3.14 Exercises

Multiple Choice

1. The voltage across the $2\ \Omega$ resistor in the circuit of Figure 3.67 is

   A. $6\ V$

   B. $16\ V$

   C. $-8\ V$

   D. $32\ V$

   E. *none of the above*

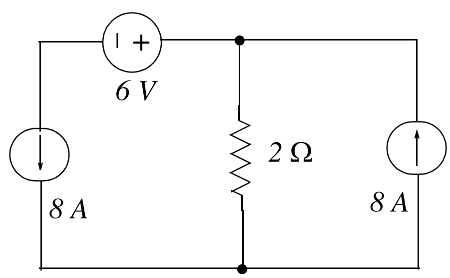


Figure 3.67. Circuit for Question 1

2. The current i in the circuit of Figure 3.68 is

   A. $-2\ A$

   B. $5\ A$

   C. $3\ A$

   D. $4\ A$

   E. *none of the above*

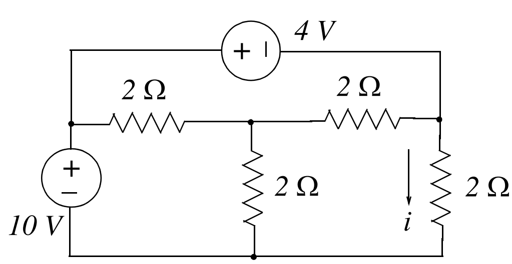


Figure 3.68. Circuit for Question 2

3. The node voltages shown in the partial network of Figure 3.69 are relative to some reference node which is not shown. The current i is

   A. *–4 A*

   B. *8/3 A*

   C. *–5 A*

   D. *–6 A*

   E. *none of the above*

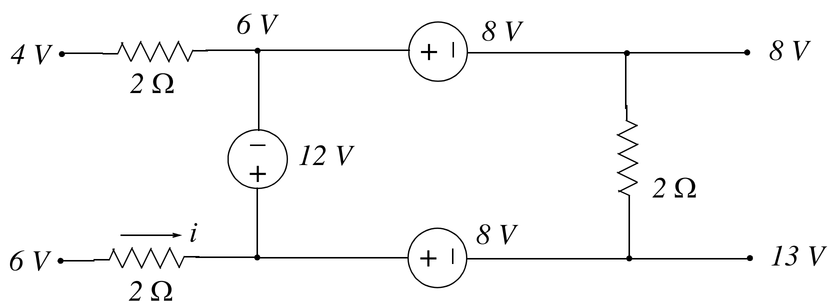


Figure 3.69. Circuit for Question 3

4. The value of the current i for the circuit of Figure 3.70 is

   A. *–3 A*

   B. *–8 A*

   C. *–9 A*

   D. *6 A*

   E. *none of the above*

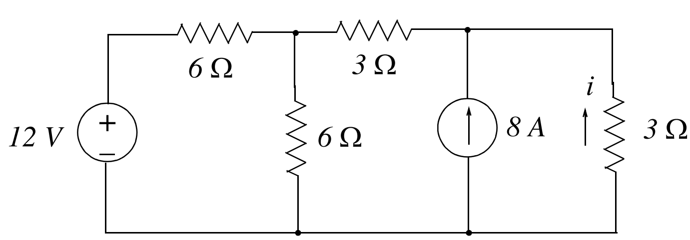


Figure 3.70. Circuit for Question 4

5. The value of the voltage v for the circuit of Figure 3.71 is

   A. *4 V*

   B. *6 V*

   C. *8 V*

   D. *12 V*

   E. *none of the above*

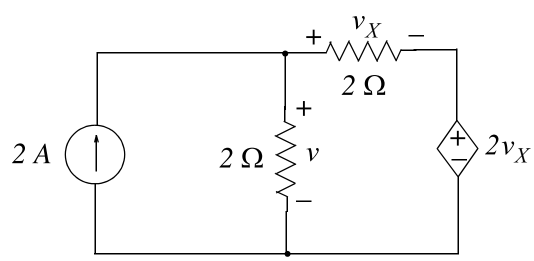


Figure 3.71. Circuit for Question 5

6. For the circuit of Figure 3.72, the value of k is dimensionless. For that circuit, no solution is possible if the value of k is

   A. *2*

   B. *1*

   C. ∞

   D. *0*

   E. *none of the above*

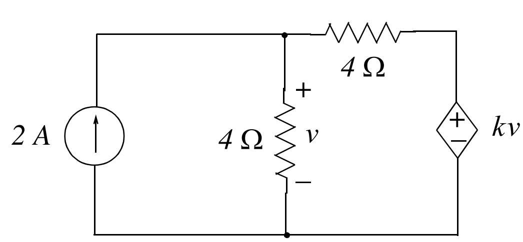


Figure 3.72. Circuit for Question 6

7. For the network of Figure 3.73, the Thevenin equivalent resistance R_{TH} to the right of terminals a and b is

   A. *1*

   B. *2*

   C. *5*

   D. *10*

   E. *none of the above*

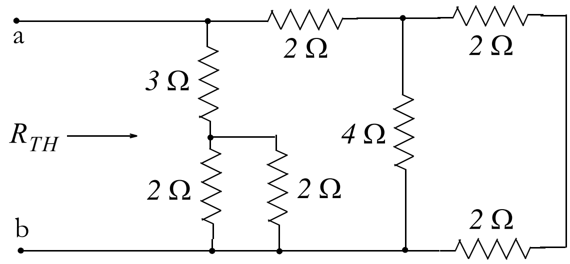


Figure 3.73. Network for Question 7

8. For the network of Figure 3.74, the Thevenin equivalent voltage V_{TH} across terminals a and b is

   A. *–3 V*

   B. *–2 V*

   C. *1 V*

   D. *5 V*

   E. *none of the above*

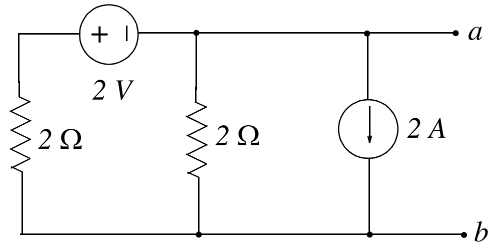


Figure 3.74. Network for Question 8

9. For the network of Figure 3.75, the Norton equivalent current source I_N and equivalent parallel resistance R_N across terminals a and b are

A. $1\ A,\ 2\ \Omega$

B. $1.5\ A,\ 25\ \Omega$

C. $4\ A,\ 2.5\ \Omega$

D. $0\ A,\ 5\Omega$

E. *none of the above*

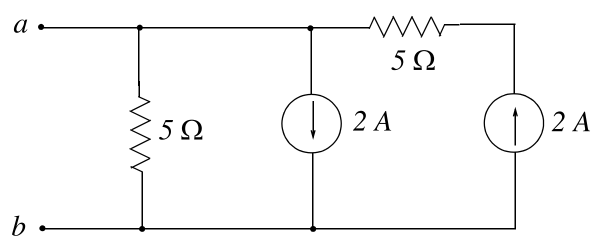


Figure 3.75. Network for Question 9

10. In applying the superposition principle to the circuit of Figure 3.76, the current i due to the $4\ V$ source acting alone is

A. $8\ A$

B. $-1\ A$

C. $4\ A$

D. $-2\ A$

E. *none of the above*

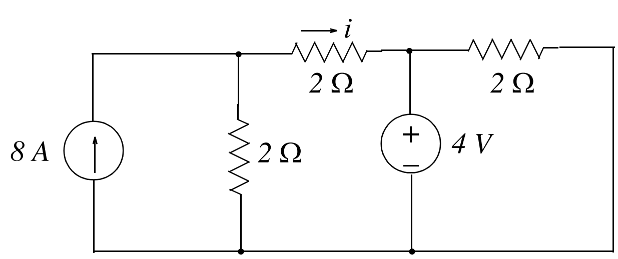


Figure 3.76. Network for Question 10

Problems

1. Use nodal analysis to compute the voltage across the 18 A current source in the circuit of Figure 3.77. Answer: *1.12 V*

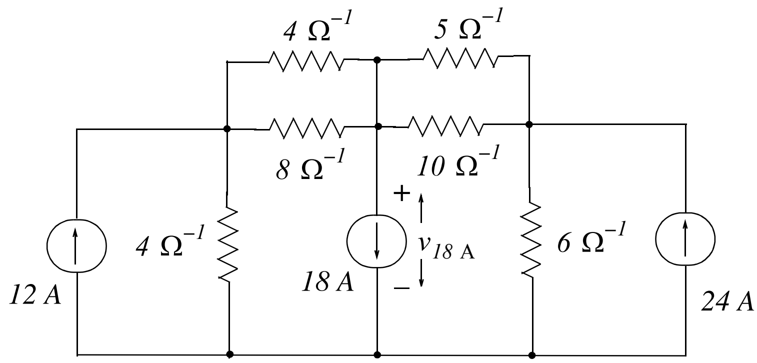


Figure 3.77. Circuit for Problem 1

2. Use nodal analysis to compute the voltage $v_{6\,\Omega}$ in the circuit of Figure 3.78. Answer: *21.6 V*

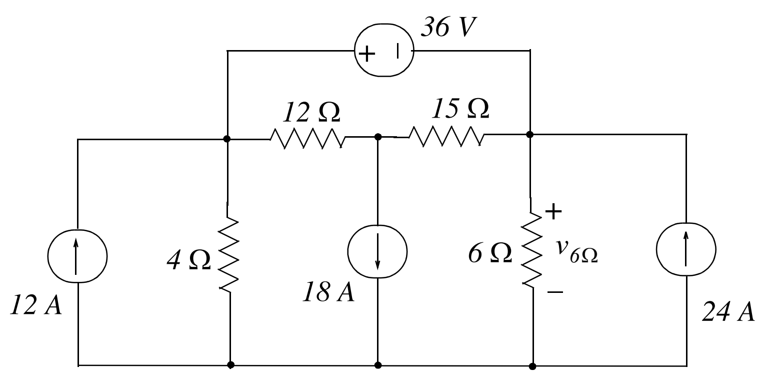


Figure 3.78. Circuit for Problem 2

3. Use nodal analysis to compute the current through the $6\ \Omega$ resistor and the power supplied (or absorbed) by the dependent source shown in Figure 3.79. Answers: *–3.9 A, –499.17 w*

4. Use mesh analysis to compute the voltage v_{36A} in Figure 3.80. Answer: *86.34 V*

5. Use mesh analysis to compute the current through the $i_{6\Omega}$ resistor, and the power supplied (or absorbed) by the dependent source shown in Figure 3.81. Answers: *–3.9 A, –499.33 w*

