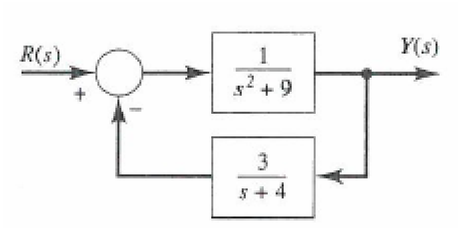
ECE311 – HW 5 Solutions

Problem 1: For the following system find the steady state error for a unit step.



Solution:

$$T(s) = \frac{\frac{1}{s^2+9}}{1+\frac{3}{s+4}\frac{1}{s^2+9}}$$

$$= \frac{s+4}{(s+4)(s^2+9)+3}$$

$$= \frac{s+4}{s^3+4s^2+9s+39}$$

Using Routh-Hurwitz we find that this transfer function is unstable so the final value does not exist and so the steady state error is ∞.

Problem 2: Find the type number and steady state error with unit step, ramp, and parabolic inputs for the following system.

$$T(s) = \frac{7}{s^2+4s+7}$$

Type given by num. of $T_E(s)$ or denom of $G_E(s)$

$$T_E(s) = 1 - \frac{7}{s^2+4s+7}$$

$$= \frac{s(s+4)}{s^2+4s+7}$$

Stable due to pos. second order coeff

$\boxed{\text{Type 1}}$

$$e_{STEP}(\infty) = T_E(0) = \boxed{0}$$

$$e_{RAMP}(\infty) = \lim_{s \to 0} \frac{T_E(s)}{s}$$

$$= \lim_{s \to 0} \frac{s+4}{s^2+4s+7} = \boxed{\frac{4}{7}}$$

$$e_{PAR}(\infty) = \lim_{s \to 0} \frac{T_E(s)}{s^2}$$

$$= \lim_{s \to 0} \frac{s+4}{s(s^2+4s+7)} = \boxed{\infty}$$

Problem 3:

Matlab script

```
s = tf('s');
T = 300*(1+s)*(1+s/40)/s^3;

cl_sys0 = T/(1+T)

% using 'minreal' will get rid of the
%   equal number of poles and zeros at 0
cl_sys = minreal(T/(1+T))

roots(cl_sys.den{1})

% Alternative method to find pole locations
cl_sys_zpk = zpk(cl_sys);
cl_sys_zpk.p{1}

margin(T)
```

1) Using ‘roots’ the closed loop poles are found:

   ```
   -3.2517 +17.0425i
   -3.2517 -17.0425i
   -0.9966 + 0.0000i
   ```

   All are stable poles.

2) See plot below resulting from the 'margin' command.

3) $\omega_C = 18.2$ rad/s

4) Phase at ω_C is above the -180 deg. line so this represents a positive phase margin.

5) $\omega_M = 6.32$ rad/s

6) Magnitude at ω_M is above the 0 dB line so this represents a negative gain margin.

7) $\omega_C = 18.2$ rad/s, $Pm = 21.3$ deg, $\omega_M = 6.32$ rad/s, $Gm = -17.7$ dB

8) $Pm > 0$ ➔ the closed loop system is stable. We do not look at the gain margin (Gm) to determine absolute stability.

9) In part (1) we found the system to be stable, by examining the closed loop system poles, and this is confirmed in part (8) using the loop gain Bode plot stability test.