Chapter 10: Sinusoidal Steady-State Analysis

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10.1 Basic Approach

• 3 Steps to Analyze AC Circuits:
  1. **Transform** the circuit to the **phasor or frequency domain**.
  2. **Solve** the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
  3. **Transform** the resulting phasor to the time domain.

- Sinusoidal Steady-State Analysis:
  Frequency **domain analysis** of AC circuit via phasors is much easier than analysis of the circuit in the time domain.
10.2 Nodal Analysis

The basic of Nodal Analysis is \textit{KCL}.

Example: Using nodal analysis, find \(v_1\) and \(v_2\) in the figure.
10.3 Mesh Analysis

The basic of Mesh Analysis is KVL.

**Example:** Find \( I_o \) in the following figure using mesh analysis.
**Table**

**Node Voltage Analysis Using the Phasor Concept to Find the Sinusoidal Steady-State Node Voltages**

1. Convert the independent sources to phasor form.
2. Select the nodes and the reference node and label the node voltages in the time domain, $V_n$, and their corresponding phasor voltages, $V_n$.
3. If the circuit contains only independent current sources, proceed to step 5; otherwise, proceed to step 4.
4. If the circuit contains a voltage source, select one of the following three cases and the associated method:
   
   **CASE**
   
   **a.** The voltage source connects node $q$ and the reference node.
   **b.** The voltage source lies between two nodes.
   **c.** The voltage source in series with an impedance lies between node $d$ and the ground, with its positive terminal at node $d$.

   **METHOD**
   
   Set $V_q = V_s$ and proceed.
   Create a supernode including both nodes.
   Replace the voltage source and series impedance with a parallel combination of an admittance $Y_1 = 1/Z_1$ and a current source $I_1 = V_S Y_1$ entering node $d$.

5. Using the known frequency of the sources, $\omega$, find the impedance of each element in the circuit.
6. For each branch at a given node, find the equivalent admittance of that branch, $Y_n$.
7. Write KCL at each node.
8. Solve for the desired node voltage $V_n$, using Cramer’s rule.
9. Convert the phasor voltage $V_n$ back to the time-domain form.

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**Table**

**Mesh Current Analysis Using the Phasor Concept to Find the Sinusoidal Steady-State Mesh Currents**

1. Convert the independent sources to phasor form.
2. Select the mesh currents and label the currents in the time domain, $i_n$, and the corresponding phasor currents, $I_n$.
3. If the circuit contains only independent voltage sources, proceed to step 5; otherwise, proceed to step 4.
4. If the circuit contains a current source, select one of the following two cases and the associated method:
   
   **CASE**
   
   **a.** The current source appears as an element of only one mesh, $n$.
   **b.** The current source is common to two meshes.

   **METHOD**
   
   Equate the mesh current $I_n$ to the current of the current source, accounting for the direction of the source current.
   Create a supermesh as the periphery of the two meshes. In step 6, write one KVL equation around the periphery of the supermesh. Also record the constraining equation incurred by the current source.

5. Using the known frequency of the sources, $\omega$, find the impedance of each element in the circuit.
6. Write KVL for each mesh.
7. Solve for the desired mesh current $I_n$, using Cramer’s rule.
8. Convert the phasor current $I_n$ back to the time-domain form.


10.4 Superposition Theorem

When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

**Example:** Calculate $v_o$ in the circuit using the superposition theorem.

![Circuit Diagram](image)
4.3 Superposition Theorem (1)

- **Superposition** states that the **voltage across** (or current through) an element in a linear circuit is the **algebraic sum** of the voltage across (or currents through) that element due to **EACH independent source acting alone**.

- The principle of superposition helps us to analyze a linear circuit with more than one independent source by **calculating the contribution of each independent source separately**.

- **Steps to Apply Superposition Principle:**
  1. **Turn off** all indep. sources except one source. Find the output (\(v\) or \(i\)) due to that active source using techniques in Chapters 2 & 3.
  2. **Repeat** Step 1 for each of the other indep. sources.
  3. Find **total** contribution by adding all contributions from indep. sources.

*Note*: In Step 1, this implies that we replace every **voltage source by 0 V** (or a **short circuit**), and **every current source by 0 A** (or an **open circuit**). Dependent sources are left intact because they are controlled by others.
10.5 Source Transformation (1)

\[ V_s = Z_s I_s \quad \Leftrightarrow \quad I_s = \frac{V_s}{Z_s} \]

Example: Find \( I_o \) using the concept of source transformation.

\[ 20 \angle -90^\circ \text{ V} \]

\[ 5 \Omega \quad 4 \Omega \quad -j13 \Omega \]

\[ 3 \Omega \quad j4 \Omega \quad 10 \Omega \]

\[ + \quad V_x \quad - \]
10.5 Source Transformation (2)

Method
Set \( I_s = \frac{V_s}{Z_s} \)

\( Z_s \) is the same in both circuits

Method
Set \( V_s = I_s Z_s \)

\( Z_s \) is the same in both circuits
4.4 Source Transformation (1)

- Like series-parallel combination and wye-delta transformation, source transformation is another tool for simplifying circuits.
- An equivalent circuit is one whose v-i characteristics are identical with the original circuit.
- A source transformation is the process of replacing a voltage source $v_s$ in series with a resistor $R$ by a current source $i_s$ in parallel with a resistor $R$, and vice versa.

