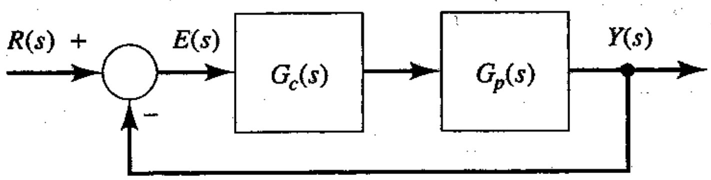
PID CONTROLLERS

P – PROPORTIONAL

I – INTEGRAL

D – DERIVATIVE



$G_p(s)$ – PLANT

$G_c(s)$ – CONTROLLER (COMPENSATOR)

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

PROPORTIONAL TERM — INTEGRAL TERM — DERIVATIVE TERM

3 TERM (CONTROLLER)

TWO APPROACHES TO PID DESIGN

1) ZIEGLER – NICHOLS

2) CHIEN – HRONES – RESWICK (CHR)

ZIEGLER – NICHOLS COMPENSATION

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

$$= \frac{K_d s^2 + K_p s + K_i}{s}$$

$$= K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$K_i = \frac{K_p}{T_i} \quad \text{AND} \quad K_d = K_p T_d$$

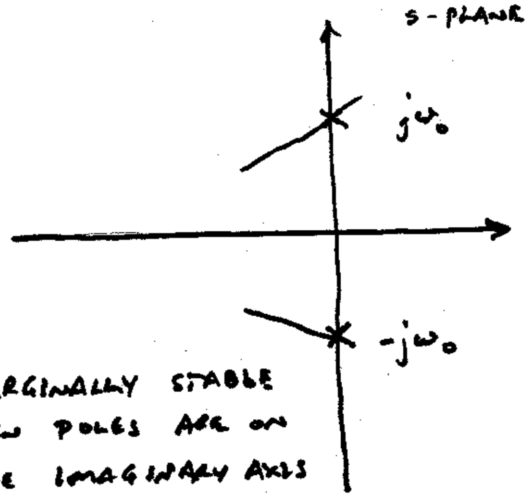
DESIGN PROCEDURE

1) SET $G_c(s) = K_p$

AND VARY K_p UNTIL MARGINALLY STABLE AT K_{po}

– PERIOD OF OSCILLATION IS T_o WHERE

$$T_o = \frac{1}{f_o} = \frac{2\pi}{\omega_o}$$



MARGINALLY STABLE WHEN POLES ARE ON THE IMAGINARY AXIS

2) USE K_{po} AND T_o TO FIND CONTROLLER PARAMETERS:

Table *Ziegler–Nichols Compensation*

Compensator	Values
P	$K_p = 0.5\,K_{po}$
PI	$K_p = 0.45\,K_{po}$ $T_i = 0.83\,T_o$
PID	$K_p = 0.6\,K_{po}$ $T_i = 0.5\,T_o$ $T_d = 0.125\,T_o$

EXAMPLE

$$G_p(s) = \frac{64}{s^3 + 14s^2 + 56s + 64}$$

1) LET $G_c(s) = K_p$ AND FIND K_{po} (THE VALUE OF K_p FOR WHICH CLOSED LOOP SYSTEM IS MARGINALLY STABLE)

$$\text{CLOSED LOOP GAIN} = \frac{Y}{R} = \frac{G_p G_c}{1 + G_p G_c}$$

$$= \frac{64}{s^3 + 14s^2 + 56s + 64(1+K_p)}$$

for $G_c = K_p$

ROUTH - HURWITZ

s^3	1	56
s^2	14	$64(1+K_p)$
s^1	x	0
s^0	$64(1+K_p)$	

$\Longrightarrow$ IMAGINARY AXIS ROOTS EXIST WHEN $x = 0$

i.e. $K = K_{po} = 11.25$

$$x = \frac{784 - 64(1+K_p)}{14}$$

FOR $K_p = K_{po} = 11.25$ ROUTH-HURWITZ TABLE BECOMES:

$$\begin{array}{c|cc} s^3 & 1 & 56 \\ s^2 & 14 & 784 \\ s^1 & 0 & 0 \\ s^0 & 784 & \end{array}$$

POLYNOMIAL DIVISOR

$$P_{DIV}(s) = 14s^2 + 784$$
$$= 14(s^2 + 56)$$
$$= 14(s + j7.483)(s - j7.483)$$

$\Rightarrow$ OSCILLATION FREQUENCY $\omega_o = 7.483$

$\Rightarrow$ OSCILLATION PERIOD $T_o = \frac{2\pi}{\omega_o} = \frac{2\pi}{7.483} = 0.8397$

