Chapter 9  Self and Mutual Inductances – Transformers

9.20 Exercises

1. For the transformer below find $v_2$ for $t > 0$.

2. For the transformer below find the phasor currents $I_1$ and $I_2$.

3. For the network below find the transfer function $G(s) = V_{OUT}(s)/V_{IN}(s)$.

4. For the transformer below find the average power delivered to the 4 $\Omega$ resistor.
5. Replace the transformer below by a Thevenin equivalent and then compute $V_1$, $V_2$, $I_1$ and $I_2$.

6. For the circuit below compute the turns ratio $a$ so that maximum power will be delivered to the 10 KΩ resistor.

7. The recorded open- and short-circuit test data for a 10KVA, 2400 / 240, 60 Hz transformer are as follows:

Open-circuit test with input to the low side: 240 V, 0.75 A, 72 W

Short-circuit test with input to the high side: 80.5 V, 5 A, 210 W

Compute the parameters for the approximate equivalent circuit shown below.

8. Repeat Exercise 7 above using per-unit values.

9. Using the data in Exercise 7 above, compute the voltage regulation for power factor 0.8 leading using per-unit values.

10. Using the data in Exercise 7 above, compute the efficiency for power factor 0.8 lagging at half load using per-unit values.
9.21 Solutions to End-of-Chapter Exercises

1.

Application of KVL in the primary yields

\[ 2i_1 + L_1 \frac{di_1}{dt} = 8u_0(t) \]

\[ 1 \cdot \frac{di_1}{dt} + 2i_1 = 8 \quad t > 0 \quad (1) \]

The total solution of \( i_1 \) is the sum of the forced component \( i_{1f} \) and the natural response \( i_{1n} \), i.e.,

\[ i_1 = i_{1f} + i_{1n} \]

From (1) we find that \( i_{1f} = 8/2 = 4 \), and \( i_{1n} \) is found from the characteristic equation \( s + 2 = 0 \) from which \( s = -2 \) and thus \( i_{1n} = Ae^{-2t} \). Then,

\[ i_1 = 4 + Ae^{-2t} \quad (2) \]

Since we are not told otherwise, we will assume that \( i_1(0^-) = 0 \) and from (2) \( 0 = 4 + Ae^0 \) or \( A = -4 \) and by substitution into (2)

\[ i_1 = 4(1 - 4e^{-2t}) \]

The voltage \( v_2 \) is found from

\[ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]

and since \( i_2 = 0 \),

\[ v_2 = 1 \cdot \frac{di_1}{dt} = \frac{d}{dt}[4(1 - 4e^{-2t})] = 8e^{-2t} \text{ V} \]
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2.

![Circuit Diagram]

The mesh equations for primary and secondary are:

\[(1 + j1)I_1 - j1I_2 = 10\angle0^\circ\]
\[-j1I_1 + (2 - j2)I_2 = 0\]

By Cramer’s rule,

\[I_1 = D_1/\Delta \quad I_2 = D_2/\Delta\]

where

\[
\Delta = \begin{bmatrix}
(1 + j1) & -j1 \\
-j1 & (2 - j2)
\end{bmatrix} = 5
\]
\[
D_1 = \begin{bmatrix}
10\angle0^\circ & -j1 \\
0 & (2 - j2)
\end{bmatrix} = 20(1 - j)
\]
\[
D_2 = \begin{bmatrix}
(1 + j1) & 10\angle0^\circ \\
-j1 & 0
\end{bmatrix} = j10
\]

Thus,

\[I_1 = \frac{20(1 - j)}{5} = 4(1 - j) = 4\sqrt{2} \angle -45^\circ \text{ A}\]
\[I_2 = \frac{j10}{5} = j2 = 2\angle90^\circ \text{ A}\]

Check with MATLAB:

```
Z=[1+j -j; -j 2-2j]; V=[10 0]; I=Z\V;
fprintf('magI1 = %5.2f A \t', abs(I(1))); fprintf('phasel1 = %5.2f deg ',angle(I(1))*180/pi);...
fprintf(' 
');...
fprintf('magI2 = %5.2f A \t', abs(I(2))); fprintf('phasel2 = %5.2f deg ',angle(I(2))*180/pi);...
fprintf(' 
')
```

```
mag I1 =  5.66 A  phasel1 = -45.00 deg
mag I2 =  2.00 A  phasel2 =  90.00 deg
```
We will find \( V_{OUT}(s) \) from \( V_{OUT}(s) = (1 \, \Omega)I_3 \). The three mesh equations in matrix form are:

\[
\begin{bmatrix}
(s + 1) & -0.5s & -0.5s \\
-0.5s & (s + 1) & -0.5s \\
-0.5s & -0.5s & (s + 1)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot V_{IN}(s)
\]