6. Use mesh analysis to compute the voltage $v_{10\Omega}$ in Figure 3.82. Answer: *0.5 V*

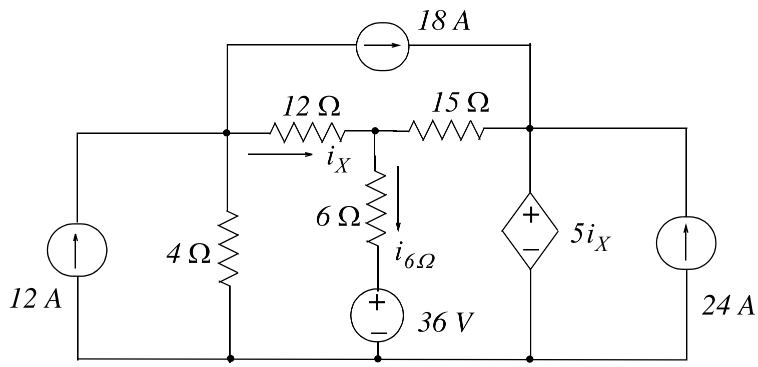


Figure 3.79. Circuit for Problem 3

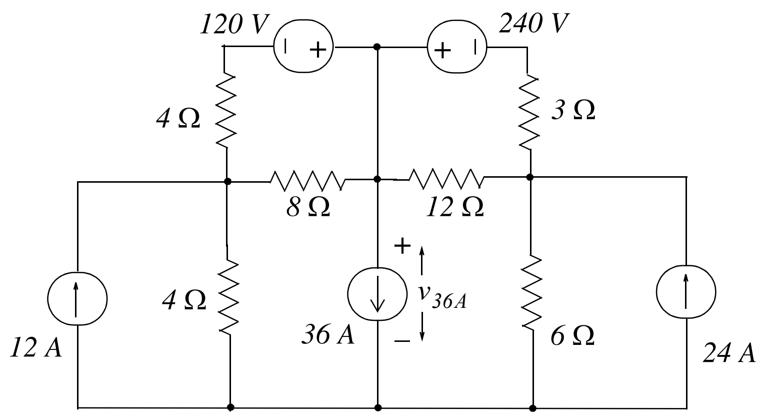


Figure 3.80. Circuit for Problem 4

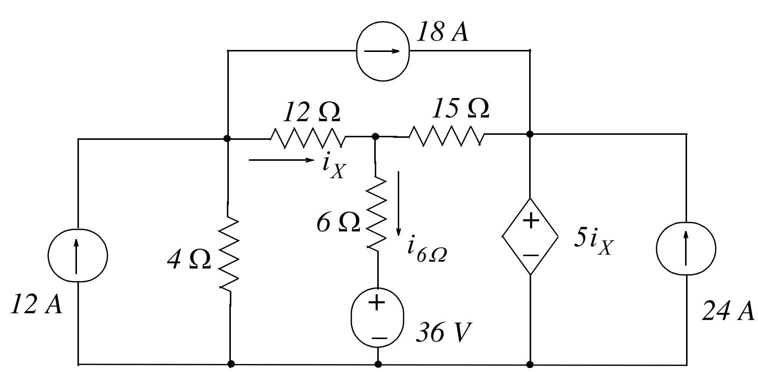


Figure 3.81. Circuit for Problem 5

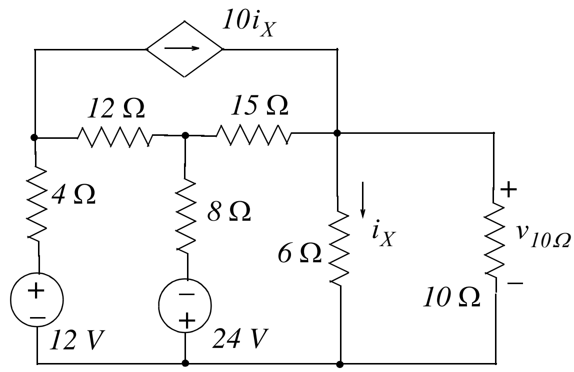


Figure 3.82. Circuit for Problem 6

7. Compute the power absorbed by the *10* Ω resistor in the circuit of Figure 3.83 using any method. Answer: *1.32 w*

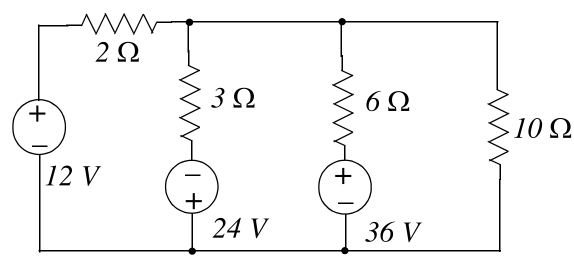


Figure 3.83. Circuit for Problem 7

8. Compute the power absorbed by the *20* Ω resistor in the circuit of Figure 3.84 using any method. Answer: *73.73 w*

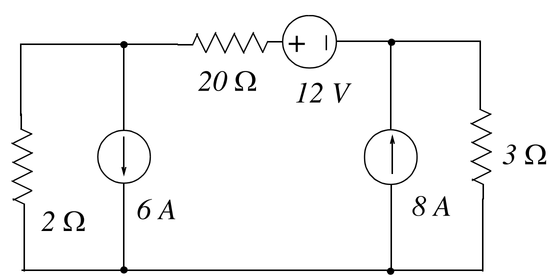


Figure 3.84. Circuit for Problem 8

9. In the circuit of Figure 3.85:

   a. To what value should the load resistor R_{LOAD} should be adjusted to so that it will absorb maximum power? Answer: *2.4* Ω

b. What would then the power absorbed by R_{LOAD} be? Answer: *135 w*

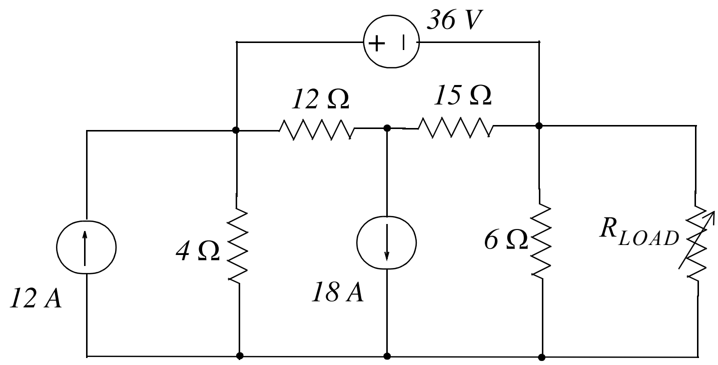


Figure 3.85. Circuit for Problem 9

10. Replace the network shown in Figure 3.86 by its Norton equivalent.
Answers: $i_N = 0$, $R_N = 23.75\ \Omega$

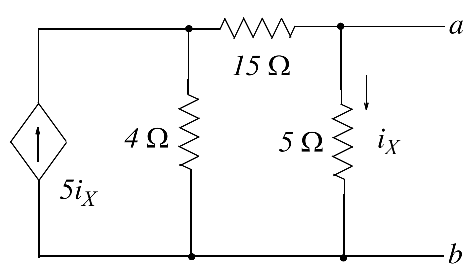


Figure 3.86. Circuit for Problem 10

11. Use the superposition principle to compute the voltage v_{18A} in the circuit of Figure 3.87.
Answer: *1.12 V*

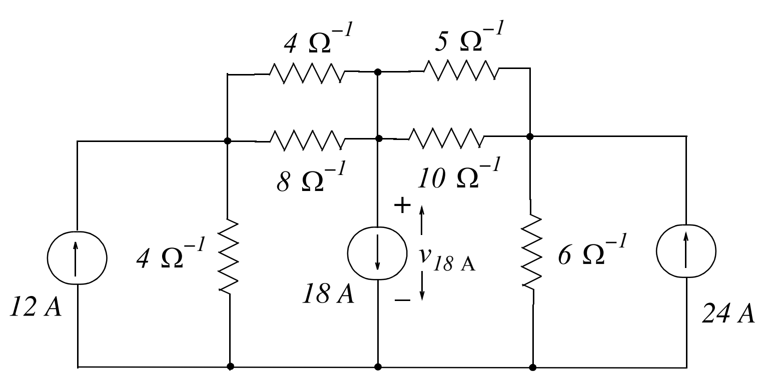


Figure 3.87. Circuit for Problem 11

12. Use the superposition principle to compute voltage $v_{6\,\Omega}$ in the circuit of Figure 3.88.
Answer: *21.6 V*

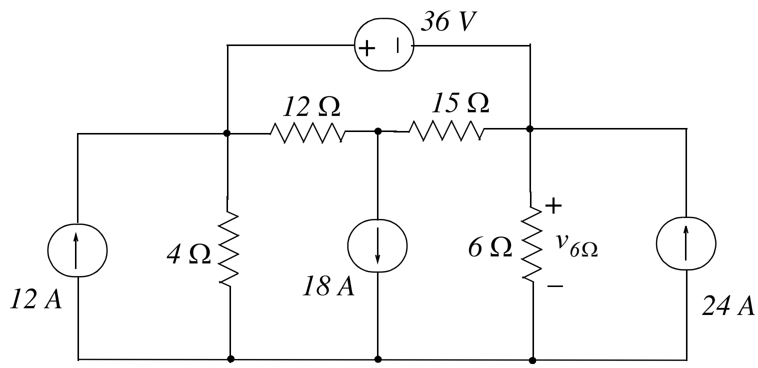


Figure 3.88. Circuit for Problem 12

13. In the circuit of Figure 3.89, v_{S1} and v_{S2} are adjustable voltage sources in the range $-50 \le V \le 50$ V, and R_{S1} and R_{S2} represent their internal resistances. Table 3.4 shows the results of several measurements. In Measurement 3 the load resistance is adjusted to the same value as Measurement 1, and in Measurement 4 the load resistance is adjusted to the same value as Measurement 2. For Measurements 5 and 6 the load resistance is adjusted to $1\ \Omega$. Make the necessary computations to fill-in the blank cells of this table.

TABLE 3.4 Table for Problem 13

Measurement	Switch S_1	Switch S_2	v_{S1} (V)	v_{S2} (V)	i_{LOAD} (A)
1	Closed	Open	48	0	16
2	Open	Closed	0	36	6
3	Closed	Open		0	–5
4	Open	Closed	0	–42	
5	Closed	Closed	15	18	
6	Closed	Closed		24	0

Answers: $-15\ V$, $-7\ A$, $11\ A$, $-24\ V$

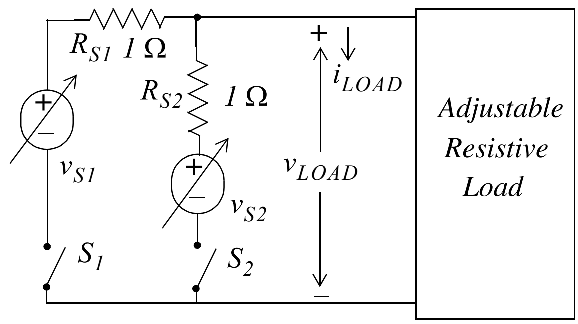


Figure 3.89. Network for Problem 13

14. Compute the efficiency of the electrical system of Figure 3.90. Answer: *76.6%*

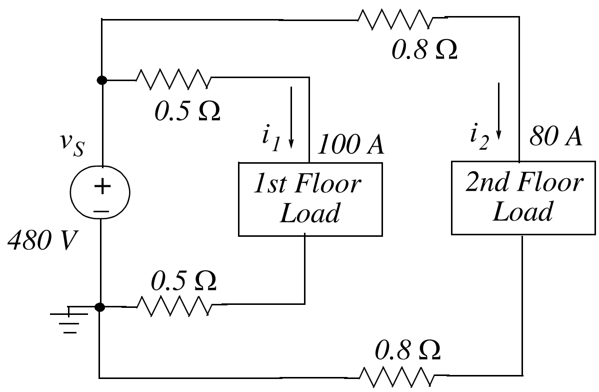


Figure 3.90. Electrical system for Problem 14

15. Compute the regulation for the 2st floor load of the electrical system of Figure 3.91. Answer: *36.4%*

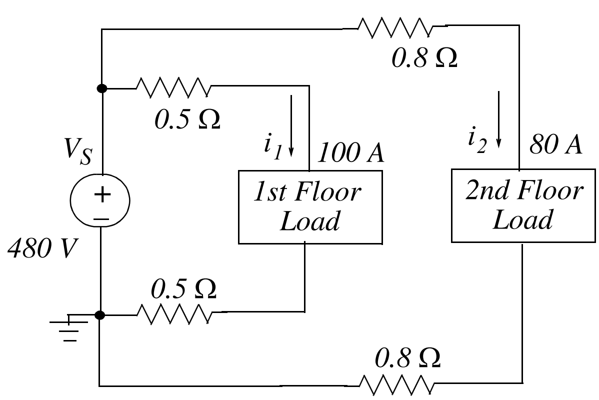


Figure 3.91. Circuit for Problem 15

16. Write a set of nodal equations and then use MATLAB to compute i_{LOAD} and v_{LOAD} for the circuit of Example 3.10 which is repeated as Figure 3.92 for convenience.
Answers: $-0.96\ A$, $-7.68\ V$

