SOLUTIONS

ECE311 Quiz 4

Problem 1: Find the total response of following systems given the transfer functions, inputs and initial conditions:

$$T(s) = \frac{4}{s^2+4s+4}, \quad r(t) = 0, \quad y(0^-) = -3, \quad y'(0^-) = 2$$

Problem 2: Find the constant k for which the system with transfer function $T(s) = \frac{10}{s^2+40s+k}$ has the second-order response property: $\zeta = 0.7$

Prob 1:

$$Y(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} R(s) + \frac{s y(0^-) + y'(0^-) + a_1 y(0^-) - b_1 r(0^-)}{s^2 + a_1 s + a_0}$$

$$\Rightarrow Y(s) = \frac{4}{s^2+4s+4}(0) + \frac{s(-3) + 2 + 4(-3) - (0)(0)}{s^2+4s+4}$$

$$= \frac{-3s - 10}{(s+2)^2} \Rightarrow \text{repeated real poles}$$

$$\frac{-3s-10}{(s+2)^2} = \frac{A}{s+2} + \frac{B}{(s+2)^2}$$

$$= \frac{A(s+2) + B}{(s+2)^2} = As + 2A + B$$

$$\Rightarrow A = -3$$

$$2A + B = -10 \Rightarrow B = -10 + 6 = -4$$

$$\Rightarrow Y(s) = \frac{-3}{s+2} - \frac{4}{(s+2)^2}$$

$$\Rightarrow \boxed{y(t) = -3e^{-2t} - 4te^{-2t}}$$

Prob. 2

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\Rightarrow \left.\begin{array}{l} \omega_n = \sqrt{k} \\ 2\zeta\omega_n = 40 \\ \zeta = 0.7 \end{array}\right\} \Rightarrow 2(0.7)\sqrt{k} = 40$$

$$\Rightarrow k = \left(\frac{40}{2\times 0.7}\right)^2$$

$$\boxed{k = 816}$$