

Name: _____

Application of Trigonometry in Engineering¹

1.1 Laboratory (Homework) Objective

The objective of this laboratory is to learn basic trigonometric functions, conversion from rectangular to polar form, and vice-versa.

1.2 Educational Objectives

After performing this experiment, students should be able to:

1. Apply appropriate basic trigonometric functions to problems.
2. Understand the concept of a unit circle
3. Recognize four quadrants.
4. Determine a proper reference angle.
5. Perform the polar to rectangular and rectangular to polar coordinate conversion.
6. Prove a few of the basic trigonometric identities.

1.3 Background

Trigonometry is a tool that mathematically forms geometrical relationships. The understanding and application of these relationships are vital for all engineering disciplines. Relevant applications include automotive, aerospace, robotics, and building design. This lab will outline a few common, but useful, trigonometric relationships.

1.3.1 Reference Angle

A reference angle is an acute angle (less than 90°) that may be used to compute the trigonometric functions of the corresponding obtuse angle (greater than 90°). Figure 1 shows the reference angle ϕ with respect to the angle θ . The reference angle is calculated using the formulas shown in the captions of each corresponding subfigure of Figure 1.

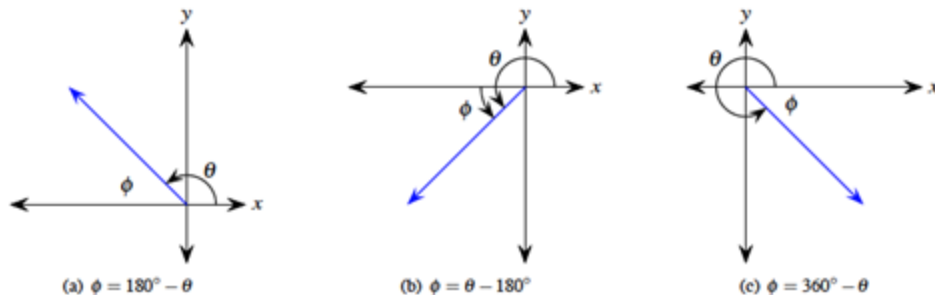


Figure 1. Reference angles in different quadrants.

¹ Adapted from The Wright State Model for Engineering Mathematics Course Laboratory.

1.3.2 Law of Cosines

The Law of cosines is a method that helps to solve triangles. Equations 1 relate the sides and interior angles of Figure 2.

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos(A) \\b^2 &= a^2 + c^2 - 2ac \cos(B) \\c^2 &= a^2 + b^2 - 2ab \cos(C)\end{aligned}\tag{1}$$

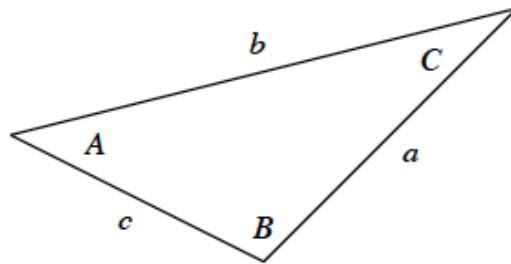


Figure 2. Law of cosines triangle.

1.3.3 Law of Sines

The Law of Sines is another method that helps to solve triangles. Using the triangle of Figure 2, Equation 2 relates the sides to the interior angles.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}\tag{2}$$

1.4 Assignment and procedures

Follow the steps outlined below. Note that you will be using a virtual “board” that is available here: <https://distance.cecs.wright.edu/#/egr1010/lab2> .

1.4.1 One Link Robot

1. **Using the virtual board** fill in Table 1 and Table 2. Pay close attention to the sign of your answer for all values. Make sure that “One-link” option is selected.

NOTE: To convert a value in degrees to radians, the multiplying factor is $\pi/180$.

2. Use equations 3 to find the calculated x and y values in Table 2. Hint: consider using fractions of π to express angles in radians, e.g. $30^\circ = \pi/6$.

$$x = l \cos(\theta) \quad \text{and} \quad y = l \sin(\theta)\tag{3}$$

Table 1. Polar to rectangular conversion – part 1.

Angle θ°	l (mm)	Measured x (mm)	Measured y (mm)
30	100		
45	100		
90	100		
135	100		
180	100		
225	100		
270	100		

Table 2. Polar to rectangular conversion – part 2.

Angle θ°	l (mm)	Reference Angle ($^\circ$)	Reference Angle (rad)	Calculated x (mm)	Calculated y (mm)
30	100				
45	100				
90	100				
135	100				
180	100				
225	100				
270	100				

3. Using the virtual board fill in Table 3.

4. Use Equations 4 to find the Calculated θ and l .

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \quad \text{and} \quad l = \sqrt{x^2 + y^2} \quad (4)$$

Table 3. Rectangular to polar conversion.

(x,y)	Measured $\theta(^\circ)$	Reference Angle ($^\circ$)	Reference Angle (rad)	Calculated $\theta(^\circ)$	Calculated l (mm)	Polar Form $l\angle\theta$
(85,50)						
(70,70)						
(0,100)						
(-70,70)						
(-100,0)						
(-70,-70)						

1.4.2 Trigonometric Identities

An identity is a trigonometric relationship that is true for all permissible values of the variable(s). Many times, trigonometric identities are used to simplify more complex problems.