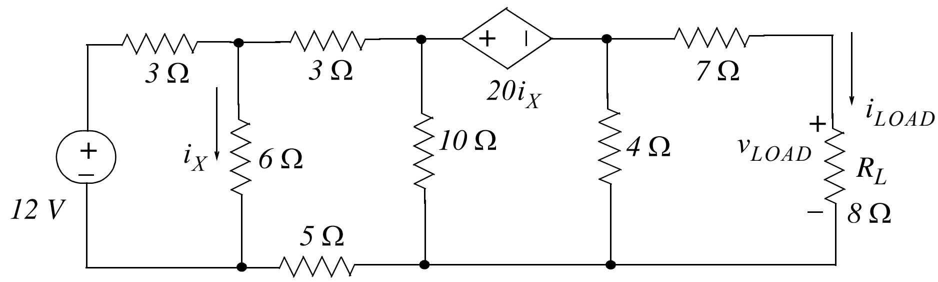
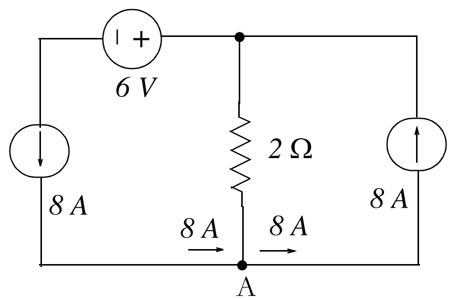


Figure 3.92. Circuit for Problem 16

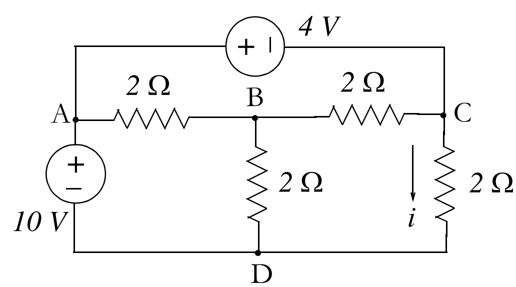
3.15 Answers to Exercises

Multiple Choice

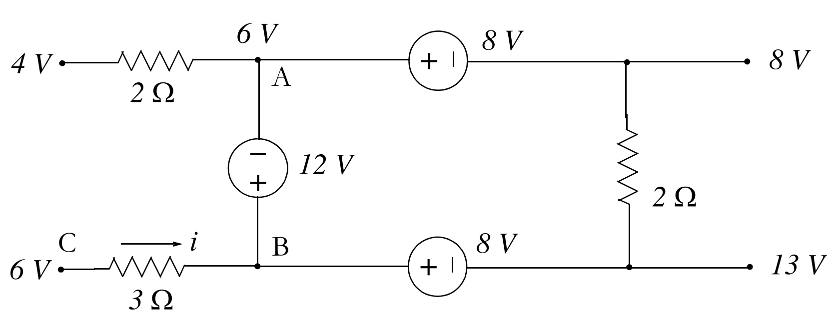
1. E The current entering Node A is equal to the current leaving that node. Therefore, there is no current through the $2\ \Omega$ resistor and the voltage across it is zero.



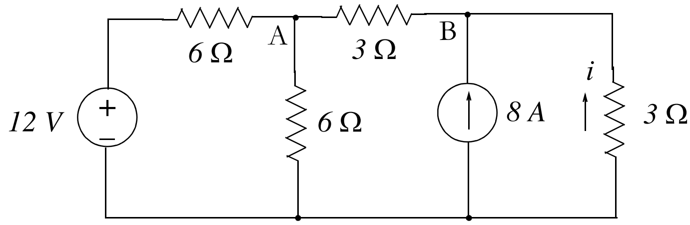
2. C From the figure below, $V_{AC} = 4\ V$. Also, $V_{AB} = V_{BC} = 2\ V$ and $V_{AD} = 10\ V$. Then, $V_{BD} = V_{AD} - V_{AB} = 10 - 2 = 8\ V$ and $V_{CD} = V_{BD} - V_{BC} = 8 - 2 = 6\ V$. Therefore, $i = 6/2 = 3\ A$.



3. A From the figure below we observe that the node voltage at A is $6\ V$ relative to the reference node which is not shown. Therefore, the node voltage at B is $6 + 12 = 18\ V$ relative to the same reference node. The voltage across the resistor is $V_{BC} = 18 - 6 = 12\ V$ and the direction of current through the $3\ \Omega$ resistor is opposite to that shown since Node B is at a higher potential than Node C. Thus $i = -12/3 = -4\ A$



4. E We assign node voltages at Nodes A and B as shown below.



At Node A

$$\frac{V_A - 12}{6} + \frac{V_A}{6} + \frac{V_A - V_B}{3} = 0$$

and at Node B

$$\frac{V_B - V_A}{3} + \frac{V_B}{3} = 8$$

These simplify to

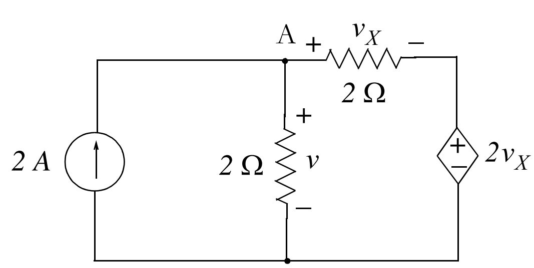
$$\frac{2}{3}V_A - \frac{1}{3}V_B = 2$$

and

$$-\frac{1}{3}V_A + \frac{2}{3}V_B = 8$$

Multiplication of the last equation by 2 and addition with the first yields $V_B = 18$ and thus $i = -18/3 = -6\ A$.

5. E Application of KCL at Node A of the circuit below yields



$$\frac{v}{2} + \frac{v - 2v_X}{2} = 2$$

or

$$v - v_X = 2$$

Also by KVL

$$v = v_X + 2v_X$$

and by substitution

$$v_X + 2v_X - v_X = 2$$

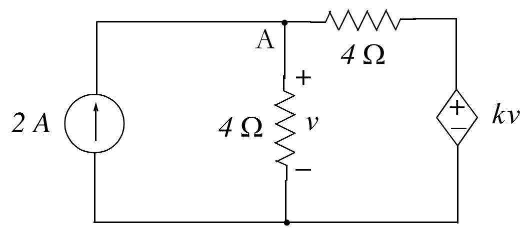
or

$$v_X = 1$$

and thus

$$v = v_X + 2v_X = 1 + 2 \times 1 = 3\ V$$

6. A Application of KCL at Node A of the circuit below yields



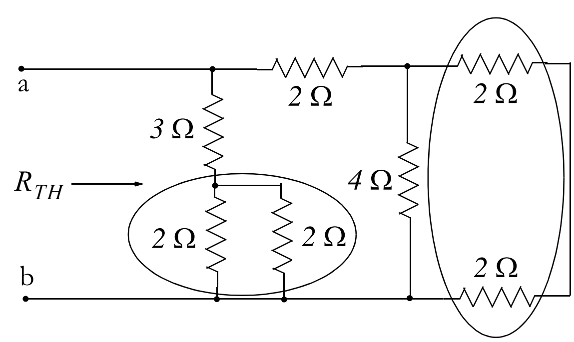
$$\frac{v}{4} + \frac{v - kv}{4} = 2$$

or

$$\frac{1}{4}(2v - kv) = 2$$

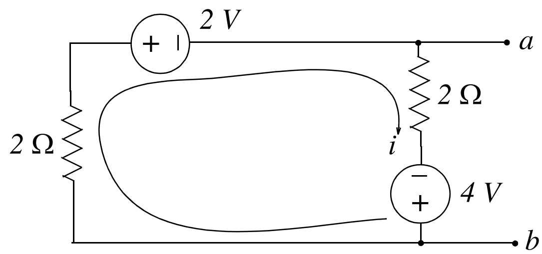
and this relation is meaningless if $k = 2$. Thus, this circuit has solutions only if $k \neq 2$.

7. B The two $2\ \Omega$ resistors on the right are in series and the two $2\ \Omega$ resistors on the left shown in the figure below are in parallel.



Starting on the right side and proceeding to the left we get $2 + 2 = 4$, $4 \| 4 = 2$, $2 + 2 = 4$, $4 \| (3 + 2 \| 2) = 4 \| (3 + 1) = 4 \| 4 = 2\ \Omega$.

8. A Replacing the current source and its $2\ \Omega$ parallel resistance with an equivalent voltage source in series with a $2\ \Omega$ resistance we get the network shown below.



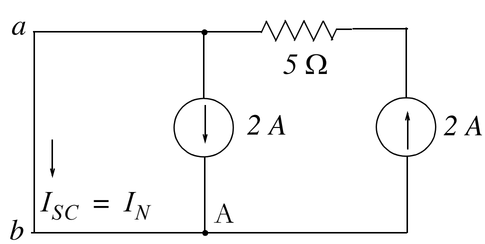
By Ohm's law,

$$i = \frac{4-2}{2+2} = 0.5\ A$$

and thus

$$v_{TH} = v_{ab} = 2 \times 0.5 + (-4) = -3\ V$$

9. D The Norton equivalent current source I_N is found by placing a short across the terminals a and b. This short shorts out the $5\ \Omega$ resistor and thus the circuit reduces to the one shown below.

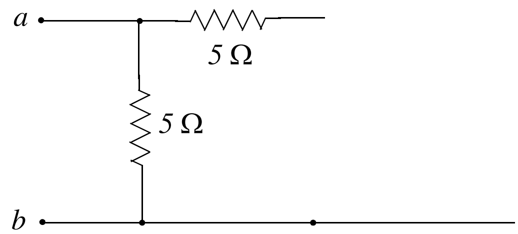


By KCL at Node A,

$$I_N + 2 = 2$$

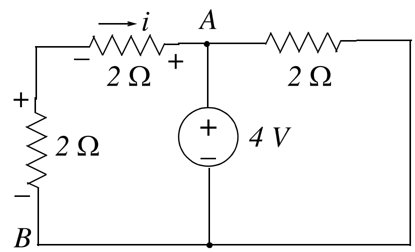
and thus $I_N = 0$

The Norton equivalent resistance R_N is found by opening the current sources and looking to the right of terminals a and b. When this is done, the circuit reduces to the one shown below.



