Change the following block diagram to a signal flow graph and subsequently determine the transfer function $C/R$.

The two forward path gains are $P_1 = G_1 G_2 G_3$ and $P_2 = G_1 G_4$. The five feedback loop gains are $P_{11} = G_1 G_2 H_1$, $P_{21} = G_1 G_3 H_2$, $P_{31} = -G_1 G_2 G_4$, $P_{41} = G_4 H_2$, and $P_{51} = -G_1 G_4$. Hence

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{31}) = 1 + G_1 G_2 G_3 - G_1 G_2 H_1 - G_2 G_3 H_2 - G_4 H_2 + G_1 G_4$$

and $\Delta_1 = \Delta_2 = 1$. Finally,

$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 G_3 - G_1 G_2 H_1 - G_2 G_3 H_2 - G_4 H_2 + G_1 G_4}$$
Problem 2:
Determine the time-domain response of a system described by:

\[ T(s) = \frac{-2}{s^2 + 4s + 13} \]

\[ r(t) = \delta(t) \]

\[ y(0^-) = 1 \]

\[ y'(0^-) = 0 \]

\[ Y(s) = \frac{\Phi(s)}{s^2 + a_1s + a_0} \]

\[ \frac{s y(0^-) + y'(0^-) + a_1 y(0^-) - b_1 y(0^-)}{s^2 + a_1 s + a_0} \]

\[ \frac{-2}{s^2 + 4s + 13} \]

\[ \frac{s + 2}{s^2 + 4s + 13} \]

\[ \frac{s + 2}{(s + 2)^2 + 3^2} \]

\[ L^{-1} \Rightarrow y(t) = e^{-2t} \cos 3t + u(t) \]
For the system below, determine the range of $K$ for stability.

\[
T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{K}{s^3 + 3s^2 + 3s + 1}}{1 + \frac{K}{s^3 + 3s^2 + 3s + 1}} = \frac{K}{s^3 + 3s^2 + 3s + 1 + K}
\]

**Routh–Hurwitz:**

\[
\begin{array}{c|ccc}
s^3 & 1 & 3 & \\
s^2 & 3 & 1+K & \\
s^1 & \frac{8-K}{3} & 0 & \\
s^0 & 1+K & 0 & \\
\end{array}
\]

For stability require:

1) \( \frac{8-K}{3} > 0 \) \( \Rightarrow \) \( K < 8 \)

2) \( 1+K > 0 \) \( \Rightarrow \) \( K > -1 \)

1) + 2) \( \Rightarrow \) \(-1 < K < 8\)
Problem 4:
We are given the following polynomial:

\[ a_3 s^3 + a_2 s^2 + a_1 s + a_0 \]

where the nominal values of the coefficients \( a_i \), \( i = 0,1,2,3 \) are given by
\[ a_3 = 4, \quad a_2 = 3, \quad a_1 = 2, \quad a_0 = 1. \]

Find upper bounds on \( a_3 \) and \( a_1 \), for which the polynomial is (robustly) stable.

(For those who want some further explanation of this question:
Keeping \( a_2 \) and \( a_0 \) constant at their respective nominal values, we allow \( a_3 \) and \( a_1 \) to simultaneously and independently vary upwards from their respective nominal values and subsequently we want to find the maximum values for these coefficients for which the polynomial remains stable.)

Apply Khon'nov's Theorem and since order of polynomial is 3rd.
only need to consider polynomial \( a_3(s) \) where

\[ a_3(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0. \]

Now \( a_3 \) and \( a_1 \) will vary upward, however only the lower bound of \( a_1 \) is used so that there is no upper bound on \( a_1 \) \( \Rightarrow \)
\[ 2 \leq a_1 < \infty \]

Stability, however does involve the upper bound on \( a_3 \),
(called \( u_3 \))

\[ a_3(s) = u_3 s^3 + 3 s^2 + 2 s + 1 \]

Routh array:

\[
\begin{array}{c|ccc}
 s^3 & u_3 & 2 \\
 s^2 & 3 & 1 \\
 s^1 & -\frac{u_3-6}{3} & 0 \\
 s^0 & 1 & \\
\end{array}
\]

Now since \( u_3 \geq 4 \) it is positive \( \Rightarrow \) require for stability
\[ \frac{u_3-6}{3} > 0 \]
\[ \text{or} \quad 6 - u_3 > 0 \]
\[ \Rightarrow \quad 6 > u_3 \]

This is the upper bound.

: range of values for \( u_3 \) for stability is
\[ 4 \leq u_3 < 6 \]