

# QUIZ #1 SOLUTION

tion of the block whose input is  $X_1(s)$ , as shown in part (b) of the figure. For each part of the figure,

$$Q(s) = F_1(s)X_1(s) + F_2(s)X_2(s)$$

**Example 6.6:**

Find the closed-loop transfer function  $T(s) = Y(s)/U(s)$  for the feedback system shown in Figure 6.17a.

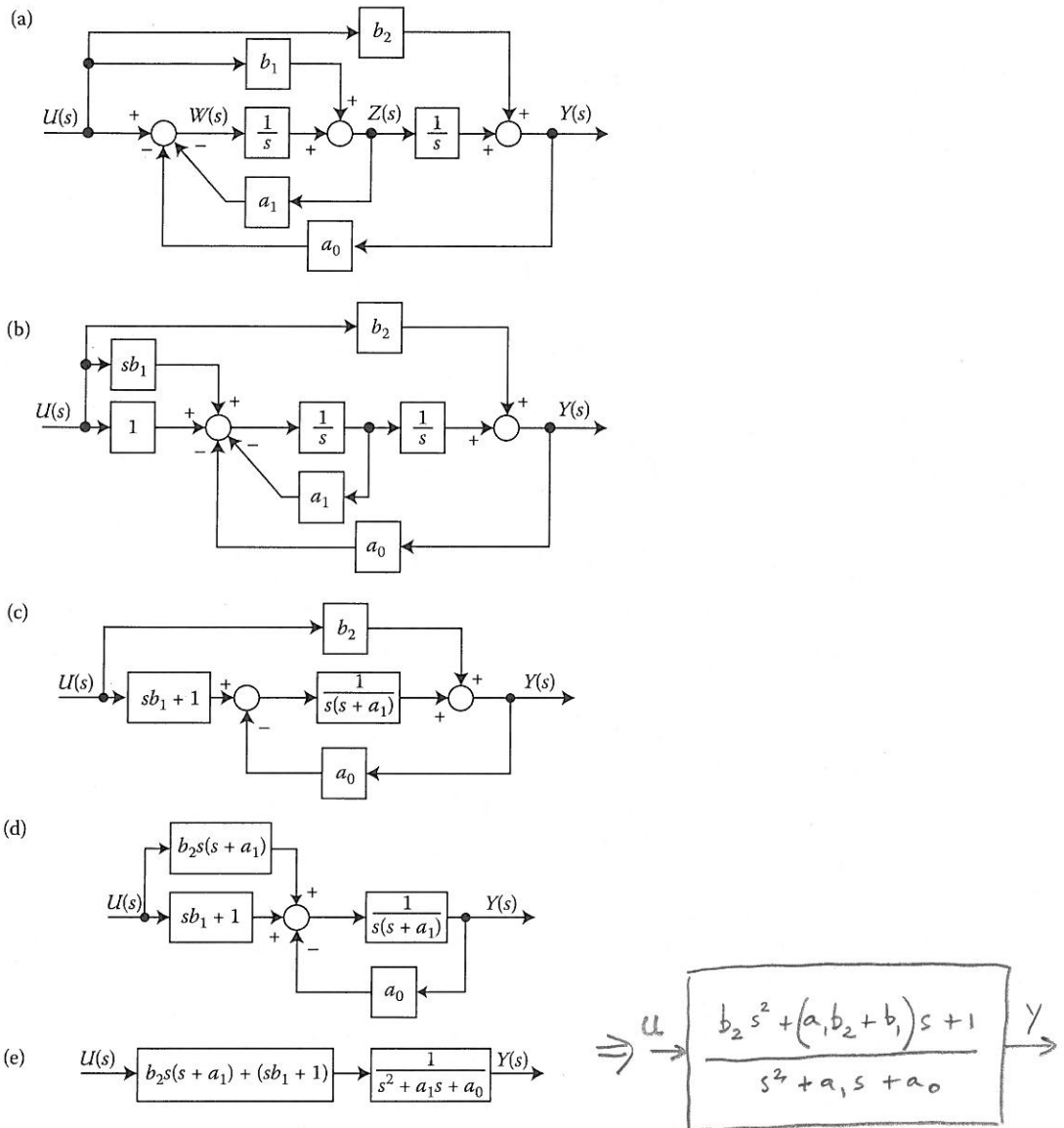
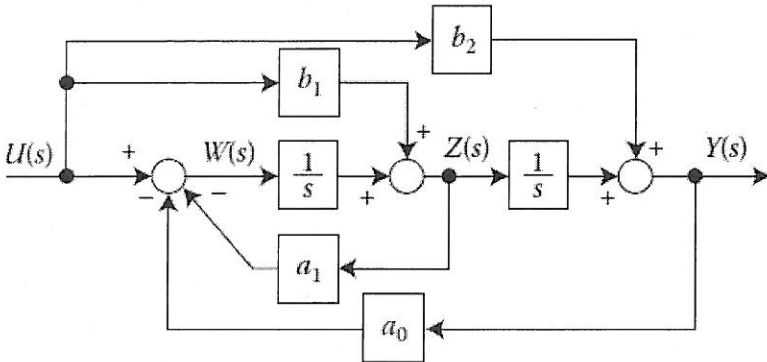


FIGURE 6.17 (a) Block diagram for Example 6.6; and (b)–(e) equivalent block diagrams.

# ALTERNATIVE SOLUTION

## ECE311 Quiz 1

**Problem:** Compute the transfer function of the depicted block diagram.



BY INSPECTION:

$$W = U - a_1 Z - a_0 Y \quad (1)$$

$$Z = b_1 U + \frac{W}{s} \quad (2)$$

$$Y = b_2 U + \frac{Z}{s} \quad (3)$$

$$(2) \rightarrow (1) \Rightarrow$$

$$Z = b_1 U + \frac{1}{s} [U - a_1 Z - a_0 Y]$$

$$Z = b_1 U + \frac{U}{s} - \frac{a_1 Z}{s} - \frac{a_0 Y}{s}$$

$$\Rightarrow Z \left(1 + \frac{a_1}{s}\right) = \left(b_1 + \frac{1}{s}\right) U - \frac{a_0 Y}{s}$$

$$\Rightarrow Z = \frac{b_1 + \frac{1}{s}}{1 + \frac{a_1}{s}} U - \frac{a_0 Y}{1 + \frac{a_1}{s}}$$

$$Z = \frac{b_1 s + 1}{s + a_1} U - \frac{a_0 Y}{s + a_1} \quad (4)$$

$$(4) \rightarrow (3) \Rightarrow$$

$$Y = b_2 U + \frac{1}{s} \left[ \frac{b_1 s + 1}{s + a_1} U - \frac{a_0 Y}{s + a_1} \right]$$

$$\Rightarrow Y \left( s + \frac{a_0}{s + a_1} \right) = b_2 s U + \left( \frac{b_1 s + 1}{s + a_1} \right) U$$

$$Y \left[ \frac{s(s + a_1) + a_0}{s + a_1} \right] = \frac{b_2 s(s + a_1) U + (b_1 s + 1) U}{s + a_1}$$

$$\Rightarrow Y (s^2 + a_1 s + a_0) = (b_2 s^2 + b_2 a_1 s + b_1 s + 1) U$$

$$\Rightarrow \frac{Y}{U} = \frac{b_2 s^2 + (b_2 a_1 + b_1) s + 1}{s^2 + a_1 s + a_0}$$