Therefore, $R_N = 5\ \Omega$ and the Norton equivalent circuit consists of just a $5\ \Omega$ resistor.

10. B With the $4\ V$ source acting alone, the circuit is as shown below.

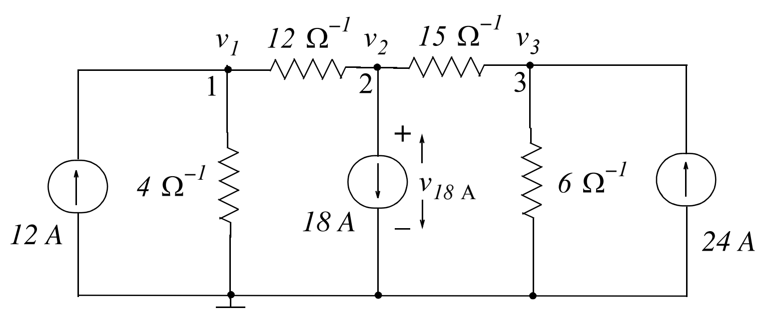


We observe that $v_{AB} = 4\ V$ and thus the voltage drop across each of the $2\ \Omega$ resistors to the left of the $4\ V$ source is $2\ V$ with the indicated polarities. Therefore,

$$i = -2/2 = -1\ A$$

Problems

1. We first replace the parallel conductances with their equivalents and the circuit simplifies to that shown below.



Applying nodal analysis at Nodes 1, 2, and 3 we get:

Node 1:

$$16v_1 - 12v_2 = 12$$

Node 2:

$$-12v_1 + 27v_2 - 15v_3 = -18$$

Node 3:

$$-15v_2 + 21v_3 = 24$$

Simplifying the above equations, we get:

$$\begin{aligned} 4v_1 - 3v_2 \qquad &= 3 \\ -4v_1 + 9v_2 - 5v_3 &= -6 \\ -5v_2 + 7v_3 &= 8 \end{aligned}$$

Addition of the first two equations above and grouping with the third yields

$$\begin{aligned} 6v_2 - 5v_3 &= -3 \\ -5v_2 + 7v_3 &= 8 \end{aligned}$$

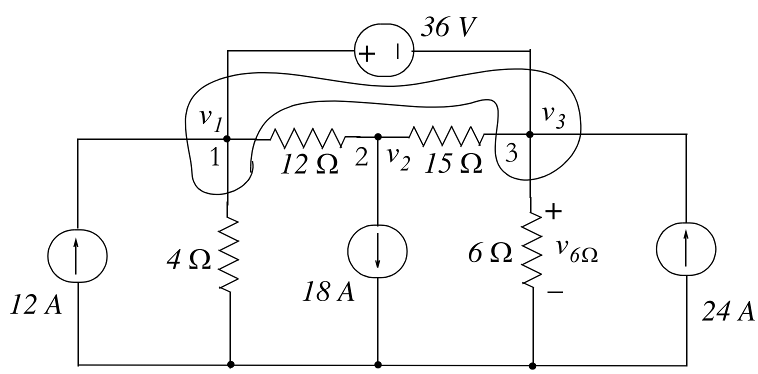
For this problem we are only interested in $v_2 = v_{18\ \mathrm{A}}$. Therefore, we will use Cramer's rule to solve for v_2. Thus,

$$v_2 = \frac{D_2}{\Delta} \qquad D_2 = \begin{bmatrix} -3 & -5 \\ 8 & 7 \end{bmatrix} = -21 + 40 = 19 \qquad \Delta = \begin{bmatrix} 6 & -5 \\ -5 & 7 \end{bmatrix} = 42 - 25 = 17$$

and

$$v_2 = v_{18\ \mathrm{A}} = 19/17 = 1.12\ V$$

2. Since we cannot write an expression for the current through the $36\ V$ source, we form a combined node as shown on the circuit below.



At Node 1 (combined node):

$$\frac{v_1}{4} + \frac{v_1 - v_2}{12} + \frac{v_3 - v_2}{15} + \frac{v_3}{6} - 12 - 24 = 0$$

and at Node 2,

$$\frac{v_2 - v_1}{12} + \frac{v_2 - v_3}{15} = -18$$

Also,

$$v_1 - v_3 = 36$$

Simplifying the above equations, we get:

$$\frac{1}{3}v_1 - \frac{3}{20}v_2 + \frac{7}{30}v_3 = 36$$

$$-\frac{1}{12}v_1 + \frac{3}{20}v_2 - \frac{1}{15}v_3 = -18$$

$$v_1 \qquad\qquad -v_3 = 36$$

Addition of the first two equations above and multiplication of the third by $-1/4$ yields

$$\frac{1}{4}v_1 + \frac{1}{6}v_3 = 18$$

$$-\frac{1}{4}v_1 + \frac{1}{4}v_3 = -9$$

and by adding the last two equations we get

$$\frac{5}{12}v_3 = 9$$

or

$$v_3 = v_{6\,\Omega} = \frac{108}{5} = 21.6V$$

Check with MATLAB:

```
format rat
R=[1/3 –3/20 7/30; –1/12 3/20 –1/15; 1 0 –1];
I=[36 –18 36]';
V=R\I;
fprintf('\n'); disp('v1='); disp(V(1)); disp('v2='); disp(V(2)); disp('v3='); disp(V(3))
```

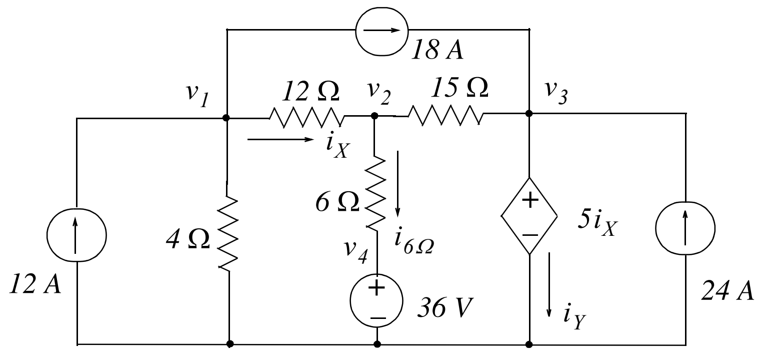
```
v1=
   288/5
v2=
  -392/5
v3=
   108/5
```

3. We assign node voltages v_1, v_2, v_3, v_4 and current i_Y as shown in the circuit below. Then,

$$\frac{v_1}{4}+\frac{v_1-v_2}{12}+18-12 = 0$$

and

$$\frac{v_2-v_1}{12}+\frac{v_2-v_3}{12}+\frac{v_2-v_4}{6} = 0$$



Simplifying the last two equations above, we get

$$\frac{1}{3}v_1-\frac{1}{12}v_2 = -6$$

and

$$-\frac{1}{12}v_1+\frac{19}{60}v_2-\frac{1}{15}v_3-\frac{1}{6}v_4 = 0$$

Next, we observe that $i_X = \frac{v_1-v_2}{12}$, $v_3 = 5i_X$ and $v_4 = 36\ V$. Then $v_3 = \frac{5}{12}(v_1-v_2)$ and by substitution into the last equation above, we get

$$-\frac{1}{12}v_1+\frac{19}{60}v_2-\frac{1}{15}\times\frac{5}{12}(v_1-v_2)-\frac{1}{6}36 = 0$$

or

$$-\frac{1}{9}v_1+\frac{31}{90}v_2 = 6$$

Thus, we have two equations with two unknowns, that is,

$$\frac{1}{3}v_1-\frac{1}{12}v_2 = -6$$

$$-\frac{1}{9}v_1+\frac{31}{90}v_2 = 6$$

Multiplication of the first equation above by $1/3$ and addition with the second yields

$$\frac{19}{60}v_2 = 4$$

or

$$v_2 = 240/19$$

We find v_1 from

$$\frac{1}{3}v_1 - \frac{1}{12}v_2 = -6$$

Thus,

$$\frac{1}{3}v_1 - \frac{1}{12} \times \frac{240}{19} = -6$$

or

$$v_1 = -282/19$$

Now, we find v_3 from

$$v_3 = \frac{5}{12}(v_1 - v_2) = \frac{5}{12}\left(\frac{-282}{19} - \frac{240}{19}\right) = -\frac{435}{38}$$

Therefore, the node voltages of interest are:

$$v_1 = -282/19\ V$$
$$v_2 = 240/19\ V$$
$$v_3 = -435/38\ V$$
$$v_4 = 36\ V$$

The current through the $6\ \Omega$ resistor is

$$i_{6\ \Omega} = \frac{v_2 - v_4}{6} = \frac{240/19 - 36}{6} = -\frac{74}{19} = -3.9\ A$$

To compute the power supplied (or absorbed) by the dependent source, we must first find the current i_Y. It is found by application of KCL at node voltage v_3. Thus,

$$i_Y - 24 - 18 + \frac{v_3 - v_2}{15} = 0$$

or

$$i_Y = 42 - \frac{-435/38 - 240/19}{15}$$

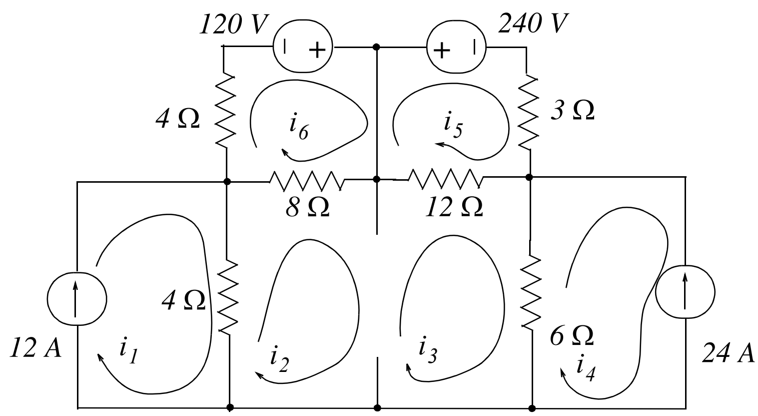
$$= 42 + \frac{915/38}{15} = \frac{1657}{38}$$

and

$$p = v_3 i_Y = -\frac{435}{38} \times \frac{1657}{38} = -\frac{72379}{145} = -499.17\ w$$

that is, the dependent source supplies power to the circuit.

4. Since we cannot write an expression for the $36\ A$ current source, we temporarily remove it and we form a combined mesh for Meshes 2 and 3 as shown below.