We will use MATLAB to find the determinant \( \Delta \) of the \( 3 \times 3 \) matrix.

\[
s = \text{sym}(s);
delta = [s + 1\end{array} -0.5s\end{array} -0.5s; -0.5s s + 1 -0.5s; -0.5s -0.5s s + 1]; \text{det}\_\text{delta} = \text{det}(\delta)
\]

\[
delta = [s + 1\end{array} -0.5s\end{array} -0.5s; -0.5s s + 1 -0.5s; 1 0 0]; \text{det}\_d3 = \text{det}(d3)
\]

\[
I_3 = \frac{\text{det}\_d3}{\text{det}\_\text{delta}}
\]

\[
|3| = \frac{\left(\frac{3}{4} s^2 + 1\right)}{\left(\frac{9}{4} s^2 + 3 s + 1\right)} \cdot \frac{s^2}{s^2 + 2}
\]

\[
\text{simplify}(I3)
\]

\[
\text{ans} = \frac{s}{3 s + 2}
\]

Therefore,

\[
V_{OUT}(s) = 1 \cdot I_3 \cdot V_{IN}(s) = \frac{s}{(3 s + 2)} \cdot V_{IN}(s)
\]

and

\[
G(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{s}{3 s + 2}
\]
For this exercise, \( P_{\text{ave} 4 \Omega} = \frac{1}{2} (I_4 \Omega)^2 4 \) and thus we need to find \( I_4 \Omega \).

At Node A,
\[
\frac{V_2}{4} + \frac{V_2 - 4\angle 0^\circ}{8} - I_2 = 0
\]
\[
\frac{3V_2}{8} - I_2 = \frac{1}{2} \quad (1)
\]

From the primary circuit,
\[
2I_1 + V_1 = 4 \quad (2)
\]

Since \( I_2/I_1 = 1/a \), \( V_2/V_1 = a \), and \( a = 2 \), it follows that \( I_1 = 2I_2 \) and \( V_1 = V_2/2 \). By substitution into (2) we obtain
\[
4I_2 + \frac{V_2}{2} = 4
\]
\[
I_2 + \frac{V_2}{8} = 1 \quad (3)
\]

Addition of (1) and (3) yields
\[
\frac{3V_2}{8} + \frac{V_2}{8} = \frac{1}{2} + 1
\]

from which \( V_2 = 3 \). Then,
\[
I_4 \Omega = \frac{V_2}{4} = \frac{3}{4}
\]

and
\[
P_{\text{ave} 4 \Omega} = \frac{1}{2} \left( \frac{3}{4} \right)^2 4 = \frac{9}{8} \text{ w}
\]
5.

Because the dot on the secondary is at the lower end, \( a = -5 \). Then,

\[
aV_s = -5 \times 12\angle 0^\circ = -60\angle 0^\circ = 60\angle 180^\circ
\]

\[
a^2Z_s = 25(2 + j3) = 50 + j75 = 90.14\angle 56.31^\circ \, \Omega
\]

\[
Z_{LD} = 100 - j75 = 125\angle -36.87^\circ \, \Omega
\]

\[
I_2 = \frac{aV_s}{a^2Z_s + Z_{LD}} = \frac{60\angle 180^\circ}{50 + j75 + 100 - j75} = \frac{60\angle 180^\circ}{150} = \frac{2}{5}\angle 180^\circ
\]

and

\[
V_2 = Z_{LD} \cdot I_2 = 125\angle -36.87^\circ \times \frac{2}{5}\angle 180^\circ = 50\angle 143.13^\circ \, V
\]

6.

From (9.102)

\[
Z_{in} = \frac{Z_{LD}}{a^2}
\]

Then,

\[
a^2 = \frac{Z_{LD}}{Z_{in}} = \frac{10000}{4} = 2500
\]

or

\[
a = 50
\]

7. We are told that open– and short–circuit test data for a 10KVA, 2400 / 240, 60 Hz transformer are as follows:

Open–circuit test with input to the low side: 240 V, 0.75 A, 72 W

Short–circuit test with input to the high side: 80.5 V, 5 A, 210 W