$\therefore$ WITH $K_{po} = 11.25$ AND $T_o = 0.8397$ THE

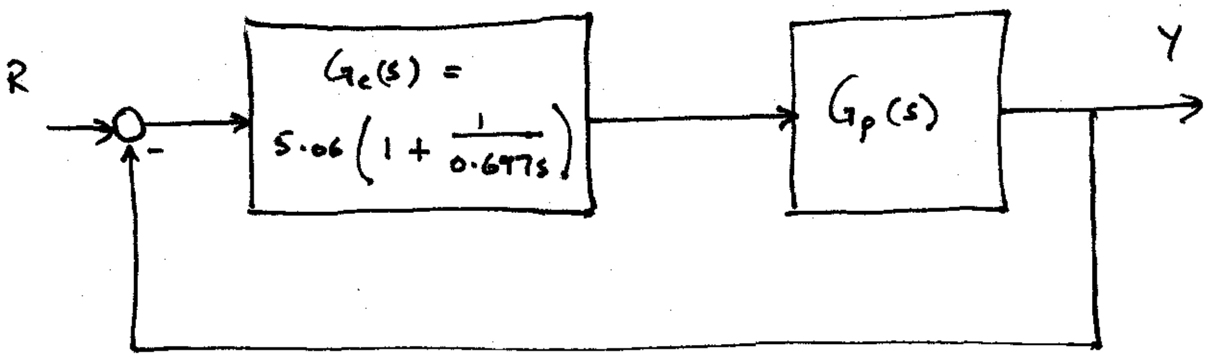
VARIOUS COMPENSATORS ARE:

Compensator	Values
P	$K_p = 0.5\,K_{po} = 5.63$
PI	$K_p = 0.45\,K_{po} = 5.06$ $T_i = 0.83\,T_o = 0.697$
PID	$K_p = 0.6\,K_{po} = 6.75$ $T_i = 0.5\,T_o = 0.420$ $T_d = 0.125\,T_o = 0.105$

FOR EXAMPLE, FOR PI COMPENSATOR

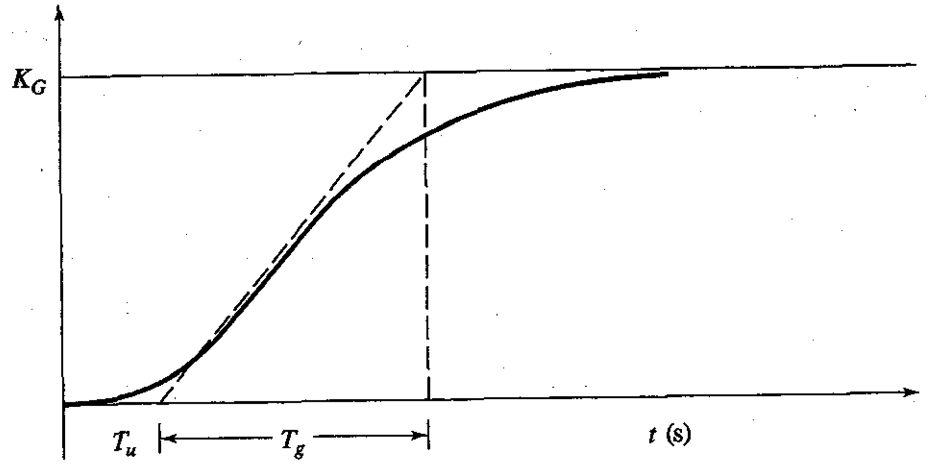
$$G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$

$$= 5.06\left(1 + \frac{1}{0.697s}\right)$$



CHIEN – HRONES – RESWICK COMPENSATION

METHOD: 1) OBTAIN THE UNIT STEP RESPONSE OF THE OPEN-LOOP PLANT AND DRAW A LINE THROUGH THE LINEAR PORTION OF THE RESPONSE



K_g IS THE FINAL VALUE OF THE STEP RESPONSE

2) DEFINE $R \triangleq \frac{T_g}{T_u}$ AND USE IT ALONG WITH K_g, T_g AND T_u TO FIND COMPENSATOR PARAMETER VALUES:

Table *Chien–Hrones–Reswick Compensator*

(a) Values for $R = T_g/T_u$

Compensator	R
P	$R > 10$
PI	$7.5 < R < 10$
PID	$3 < R < 7.5$
Higher order	$R < 3$

(b) CHR Compensation

Compensator	Overdamped	20% Overshoot
P	$K_p = 0.3R/K_g$	$K_p = 0.7R/K_g$
PI	$K_p = 0.35R/K_g$ $T_i = 1.2T_g$	$K_p = 0.6R/K_g$ $T_i = T_g$
PID	$K_p = 0.6R/K_g$ $T_i = T_g$ $T_d = 0.5T_u$	$K_p = 0.95R/K_g$ $T_i = 1.35T_g$ $T_d = 0.47T_u$