Mesh 1:

$$i_1 = 12$$

Combined mesh (2 and 3):

$$-4i_1 + 12i_2 + 18i_3 - 6i_4 - 8i_5 - 12i_6 = 0$$

or

$$-2i_1 + 6i_2 + 9i_3 - 3i_4 - 4i_5 - 6i_6 = 0$$

We now re-insert the $36\ A$ current source and we write the third equation as

$$i_2 - i_3 = 36$$

Mesh 4:

$$i_4 = -24$$

Mesh 5:

$$-8i_2 + 12i_5 = 120$$

or

$$-2i_2 + 3i_5 = 30$$

Mesh 6:

$$-12i_3 + 15i_6 = -240$$

or

$$-4i_3 + 5i_6 = -80$$

Thus, we have the following system of equations:

$$\begin{aligned}
i_1 \quad & = 12 \\
-2i_1 + 6i_2 + 9i_3 - 3i_4 - 4i_5 - 6i_6 & = 0 \\
i_2 - i_3 \quad & = 36 \\
i_4 \quad & = -24 \\
-2i_2 + 3i_5 & = 30 \\
-4i_3 + 5i_6 & = -80
\end{aligned}$$

and in matrix form

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -2 & 6 & 9 & -3 & -4 & -6 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 3 & 0 \\ 0 & 0 & -4 & 0 & 0 & 5 \end{bmatrix}}_{R} \cdot \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} 12 \\ 0 \\ 36 \\ -24 \\ 30 \\ -80 \end{bmatrix}}_{V}$$

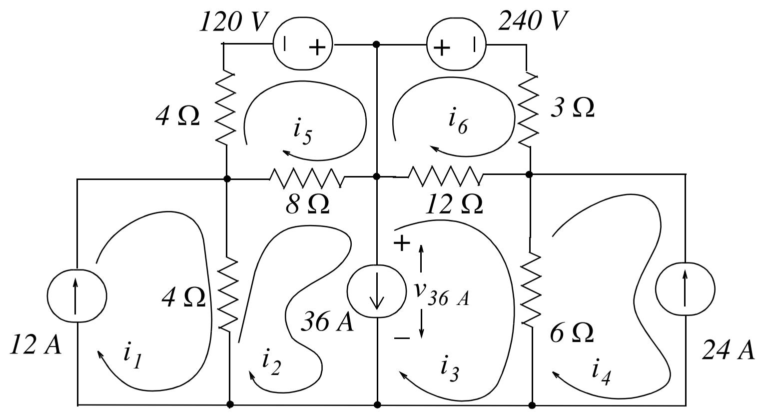
We find the currents i_1 through i_6 with the following MATLAB code:

```
R=[1  0  0  0  0  0;  –2  6  9  –3  –4  –6;...
     0  1  –1  0  0  0; 0  0  0  1  0  0;...
     0  –2  0  0  3  0; 0  0  –4  0  0  5];
V=[12  0  36  –24  30  –80]';
I=R\V;
fprintf('\n');...
  fprintf('i1=%7.2f A \t', I(1));...
  fprintf('i2=%7.2f A \t', I(2));...
  fprintf('i3=%7.2f A \t', I(3));...
  fprintf('\n');...
```

```
fprintf('i4=%7.2f A \t', I(4));...
fprintf('i5=%7.2f A \t', I(5));...
fprintf('i6=%7.2f A \t', I(6));...
fprintf('\n')
```

```
i1= 12.00 A   i2=  6.27 A   i3= -29.73 A
i4= -24.00 A  i5=  14.18 A  i6= -39.79 A
```

Now, we can find the voltage $v_{36\ A}$ by application of KVL around Mesh 3. Thus,



$$v_{36\ A} = v_{12\ \Omega} + v_{6\ \Omega} = 12 \times [(-29.73) - (-39.79)] + 6 \times [(-29.73) - (24.00)]$$

or

$$v_{36\ A} = 86.34\ V$$

To verify that this value is correct, we apply KVL around Mesh 2. Thus, we must show that

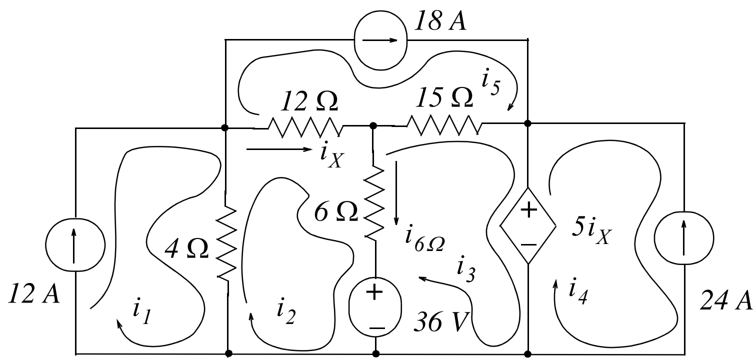
$$v_{4\ \Omega} + v_{8\ \Omega} + v_{36\ A} = 0$$

By substitution of numerical values, we find that

$$4 \times [6.27 - 12] + 8 \times [6.27 - 14.18] + 86.34 = 0.14$$

5. This is the same circuit as that of Problem 3. We will show that we obtain the same answers using mesh analysis.

   We assign mesh currents as shown below.



Mesh 1:

$$i_1 = 12$$

Mesh 2:

$$-4i_1 + 22i_2 - 6i_3 - 12i_5 = -36$$

or

$$-2i_1 + 11i_2 - 3i_3 - 6i_5 = -18$$

Mesh 3:

$$-6i_2 + 21i_3 - 15i_5 + 5i_X = 36$$

and since $i_X = i_2 - i_5$, the above reduces to

$$-6i_2 + 21i_3 - 15i_5 + 5i_2 - 5i_5 = 36$$

or

$$-i_2 + 21i_3 - 20i_5 = 36$$

Mesh 4:

$$i_4 = -24$$

Mesh 5:

$$i_5 = 18$$

Grouping these five independent equations we get:

$$\begin{aligned} i_1 \qquad\qquad\qquad\qquad\qquad &= 12 \\ -2i_1 + 11i_2 - 3i_3 \qquad -6i_5 &= -18 \\ -i_2 + 21i_3 \quad -20i_5 &= 36 \\ i_4 \qquad\qquad &= -24 \\ i_5 &= 18 \end{aligned}$$

and in matrix form,

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 11 & -3 & 0 & -6 \\ 0 & -1 & 21 & 0 & -20 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{R} \cdot \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} 12 \\ -18 \\ 36 \\ -24 \\ 18 \end{bmatrix}}_{V}$$

We find the currents i_1 through i_5 with the following MATLAB code:

```
R=[1  0  0  0  0 ;  -2  11 -3  0  -6;  0  -1  21  0  -20; ...
    0  0  0  1  0; 0  0  0  0  1];
V=[12  -18  36  -24  18]';
I=R\V;
fprintf('\n');...
  fprintf('i1=%7.2f A \t', I(1));...
  fprintf('i2=%7.2f A \t', I(2));...
  fprintf('i3=%7.2f A \t', I(3));...
  fprintf('\n');...
  fprintf('i4=%7.2f A \t', I(4));...
  fprintf('i5=%7.2f A \t', I(5));...
  fprintf('\n')
```

```
i1=  12.00 A   i2=  15.71 A   i3=  19.61 A
i4= -24.00 A   i5=  18.00 A
```

By inspection,

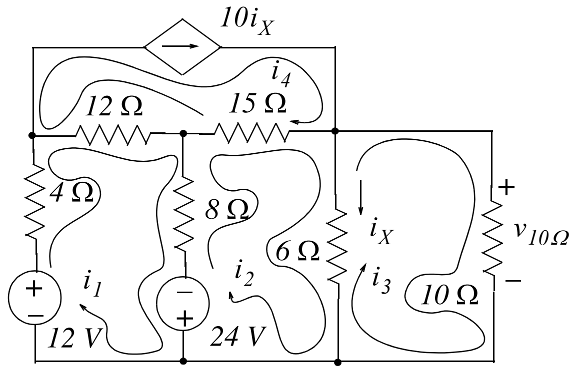
$$i_{6\ \Omega} = i_2 - i_3 = 15.71 - 19.61 = -3.9\ A$$

Next,

$$p_{5i_X} = 5i_X(i_3 - i_4) = 5(i_2 - i_5)(i_3 - i_4)$$
$$= 5(15.71 - 18.00)(19.61 + 24.00) = -499.33\ w$$

These are the same answers as those we found in Problem 3.

6. We assign mesh currents as shown below and we write mesh equations.



Mesh 1:

$$24i_1 - 8i_2 - 12i_4 - 24 - 12 = 0$$

or

$$6i_1 - 2i_2 - 3i_4 = 9$$

Mesh 2:

$$-8i_1 + 29i_2 - 6i_3 - 15i_4 = -24$$

Mesh 3:

$$-6i_2 + 16i_3 = 0$$

or

$$-3i_2 + 8i_3 = 0$$

Mesh 4:

$$i_4 = 10i_X = 10(i_2 - i_3)$$

or

$$10i_2 - 10i_3 - i_4 = 0$$

Grouping these four independent equations we get:

$$\begin{aligned} 6i_1 - 2i_2 \qquad\quad - 3i_4 &= 9 \\ -8i_1 + 29i_2 - 6i_3 - 15i_4 &= -24 \\ -3i_2 + 8i_3 \qquad\quad &= 0 \\ 10i_2 - 10i_3 - i_4 &= 0 \end{aligned}$$

and in matrix form,

$$\underbrace{\begin{bmatrix} 6 & -2 & 0 & -3 \\ -8 & 29 & -6 & -15 \\ 0 & -3 & 8 & 0 \\ 0 & 10 & -10 & -1 \end{bmatrix}}_{R} \cdot \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} 9 \\ -24 \\ 0 \\ 0 \end{bmatrix}}_{V}$$

We find the currents i_1 through i_4 with the following MATLAB code:

```
R=[6 –2 0 –3; –8 29 –6 –15; 0 –3 8 0 ; 0 10 –10 –1];
V=[9 –24 0 0]';
I=R\V;
fprintf('\n');...
  fprintf('i1=%7.2f A \t', I(1));...
  fprintf('i2=%7.2f A \t', I(2));...
  fprintf('i3=%7.2f A \t', I(3));...
  fprintf('i4=%7.2f A \t', I(4));...
   fprintf('\n')
```

```
i1= 1.94 A   i2= 0.13 A   i3= 0.05 A   i4= 0.79 A
```

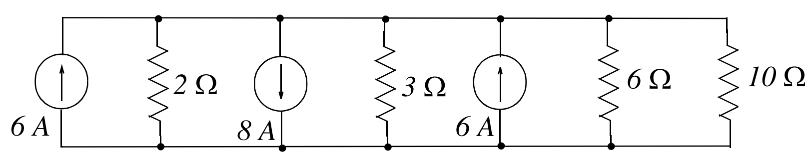
Now, we find $v_{10\Omega}$ by Ohm's law, that is,

$$v_{10\Omega} = 10i_3 = 10 \times 0.05 = 0.5\ V$$

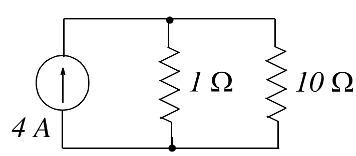
The same value is obtained by computing the voltage across the $6\ \Omega$ resistor, that is,

$$v_{6\Omega} = 6(i_2 - i_3) = 6(0.13 - 0.05) = 0.48\ V$$

7. Voltage-to-current source transformation yields the circuit below.



By combining all current sources and all parallel resistors except the $10\ \Omega$ resistor, we obtain the simplified circuit below.



