $n - m$ or $m - n$ angles are obtained, where n is the number of poles and m is the number of zeros of GH.

4. The starting point of the asymptotes, the centroid of the pole–zero plot, is on the real axis at

$$\sigma = \frac{\sum \text{pole values of } GH - \sum \text{zero values of } GH}{n - m}$$

5. Loci leave the real axis at a gain K that is the maximum K in that region of the real axis. Loci enter the real axis at the minimum value of K in that region of the real axis. These points are termed *breakway points* and *entry points*, respectively. A pair of locus segments leave or enter the real axis at angles of $\pm 90°$.
6. The angle of departure ϕ of a locus branch from a complex pole is given by

$$\phi = -\sum \text{other } GH \text{ pole angles } + \sum GH \text{zero angles} + 180°$$

The angle of approach ϕ' of a locus branch to a complex zero is given by

$$\phi' = \sum GH \text{ pole angles} - \sum \text{other } GH \text{ zero angles} - 180°$$

where each GH pole angle and GH zero angle is calculated to the complex pole for ϕ and to the complex zero for ϕ'.

If the complex pole or zero is of order m, the m angles of departure and approach are given by

$$\phi = \frac{-\sum \text{other} GH \text{pole angles} + \sum GH \text{ zero angles} + (1 + 2i)180°}{m}$$

$$\phi' = \frac{\sum GH \text{ pole angles} - \sum \text{other } GH \text{ zero angles} - (1 + 2i)180°}{m}$$

for $i = 0, 1, 2, \ldots, (m - 1)$.