SOLUTIONS

ECE311

Homework 4

Problem 1: Find the total response of following systems given the transfer functions, inputs and initial conditions:

a. $T(s) = \frac{10}{s+4}$, $\quad r(t) = \delta(t)$, $\quad y(0^-) = 0$

b. $T(s) = \frac{s-5}{s^2+3s+2}$, $\quad r(t) = u(t)$, $\quad y(0^-) = -3$, $\quad y'(0^-) = 4$

Problem 2: Consider the second order system $T(s) = \frac{100}{s^2+3s+13}$, find the a) undamped natural frequency, b) the damping ratio, and c) the true (damped) oscillation frequency.

PROB 1 a

$$Y(s) = \frac{b_1 s + b_0}{s+a_0} R(s) + \frac{y(0^-) - \frac{b_1}{1} r(0^-)}{s+a_0}$$

$$\Rightarrow Y(s) = \frac{10}{s+4}(1) + \frac{0 - (0)(0)}{s+4}$$

$$= \frac{10}{s+4} \Rightarrow \boxed{y(t) = 10e^{-4t} \quad \text{OR} \quad y(t) = 10e^{-4t}u(t)}$$

PROB 1 b

$$Y(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} R(s) + \frac{s\,y(0^-) + y'(0) + a_1 y(0^-) + b_1 r(0^-)}{s^2 + a_1 s + a_0}$$

$$\Rightarrow Y(s) = \frac{s-5}{s^2+3s+2}\left(\frac{1}{s}\right) + \frac{s(-3) + 4 + 3(-3) + (1)(0)}{s^2+3s+2}$$

$$= \frac{s-5}{s^2+3s+2}\frac{1}{s} + \frac{-3s-5}{s^2+3s+2}$$

$$= -\frac{+3s^2+4s+5}{s(s^2+3s+2)} = -\left[\frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+1}\right]$$

$$= \frac{K_1(s^2+3s+2) + K_2(s^2+s) + K_3(s^2+2s)}{s(s^2+3s+2)}$$

$$\Rightarrow \begin{cases} K_1 + K_2 + K_3 = 3 \\ 3K_1 + K_2 + 2K_3 = 4 \\ 2K_1 = 5 \end{cases} \Rightarrow K_1 = \frac{5}{2},\ K_2 = \frac{9}{2},\ K_3 = -4$$

$$Y(s) = \frac{-\frac{5}{2}}{s} - \frac{\frac{9}{2}}{s+2} + \frac{4}{s+1}$$

$$\Rightarrow \boxed{y(t) = -\frac{5}{2} - \frac{9}{2}e^{-2t} + 4e^{-t} \quad \text{OR} \quad y(t) = \left[-\frac{5}{2} - \frac{9}{2}e^{-2t} + 4e^{-t}\right]u(t)}$$

PROB. 2.

$$T(s) = \frac{100}{s^2+3s+13}$$

STANDARD FORM 2ND ORDER POLYNOMIAL: $s^2 + 2\zeta\omega_n s + \omega_n^2$

a) $\omega_n = \sqrt{13} = 3.6$

b) $2\zeta\omega_n = 3 \Rightarrow \zeta = \frac{3}{2\omega_n} = \frac{3}{7.2} = 0.42$

c) $\omega = \omega_n\sqrt{1-\zeta^2}$

$= 3.6(0.9)$

$= 3.27$