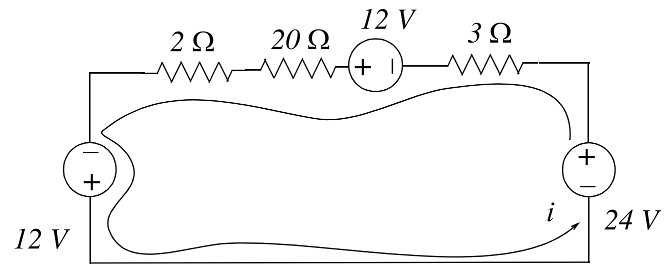
Applying the current division expression, we get

$$i_{10\ \Omega} = \frac{1}{1+10} \times 4 = \frac{4}{11}\ A$$

and thus

$$p_{10\ \Omega} = i_{10\ \Omega}^2(10) = \left(\frac{4}{11}\right)^2 \times 10 = \frac{16}{121} \times 10 = \frac{160}{121} = 1.32\ w$$

8. Current-to-voltage source transformation yields the circuit below.



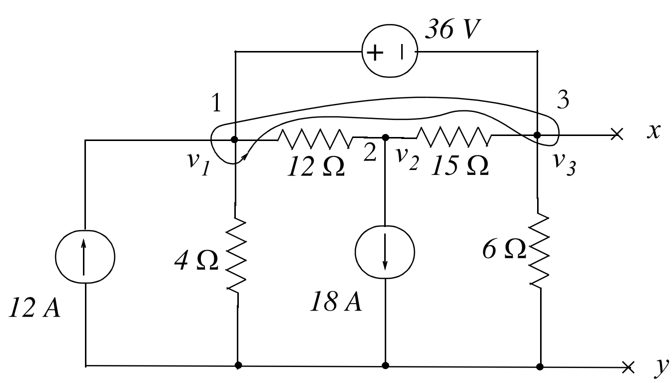
From this series circuit,

$$i = \frac{\Sigma v}{\Sigma R} = \frac{48}{25}\ A$$

and thus

$$p_{20\ \Omega} = i^2(20) = \left(\frac{48}{25}\right)^2 \times 20 = \frac{2304}{625} \times 20 = 73.73\ w$$

9. We remove R_{LOAD} from the rest of the rest of the circuit and we assign node voltages v_1, v_2, and v_3. We also form the combined node as shown on the circuit below.



Node 1:

$$\frac{v_1}{4} + \frac{v_1 - v_2}{12} - 12 + \frac{v_3 - v_2}{15} + \frac{v_3}{6} = 0$$

or

$$\frac{1}{3}v_1 - \frac{3}{20}v_2 + \frac{7}{30}v_3 = 12$$

Node 2:

$$\frac{v_2 - v_1}{12} + \frac{v_2 - v_3}{15} = -18$$

or

$$-\frac{1}{12}v_1 + \frac{3}{20}v_2 - \frac{1}{15}v_3 = -18$$

Also,

$$v_1 - v_3 = 36$$

For this problem, we are interested only in the value of v_3 which is the Thevenin voltage v_{TH}, and we could find it by Gauss's elimination method. However, for convenience, we will group these three independent equations, express these in matrix form, and use MATLAB for their solution.

$$\begin{aligned} \frac{1}{3}v_1 - \frac{3}{20}v_2 + \frac{7}{30}v_3 &= 12 \\ -\frac{1}{12}v_1 + \frac{3}{20}v_2 - \frac{1}{15}v_3 &= -18 \\ v_1 \qquad\qquad - v_3 &= 36 \end{aligned}$$

and in matrix form,

$$\underbrace{\begin{bmatrix} \frac{1}{3} & -\frac{3}{20} & \frac{7}{30} \\ -\frac{1}{12} & \frac{3}{20} & -\frac{1}{15} \\ 1 & 0 & -1 \end{bmatrix}}_{G} \cdot \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} 12 \\ -18 \\ 36 \end{bmatrix}}_{I}$$

We find the voltages v_1 through v_3 with the following MATLAB code:

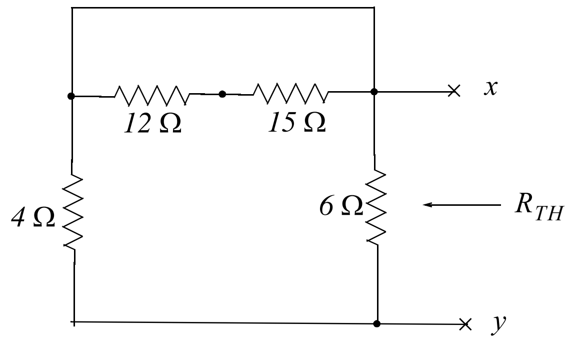
```
G=[1/3 –3/20 7/30; –1/12 3/20 –1/15; 1 0 –1];
I=[12 –18 36]'; V=G\I;
fprintf('\n');...
  fprintf('v1=%7.2f V \t', V(1)); fprintf('v2=%7.2f V \t', V(2)); fprintf('v3=%7.2f V \t', V(3));
  fprintf('\n')
```

```
v1= 0.00 V   v2= -136.00 V   v3= -36.00 V
```

Thus,

$$v_{TH} = v_3 = -36\ V$$

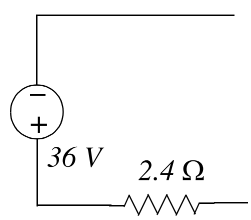
To find R_{TH} we short circuit the voltage source and we open the current sources. The circuit then reduces to the resistive network below.



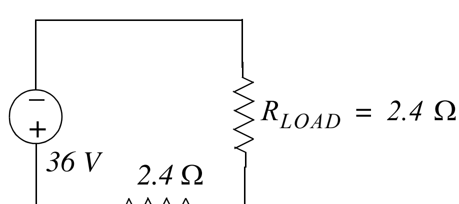
We observe that the resistors in series are shorted out and thus the Thevenin resistance is the parallel combination of the $4\ \Omega$ and $6\ \Omega$ resistors, that is,

$$4\ \Omega \| 6\ \Omega = 2.4\ \Omega$$

and the Thevenin equivalent circuit is as shown below.



Now, we connect the load resistor R_{LOAD} at the open terminals and we get the simple series circuit shown below.



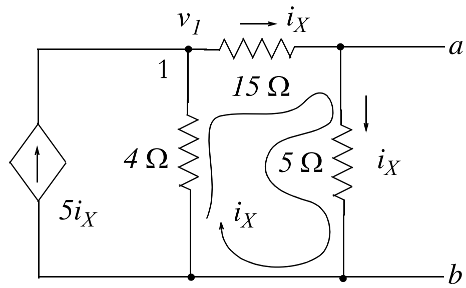
a. For maximum power transfer,

$$R_{LOAD} = 2.4\ \Omega$$

b. Power under maximum power transfer condition is

$$p_{MAX} = i^2 R_{LOAD} = \left(\frac{36}{2.4 + 2.4}\right)^2 \times 2.4 = 7.5^2 \times 2.4 = 135\ w$$

10. We assign a node voltage Node 1 and a mesh current for the mesh on the right as shown below.



At Node 1:

$$\frac{v_1}{4} + i_X = 5i_X$$

Mesh on the right:

$$(15 + 5)i_X = v_1$$

and by substitution into the node equation above,

$$\frac{20i_X}{4} + i_X = 5i_X$$

or

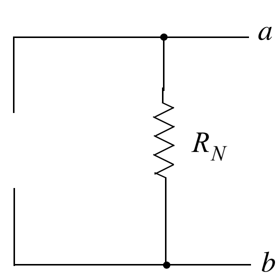
$$6i_X = 5i_X$$

but this can only be true if $i_X = 0$.

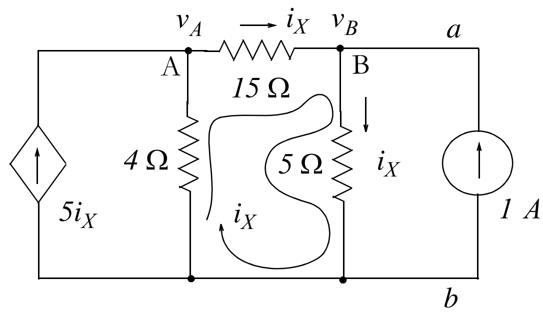
Then,

$$i_N = \frac{v_{OC}}{R_N} = \frac{v_{ab}}{R_N} = \frac{5 \times i_X}{R_N} = \frac{5 \times 0}{R_N} = 0$$

Thus, the Norton current source is open as shown below.



To find R_N we insert a $1\ A$ current source as shown below.



At Node A:

$$\frac{v_A}{4} + \frac{v_A - v_B}{15} = 5i_X$$

But

$$v_B = (5\ \Omega) \times i_X = 5i_X$$

and by substitution into the above relation

$$\frac{v_A}{4} + \frac{v_A - v_B}{15} = v_B$$

or

$$\frac{19}{60}v_A - \frac{16}{15}v_B = 0$$

At Node B:

$$\frac{v_B - v_A}{15} + \frac{v_B}{5} = 1$$

or

$$-\frac{1}{15}v_A + \frac{4}{15}v_B = 1$$

For this problem, we are interested only in the value of v_B which we could find by Gauss's elimination method. However, for convenience, we will use MATLAB for their solution.

$$\frac{19}{60}v_A - \frac{16}{15}v_B = 0$$

$$-\frac{1}{15}v_A + \frac{4}{15}v_B = 1$$

and in matrix form,

$$\underbrace{\begin{bmatrix} \frac{19}{60} & -\frac{16}{15} \\ -\frac{1}{15} & \frac{4}{15} \end{bmatrix}}_{G} \cdot \underbrace{\begin{bmatrix} v_A \\ v_B \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{I}$$

We find the voltages v_1 and v_2 with the following MATLAB code:

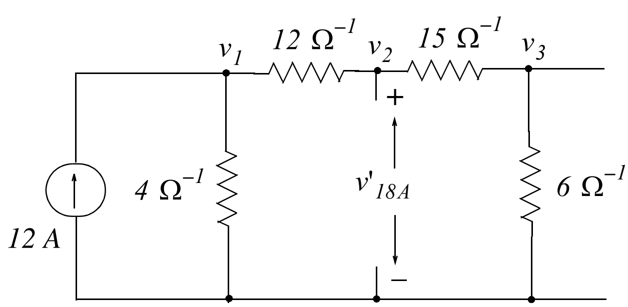
```
G=[19/60  −16/15;  −1/15  4/15];
I=[0  1]'; V=G\I;
fprintf('\n');...
  fprintf('vA=%7.2f V \t', V(1)); fprintf('vB=%7.2f V \t', V(2));
  fprintf('\n')
```

```
vA= 80.00 V   vB= 23.75 V
```

Now, we can find the Norton equivalent resistance from the relation

$$R_N = \frac{V_{ab}}{I_{SC}} = \frac{V_B}{1} = 23.75\ \Omega$$

11. This is the same circuit as that of Problem 1. Let v'_{18A} be the voltage due to the $12\ A$ current source acting alone. The simplified circuit with assigned node voltages is shown below where the parallel conductances have been replaced by their equivalents.



