Problem 3:
Given the following loop gain:

\[ T(s) = \frac{A(1 + \frac{s}{\omega_1})}{(1 + \frac{s}{Q\omega_2} + \frac{s^2}{\omega_2^2})} \]

where

\[ A = 50 \]
\[ \omega_1 = 150 \text{ rds/s} \]
\[ \omega_2 = 8 \text{ rds/s} \]
\[ Q = 3 \]

Using asymptotic approximations only,

a) Sketch the Bode magnitude and phase plots (on the following blank page). Be sure to label all break frequencies, slopes of sloping line, gains of sloping lines and gain and phase levels on zero slope lines.

b) Find the maximum gain (as an absolute value) and the frequency or range of frequencies at which it occurs.

c) Using your plots determine the phase margin and associated crossover frequency.

d) Using your plots determine the gain margin and associated crossover frequency.

e) Determine whether the closed loop system is stable.

f) With the assumption that \( \omega_1 \) may be moved, what value should it take to achieve a 45° phase margin.

\[ \phi = -180 + \arctan\left(\frac{\omega_c}{\omega_1}\right) \]
\[ \Rightarrow \omega_1 = \frac{\omega_2 \sqrt{A}}{4\pi + 45} \]
\[ = \omega_2 \sqrt{A} \]
\[ = 56.57 \text{ rds/sec} \]

\[ \phi = -180 + \arctan\left(\frac{\omega_c}{\omega_1}\right) \]
\[ \Rightarrow \phi = -135° \]
\[ \Rightarrow -135° = -180 + \arctan\left(\frac{\omega_2 \sqrt{A}}{\omega_1}\right) \]
b) Max gain occurs at $w_2 = 8 \text{ kHz}$.

$A_A = 3 \times 50 = 150$

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Problem 9: \[ T(s) = \frac{10}{s^2 + 3s + 10}, \quad r(t) = 8 \cos(2t + 70^\circ) \]

\[ \omega = 2 \]

\[ F(\omega) \bigg|_{\omega = 2} = 1.18 \quad \text{(From MATLAB or calculation)} \]

\[ \theta = -45^\circ \]

Input amp. = 8 \quad \Rightarrow \quad \text{Output amp.} = 8 \times 1.18

\[ = 9.43 \]

Input phase = 70 \quad \Rightarrow \quad \text{Output phase} = 70 - 45^\circ

\[ = 25^\circ \]

\[ \Rightarrow \quad y(t) = 9.43 \cos(2t + 25^\circ) \]