1. Using MATLAB, fill in Table 4 and Table 5.
 - a. The first column of Table 4 comes from Table 3.
 - b. Define this column as a vector in MATLAB and perform element by element calculations on it to get the other columns.

NOTE: All calculations should be done with MATLAB. No calculator use! MATLAB functions are: `sind()`, `cosd()`, `tand()`, `secd()`. If you are using vector `x` to store more than one variable then you can simply use `sind(x)`, `cosd(x)`, `tand(x)`, `secd(x)`. However, for division, multiplication and power operations you should use the dot “.” operator. For example, `sind(x)./cosd(x)` and `sind(x).^2 + cosd(x).^2`.

Table 4. Using MATLAB to calculate trigonometric functions - part 1.

Calculated $\theta(^{\circ})$	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\sec(\theta)$

Table 5. Using MATLAB to calculate trigonometric functions - part 2.

$\frac{\sin(\theta)}{\cos(\theta)}$	$\sin^2(\theta) + \cos^2(\theta)$	$1 + \tan^2(\theta)$	$\sec^2(\theta)$

1.4.3 Two Link Robot

1. **Using the virtual board** fill in the Measured Values columns of Table 6. Make sure that “Two-link” option is selected. NOTE: $l_1 = l_2 = 50\text{mm}$
2. Write a MATLAB code to calculate X and Y in the last two columns by adding the components of each link. Recall the following equations from class. Remember to use `.*` for multiplication!

$$x_1 = l_1 \cos(\theta_1)$$

$$y_1 = l_1 \sin(\theta_1)$$

$$x_2 = l_2 \cos(\theta_1 + \theta_2)$$

$$y_2 = l_2 \sin(\theta_1 + \theta_2)$$

$$X = x_1 + x_2$$

$$Y = y_1 + y_2$$

Table 6. Data for two-link robot arm.

		Measured Values						Calculated Values	
$\theta_1(^{\circ})$	$\theta_2(^{\circ})$	x_1	y_1	x_2	y_2	$X(=x_1+x_2)$	$Y(=y_1+y_2)$	X	Y
0	0								
0	90								
30	45								
30	60								
180	0								
270	30								
360	90								

1.4.4 Solve a Triangle Using Law of Cosines and Law of Sines

In some cases, the laws of sines and cosines must both be used to solve a triangle. Figure 3 is one such case where the lengths l_1 and l_2 along with the final ending point P of the two links are known and the θ values are not. Both laws are needed to solve this triangle.

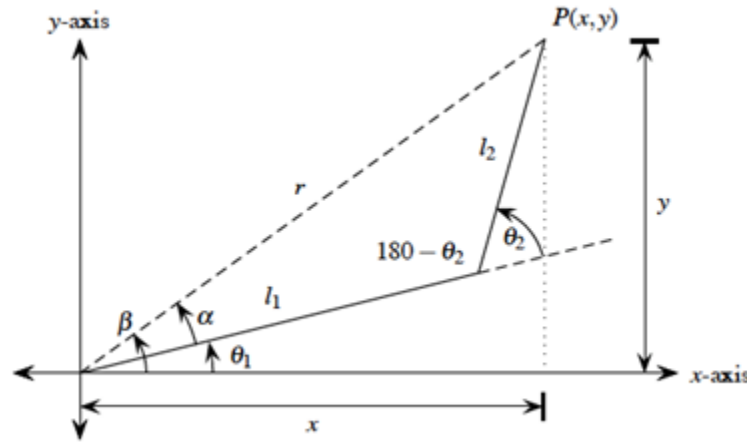


Figure 3. General two-link robot arm problem.

1. The radius r is found by:

$$r = \sqrt{x^2 + y^2} \quad (5)$$

2. Using the Law of Cosines, θ_2 is found by the following equation:

$$r^2 = l_1^2 + l_2^2 - 2l_1l_2\cos(180 - \theta_2)$$

$$\theta_2 = 180 - \cos^{-1}\left(\frac{r^2 - l_1^2 - l_2^2}{-2l_1l_2}\right) \quad (6)$$

3. Using the Law of Sines, α is found by the following equation:

$$\frac{\sin(\theta_2)}{r} = \frac{\sin(\alpha)}{l_2}$$

$$\alpha = \sin^{-1}\left(\frac{l_2 \sin(\theta_2)}{r}\right) \quad (7)$$

4. θ_1 is now found by the equation:

$$\beta = \tan^{-1}\left(\frac{y}{x}\right) \quad (8)$$

$$\theta_1 = \beta - \alpha \quad (9)$$

Table 7. Application of sine and cosine laws.

P(x,y)	$\theta_2(^{\circ})$	$\alpha(^{\circ})$	$\beta(^{\circ})$	$\theta_1 (^{\circ})$
(55, 75)				
(75, 60)				
(15, 63)				
(32, 14)				
(71, 70)				

1.5 Submission requirements

1. Complete Tables 1 to 7.
2. Show a sample of hand calculations for Tables 1, 2 and 3. Insert after this page by including a photo of it.
3. Combine all MATLAB code into one m-file for Tables 4 to 7. It should be possible to run this file without any modifications and generate the numbers you entered into the tables. Upload the m-file on D2L
4. Answer the following questions.
 - a) Based on your results for Tables 4 and 5, write down the three trigonometric identities that were verified?