The nodal equations at the three nodes are

$$\begin{aligned} 16v_1 - 12v_2 \qquad\quad &= 12 \\ -12v_1 + 27v_2 - 15v_3 &= 0 \\ -15v_2 + 21v_3 &= 0 \end{aligned}$$

or

$$\begin{aligned} 4v_1 - 3v_2 \qquad\quad &= 3 \\ -4v_1 + 9v_2 - 5v_3 &= 0 \\ -5v_2 + 7v_3 &= 0 \end{aligned}$$

Since $v_2 = v'_{18A}$, we only need to solve for v_2. Adding the first 2 equations above and grouping with the third we obtain

$$\begin{aligned} 6v_2 - 5v_3 &= 3 \\ -5v_2 + 7v_3 &= 0 \end{aligned}$$

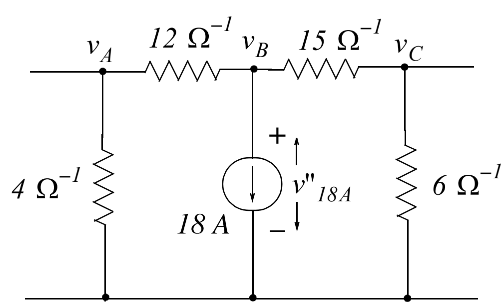
Multiplying the first by 7 and the second by 5 we get

$$\begin{aligned} 42v_2 - 35v_3 &= 21 \\ -25v_2 + 35v_3 &= 0 \end{aligned}$$

and by addition of these we get

$$v_2 = v'_{18A} = \frac{21}{17}\ V$$

Next, we let v''_{18A} be the voltage due to the $18\ A$ current source acting alone. The simplified circuit with assigned node voltages is shown below where the parallel conductances have been replaced by their equivalents.



The nodal equations at the three nodes are

$$\begin{aligned} 16v_A - 12v_B \qquad\quad &= 0 \\ -12v_A + 27v_B - 15v_C &= -18 \\ -15v_B + 21v_C &= 0 \end{aligned}$$

or

$$\begin{aligned} 4v_A - 3v_B \qquad\quad &= 0 \\ -4v_A + 9v_B - 5v_C &= -6 \\ -5v_B + 7v_C &= 0 \end{aligned}$$

Since $v_B = v''_{18A}$, we only need to solve for v_B. Adding the first 2 equations above and grouping with the third we obtain

$$\begin{aligned} 6v_B - 5v_C &= -6 \\ -5v_B + 7v_C &= 0 \end{aligned}$$

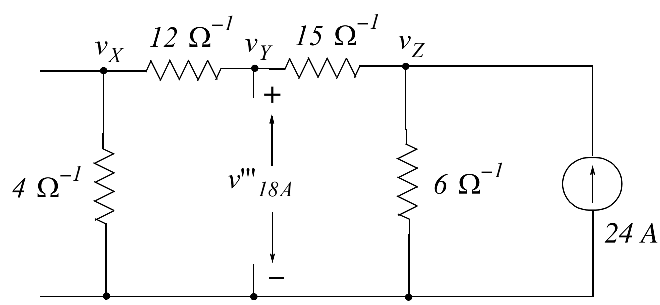
Multiplying the first by 7 and the second by 5 we get

$$\begin{aligned} 42v_B - 35v_C &= -42 \\ -25v_B + 35v_C &= 0 \end{aligned}$$

and by addition of these we get

$$v_B = v''_{18A} = \frac{-42}{17}\ V$$

Finally, we let v'''_{18A} be the voltage due to the $24\ A$ current source acting alone. The simplified circuit with assigned node voltages is shown below where the parallel conductances have been replaced by their equivalents.



The nodal equations at the three nodes are

$$\begin{aligned} 16v_X - 12v_Y \qquad\quad &= 0 \\ -12v_A + 27v_Y - 15v_Z &= 0 \\ -15v_B + 21v_Z &= 24 \end{aligned}$$

or

$$\begin{aligned} 4v_X - 3v_Y \quad\quad &= 0 \\ -4v_X + 9v_Y - 5v_Z &= 0 \\ -5v_Y + 7v_Z &= 8 \end{aligned}$$

Since $v_Y = v'''_{18A}$, we only need to solve for v_Y. Adding the first 2 equations above and grouping with the third we obtain

$$\begin{aligned} 6v_Y - 5v_Z &= 0 \\ -5v_Y + 7v_Z &= 0 \end{aligned}$$

Multiplying the first by 7 and the second by 5 we get

$$\begin{aligned} 42v_Y - 35v_Z &= 0 \\ -25v_Y + 35v_Z &= 40 \end{aligned}$$

and by addition of these we get

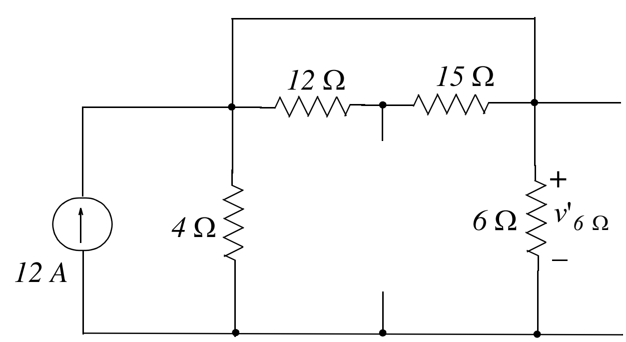
$$v_Y = v'''_{18A} = \frac{40}{17}\ V$$

and thus

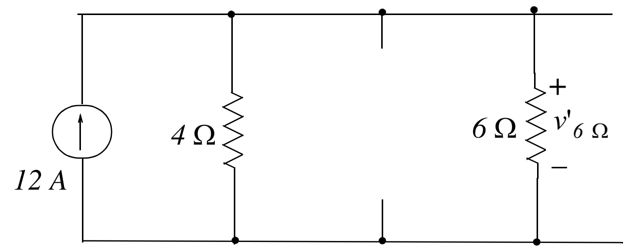
$$v_{18A} = v'_{18A} + v''_{18A} + v'''_{18A} = \frac{21}{17} + \frac{-42}{17} + \frac{40}{17} = \frac{19}{17} = 1.12\ V$$

This is the same answer as in Problem 1.

12. This is the same circuit as that of Problem 2. Let $v'_{6\ \Omega}$ be the voltage due to the $12\ A$ current source acting alone. The simplified circuit is shown below.



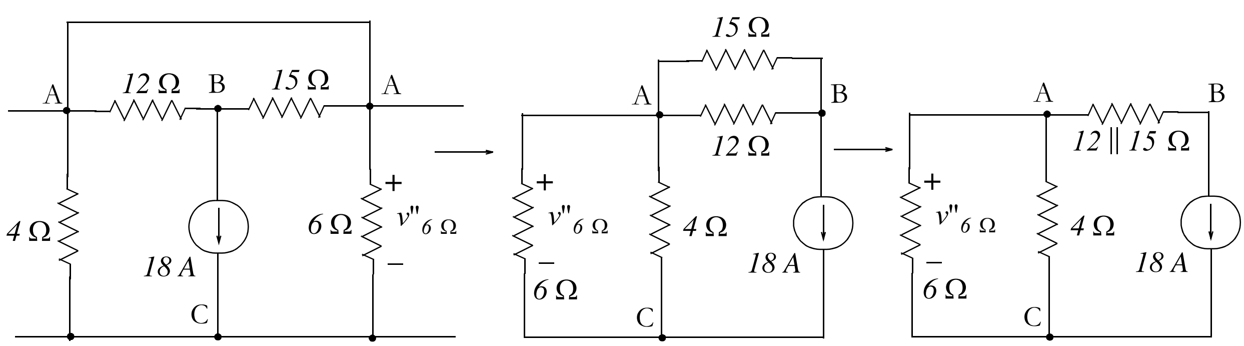
The $12\ \Omega$ and $15\ \Omega$ resistors are shorted out and the circuit is further simplified to the one shown below.



The voltage $v'_{6\ \Omega}$ is computed easily by application of the current division expression and multiplication by the $6\ \Omega$ resistor. Thus,

$$v'_{6\ \Omega} = \left(\frac{4}{4+6} \times 12\right) \times 6 = \frac{144}{5}\ V$$

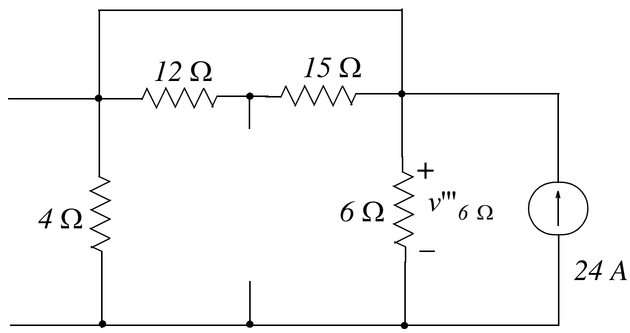
Next, we let $v''_{6\ \Omega}$ be the voltage due to the $18\ A$ current source acting alone. The simplified circuit is shown below. The letters A, B, and C are shown to visualize the circuit simplification process.



The voltage $v''_{6\ \Omega}$ is computed easily by application of the current division expression and multiplication by the $6\ \Omega$ resistor. Thus,

$$v''_{6\ \Omega} = \left[\frac{4}{4+6} \times (-18)\right] \times 6 = \frac{-216}{5}\ V$$

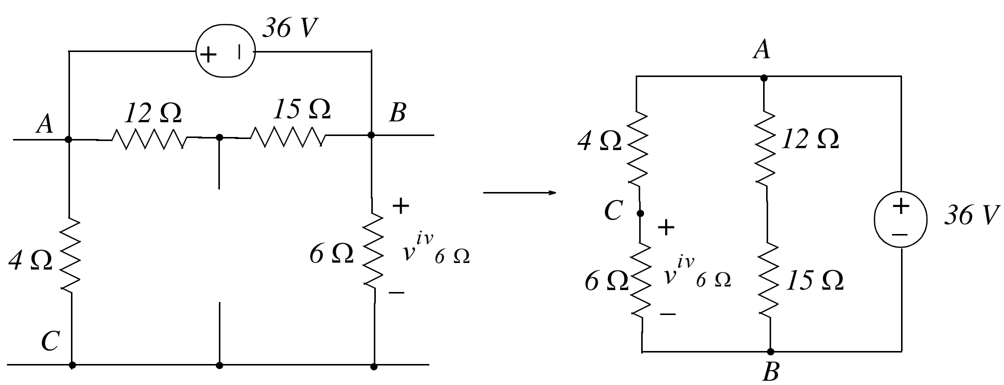
Now, we let $v'''_{6\ \Omega}$ be the voltage due to the $24\ A$ current source acting alone. The simplified circuit is shown below.



The $12\ \Omega$ and $15\ \Omega$ resistors are shorted out and voltage $v'''_{6\ \Omega}$ is computed by application of the current division expression and multiplication by the $6\ \Omega$ resistor. Thus,

$$v'''_{6\ \Omega} = \left(\frac{4}{4+6} \times 24\right) \times 6 = \frac{288}{5}\ V$$

Finally, we let $v^{iv}_{6\ \Omega}$ be the voltage due to the $36\ V$ voltage source acting alone. The simplified circuit is shown below.



By application of the voltage division expression we find that

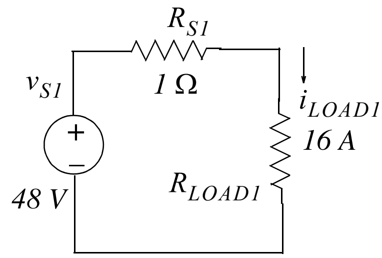
$$v^{iv}_{6\ \Omega} = \frac{6}{4+6} \times (-36) = -\frac{108}{5}$$

Therefore,

$$v_{6\ \Omega} = v'_{6\ \Omega} + v''_{6\ \Omega} + v'''_{6\ \Omega} + v^{iv}_{6\ \Omega} = \frac{144}{5} - \frac{216}{5} + \frac{288}{5} - \frac{108}{5} = \frac{108}{5} = 21.6\ V$$

This is the same answer as that of Problem 2.