- **Transformation of independent sources**
  - The arrow of the current source is directed toward the positive terminal of the voltage source.

  ![Diagram of independent source transformation]

- **Transformation of dependent sources**
  - The source transformation is not possible when $R = 0$ for voltage source and $R = \infty$ for current source.
A voltage source $v_s$ connected in series with a resistor $R_s$ and a current source $i_s$ is connected in parallel with a resistor $R_p$ are equivalent circuits provided that

$$R_p = R_s \quad \text{and} \quad v_s = R_s i_s$$
10.6 Thevenin & Norton Equivalent Circuits

\[ V_{Th} = Z_N I_N, \quad Z_{Th} = Z_N \]

**Thevenin Equivalent**

**Norton Equivalent**

**Example:** Find the Thevenin equivalent at terminals \( a-b \).
4.5 Thevenin’s Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source $V_{Th}$ in series with a resistor $R_{Th}$, where

- $V_{Th}$ is the open-circuit voltage at the terminals.
- $R_{Th}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.
To find $R_{Th}$:

**Case 1:** If the network has no dependent sources, we turn off all indep. Source. $R_{Th}$ is the input resistance of the network looking btw terminals $a$ & $b$.

**Case 2:** If the network has depend. Sources. Depend. sources are not to be turned off because they are controlled by circuit variables. (a) Apply $v_o$ at $a$ & $b$ and determine the resulting $i_o$. Then $R_{Th} = v_o/i_o$. Alternatively, (b) insert $i_o$ at $a$ & $b$ and determine $v_o$. Again $R_{Th} = v_o/i_o$.

![Circuit Diagrams](image)

(a) Circuit with all independent sources set equal to zero

$$R_{Th} = \frac{v_o}{i_o}$$

(b) Circuit with all independent sources set equal to zero

$$R_{Th} = \frac{v_o}{i_o}$$
4.6 Norton’s Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a current source $I_N$ in parallel with a resistor $R_N$,

where

- $I_N$ is the short-circuit current through the terminals.
- $R_N$ is the input or equivalent resistance at the terminals when the indepen. sources are turned off.

$R_N = R_{Th}$

$R_{Th} = R_{in}$

Linear circuit with all independent sources set equal to zero
The key to analyzing op amp circuits is to keep two important properties of an ideal op amp in mind:
• No current enters either of its input terminals.
• The voltage across its input terminals is zero.

Example: Compute the closed-loop gain and phase shift. Assume that $R_1 = R_2 = 10 \, \text{k}\Omega$, $C_1 = 2 \, \mu\text{F}$, $C_2 = 1 \, \mu\text{F}$, and $\omega = 200 \, \text{rad/s}$.

\[ Z_f = R_2 \left\| \frac{1}{j\omega C_2} \right\| = \frac{R_2}{1 + j\omega R_2 C_2} \]

\[ Z_i = R_1 + \frac{1}{j\omega C_1} = \frac{1}{j\omega C_1} \]

\[ G = \frac{V_o}{V_s} = \frac{Z_f}{Z_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)} \]

Substituting the given values of $R_1$, $R_2$, $C_1$, $C_2$, and $\omega$, we obtain

\[ G = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434 \angle 130.6^\circ \]

Thus the closed-loop gain is 0.434 and the phase shift is 130.6°.
With the pervasive use of ac electric power in the home and industry, it is important for engineers to analyze circuits with sinusoidal independent sources.

The steady-state response of a linear circuit to a sinusoidal input is itself a sinusoid having the same frequency as the input signal.

Circuits that contain inductors and capacitors are represented by differential equations. When the input to the circuit is sinusoidal, the phasors and impedances can be used to represent the circuit in the frequency domain. In the frequency domain, the circuit is represented by algebraic equations.

The steady-state response of a linear circuit with a sinusoidal input is obtained as follows:
1. Transform the circuit into the frequency domain, using phasors and impedances.
2. Represent the frequency-domain circuit by algebraic equation, for example, mesh or node equations.
3. Solve the algebraic equations to obtain the response of the circuit.
4. Transform the response into the time domain, using phasors.

- A circuit contains several sinusoidal sources, two cases:
  - When all of the sinusoidal sources have the same frequency, the response will be a sinusoid with that frequency, and the problem can be solved in the same way that it would be if there was only one source.
  - When the sinusoidal sources have different frequencies, superposition is used to break the time-domain circuit up into several circuits, each with sinusoidal inputs all at the same frequency. Each of the separate circuits is analyzed separately and the responses are summed in the time domain.