EXAMPLE

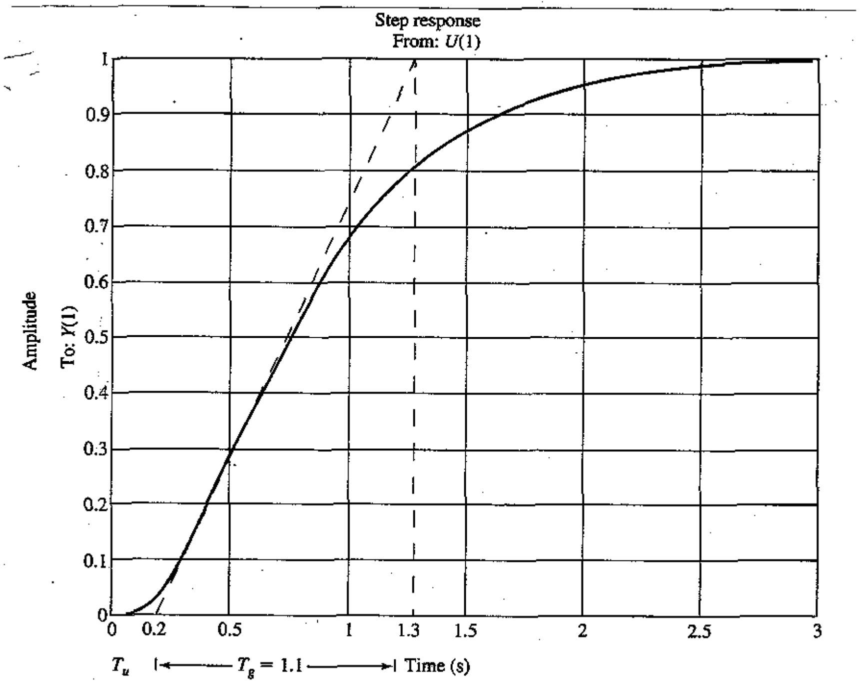
$$G_p(s) = \frac{64}{s^3 + 14s^2 + 56s + 64}$$

UNIT STEP INPUT $\xrightarrow{U(s)}$ $\boxed{G_p(s)}$ $\xrightarrow{Y(s)}$

$$Y(s) = G_p(s)\, U(s)$$
$$= G_p(s)\, \frac{1}{s}$$

SYSTEM IS STABLE SO USING THE FINAL VALUE THEOREM

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\, Y(s)$$
$$= \lim_{s \to 0} \frac{s\, G_p(s)}{s}$$
$$= G_p(0)$$



$$K_G = G_p(0) = \frac{64}{64} = 1$$

(AS SEEN IN STEP RESPONSE)

ALSO

$$T_u = 0.2$$
$$T_g = 1.3 - 0.2 = 1.1$$

$$\Rightarrow R = \frac{T_g}{T_u} = \frac{1.1}{0.2} = 5.5$$

FOR $R = 5.5 \Rightarrow$ USE A FULL THREE TERM CONTROLLER

THE CONTROLLER PARAMETERS FOR THE TWO CATEGORIES OF CONTROLLERS ARE :

PID compensator	
Over damped	$K_p = 3.31$ $T_i = 1.1$ $T_d = 0.10$
20% Overshoot	$K_p = 5.22$ $T_i = 1.49$ $T_d = 0.094$