13. The circuit for Measurement 1 is shown below.



Let $R_{eq1} = R_{S1} + R_{LOAD1}$. Then,

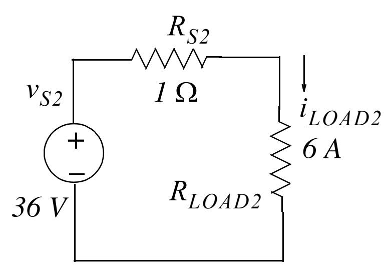
$$R_{eq1} = \frac{v_{S1}}{i_{LOAD1}} = \frac{48}{16} = 3\ \Omega$$

For Measurement 3 the load resistance is the same as for Measurement 1 and the load current is given as $-5\ A$. Therefore, for Measurement 3 we find that

$$v_{S1} = R_{eq1}(-5) = 3 \times (-5) = -15\ V$$

and we enter this value in the table below.

The circuit for Measurement 2 is shown below.



Let $R_{eq2} = R_{S1} + R_{LOAD2}$. Then,

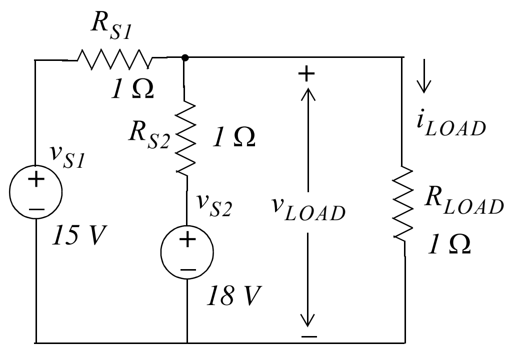
$$R_{eq2} = \frac{v_{S2}}{i_{LOAD2}} = \frac{36}{6} = 6\ \Omega$$

For Measurement 4 the load resistance is the same as for Measurement 2 and v_{S2} is given as $-42\ V$. Therefore, for Measurement 4 we find that

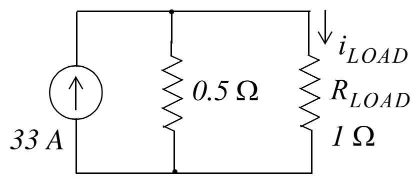
$$i_{LOAD2} = \frac{v_{S2}}{R_{eq2}} = -\frac{42}{6} = -7\ \mathrm{A}$$

and we enter this value in the table below.

The circuit for Measurement 5 is shown below.



Replacing the voltage sources with their series resistances to their equivalent current sources with their parallel resistances and simplifying, we get the circuit below.

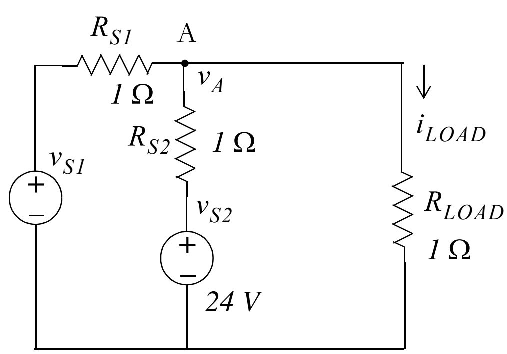


Application of the current division expression yields

$$i_{LOAD} = \frac{0.5}{0.5+1} \times 33 = 11\ A$$

and we enter this value in the table below.

The circuit for Measurement 6 is shown below.



We observe that i_{LOAD} will be zero if $v_A = 0$ and this will occur when $v_{S1} = -24$. This can be shown to be true by writing a nodal equation at Node A. Thus,

$$\frac{v_A - (-24)}{1} + \frac{v_A - 24}{1} + 0 = 0$$

or $v_A = 0$

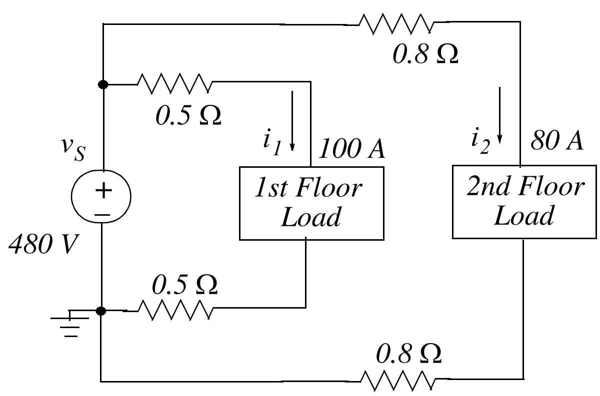
Measurement	Switch S_1	Switch S_2	v_{S1} (V)	v_{S2} (V)	i_L (A)
1	Closed	Open	48	0	16
2	Open	Closed	0	36	6
3	Closed	Open	-15	0	–5
4	Open	Closed	0	–42	–7
5	Closed	Closed	15	18	11
6	Closed	Closed	–24	24	0

14. The power supplied by the voltage source is

$$p_S = v_S(i_1 + i_2) = 480(100 + 80) = 86,400\ w = 86.4\ Kw$$

The power loss on the 1st floor is

$$p_{LOSS1} = i_1^2(0.5 + 0.5) = 100^2 \times 1 = 10,000\ w = 10\ Kw$$



The power loss on the 2nd floor is

$$p_{LOSS2} = i_2^2(0.8 + 0.8) = 80^2 \times 1.6 = 10,240\ w = 10.24\ Kw$$

and thus the total loss is

$$Total\ loss = 10 + 10.24 = 20.24\ Kw$$

Then,

$$Output\ power = Input\ power - power\ losses = 86.4 - 20.24 = 66.16Kw$$

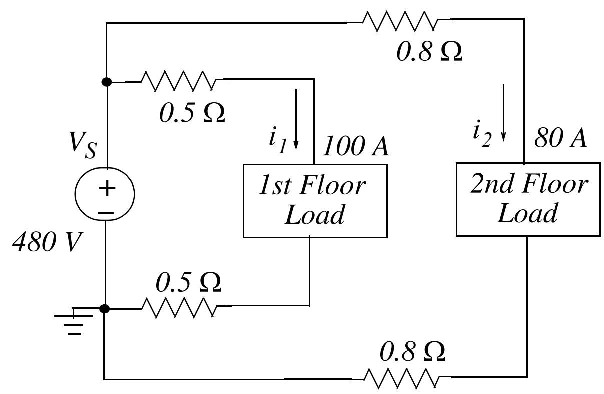
and

$$\%\ Efficiency = \eta = \frac{Output}{Input} \times 100 = \frac{66.16}{86.4} \times 100 = 76.6\%$$

This is indeed a low efficiency.

15. The voltage drop on the second floor conductor is

$$v_{cond} = R_T i_2 = 1.6 \times 80 = 128\ V$$



and thus the full-load voltage is

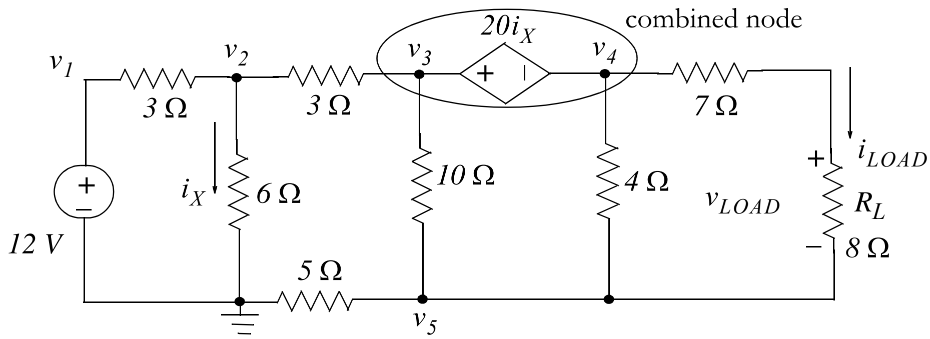
$$v_{FL} = 480 - 128 = 352\ V$$

Then,

$$\%\ Regulation = \frac{v_{NL} - v_{FL}}{v_{FL}} \times 100 = \frac{480 - 352}{352} \times 100 = 36.4\%$$

This is a very poor regulation.

16. We assign node voltages and we write nodal equations as shown below.



$$v_1 = 12$$

$$\frac{v_2 - v_1}{3} + \frac{v_2}{6} + \frac{v_2 - v_3}{3} = 0$$

$$\frac{v_3 - v_2}{3} + \frac{v_3 - v_5}{10} + \frac{v_4 - v_5}{4} + \frac{v_4 - v_5}{7+8} = 0$$

$$v_3 - v_4 = 20i_X$$

where $i_X = \dfrac{v_2}{6}$ and thus

$$v_5 = \frac{10}{3}v_2$$

$$\frac{v_5}{5} + \frac{v_5 - v_3}{10} + \frac{v_5 - v_4}{4} + \frac{v_5 - v_4}{7+8} = 0$$

Collecting like terms and rearranging we get

$$\begin{aligned}
v_1 \qquad\qquad\qquad\qquad &= 12 \\
\frac{-1}{3}v_1 + \frac{5}{6}v_2 + \frac{-1}{3}v_3 \qquad\qquad &= 0 \\
\frac{-1}{3}v_2 + \frac{13}{30}v_3 + \frac{19}{60}v_4 - \frac{19}{60}v_5 &= 0 \\
-\frac{10}{3}v_2 \quad + v_3 \quad - v_4 \qquad\qquad &= 0 \\
-\frac{1}{10}v_3 - \frac{19}{60}v_4 + \frac{37}{60}v_5 &= 0
\end{aligned}$$

and in matrix form

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & \frac{5}{6} & \frac{-1}{3} & 0 & 0 \\ 0 & \frac{-1}{3} & \frac{13}{30} & \frac{19}{60} & -\frac{19}{60} \\ 0 & -\frac{10}{3} & 1 & -1 & 0 \\ 0 & 0 & -\frac{1}{10} & -\frac{19}{60} & \frac{37}{60} \end{bmatrix}}_{G} \cdot \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{I}$$

We will use MATLAB to solve the above.

```
 G=[1  0  0  0  0;...
–1/3  5/6  –1/3  0  0;...
0  –1/3  13/30  19/60  –19/60;...
0  –10/3  1  –1  0;...
0  0  –1/10  –19/60  37/60];
 I=[12  0  0  0  0]'; V=G\I;
 fprintf('\n');...
   fprintf('v1 = %7.2f V \n',V(1));...
   fprintf('v2 = %7.2f V \n',V(2));...
   fprintf('v3 = %7.2f V \n',V(3));...
   fprintf('v4 = %7.2f V \n',V(4));...
   fprintf('v5 = %7.2f V \n',V(5));...
   fprintf('\n'); fprintf('\n')
```

```
v1 =    12.00 V
v2 =    13.04 V
v3 =    20.60 V
v4 =   -22.87 V
v5 =    -8.40 V
```

Now,

$$i_{LOAD} = \frac{v_4 - v_5}{8+7} = \frac{-22.87-(-8.40)}{15} = -0.96\ A$$

and

$$v_{LOAD} = 8i_{LOAD} = 8 \times (-0.96) = -7.68\ V$$