FIG. 1 – ZN P

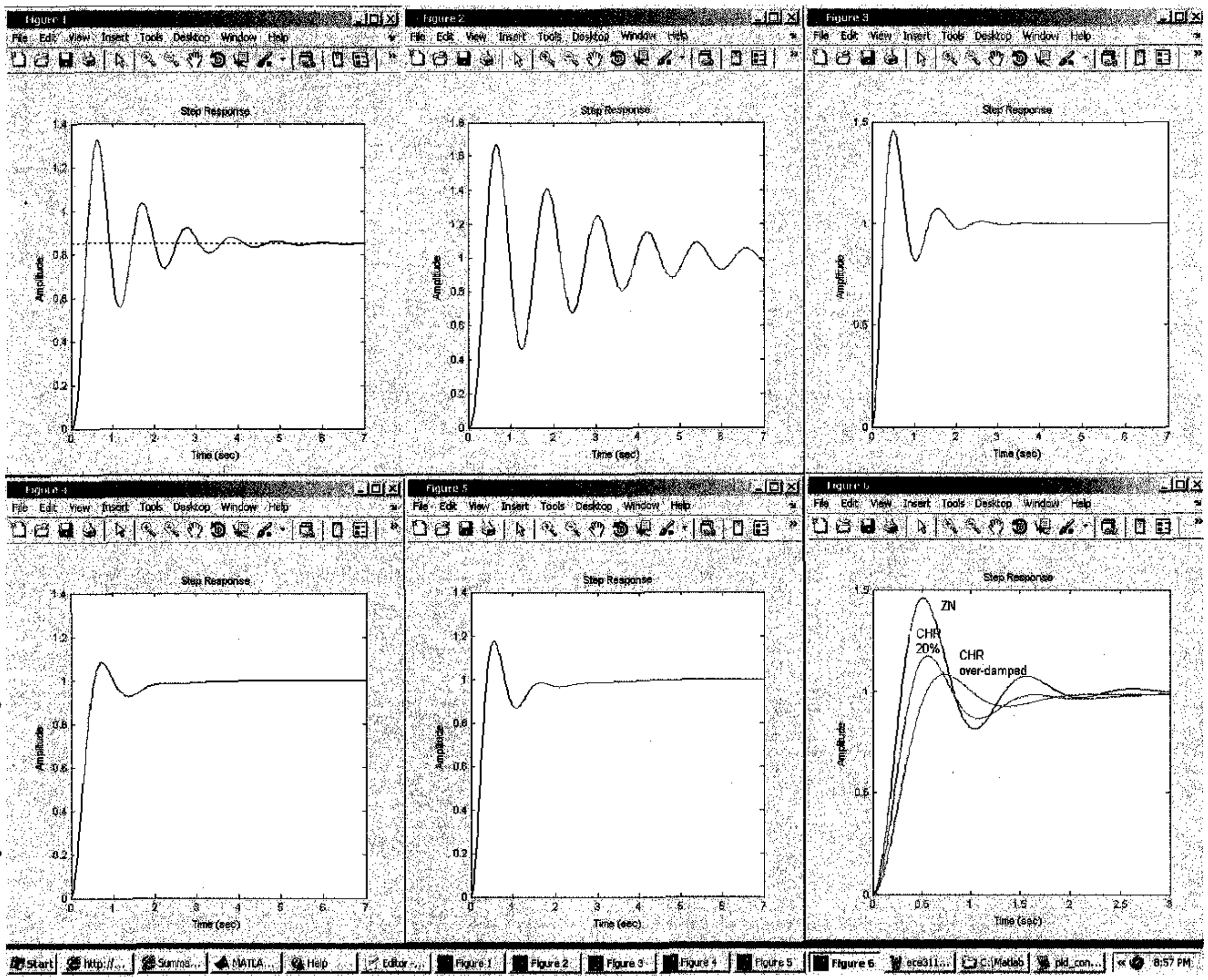
FIG 2 – ZN PI

FIG 3 – ZN PID

FIG. 4 – CHR OD

FIG. 5 – CHR 20%

FIG. 6 – comparison of
ZN PID
CHR OD
CHR 20%



pid_controllers.m

```
clear

Gp = tf(64, [1, 14, 56, 64]);

syms s;
s = tf('s');

%%%%%%%%%

%  Ziegler - Nichols

Gc_zn_p   = 5.63;
Gc_zn_pi  = 5.06*(1 + 1/(0.697*s));
Gc_zn_pid = 6.75*(1 + 1/(0.42*s) + 0.105*s);
tf = 7;

G_zn_p = Gc_zn_p*Gp/(1 + Gc_zn_p*Gp);
figure(1)
step(G_zn_p, tf)

G_zn_pi = Gc_zn_pi*Gp/(1 + Gc_zn_pi*Gp);
figure(2)
step(G_zn_pi, tf)

G_zn_pid = Gc_zn_pid*Gp/(1 + Gc_zn_pid*Gp);
figure(3)
step(G_zn_pid, tf)

%%%%%%%%%%%%%%%%%%%%%%%%%

% Chien - Hrones - Reswick

Gc_chr_pid_od = 3.31*(1 + 1/(1.1*s) + 0.1*s);

Gc_chr_pid_20 = 5.22*(1 + 1/(1.49*s) + 0.094*s);

G_chr_pid_od = Gc_chr_pid_od*Gp/(1 + Gc_chr_pid_od*Gp);
figure(4)
step(G_chr_pid_od, tf)

G_chr_pid_20 = Gc_chr_pid_20*Gp/(1 + Gc_chr_pid_20*Gp);
figure(5)
step(G_chr_pid_20, tf)

%%%%%%%%%%%%%%%%%%%%%%%%

%  comparison

tf_comp = 3;
figure(6)
step(G_zn_pid, G_chr_pid_od, G_chr_pid_20, tf_comp)
```