ECE311

Homework 2 - SOLUTIONS

Problems

Determine the stability of the systems with the following characteristic equations:

a. $s^5 + 3s^4 + 7s^3 + 20s^2 + 6s + 15 = 0$

b. $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$

c. $s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$

d. $s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$

Solution

a. The characteristic equation is

$$s^5 + 3s^4 + 7s^3 + 20s^2 + 6s + 15 = 0 \qquad \text{(P5.1.1)}$$

Routh's tabulation is

$$\begin{array}{c|ccc} s^5 & 1 & 7 & 6 \\ s^4 & 3 & 20 & 15 \\ s^3 & 1/3 & 1 & \\ s^2 & 11 & 15 & \\ s^1 & 6/11 & & \\ s^0 & 15 & & \end{array}$$

There is no change of sign in the first column of Routh's tabulation. Hence, the system is stable.

b. The characteristic equation is

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0 \qquad \text{(P5.1.2)}$$

Routh's tabulation is written as

$$\begin{array}{c|ccc} s^4 & 2 & 3 & 10 \\ s^3 & 1 & 5 & \\ s^2 & -7 & 10 & \\ s^1 & 6.43 & & \\ s^0 & 10 & & \end{array}$$

There are two sign changes in the first column of Routh's tabulation ($1 \to -7 \to 6.43$); hence, the characteristic equation has two roots in the right-half s-plane, and the system is unstable.

c. The characteristic equation is

$$s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0 \qquad \text{(P5.1.3)}$$

Routh's tabulation is written as

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & \varepsilon & 6 & \\ s^2 & \dfrac{4\varepsilon - 12}{\varepsilon} & 10 & \\ s^1 & 6 + \dfrac{10}{12}\varepsilon^2 & & \\ s^0 & 10 & & \end{array}$$

The first term in row s^3 was zero and it is replaced by a very small number ε (where, $\varepsilon > 0$ and $\lim \varepsilon \to 0$).

We have $4\varepsilon - 12/\varepsilon < 0$ and $6 + (10/12)\varepsilon^2 > 0$.

Hence, the system is unstable, and the characteristic equation has two roots in the right-half s-plane.

d. The characteristic equation is

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0 \qquad \text{(P5.1.4)}$$

The system is unstable, because the polynomial $s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50$ has two coefficients of different sign. By applying Routh's criterion we obtain the same conclusion.

Routh's tabulation is

s^5	1	24	–25
s^4	2	48	–50
s^3	8	96	
s^2	24	–50	
s^1	112.7		
s^0	–50		

All coefficients of row s^3 were zero, thus, they have been replaced by the terms of the differentiated auxiliary equation of row s^4.

We have

$$2s^4 + 48s^2 - 50 = 0 \Rightarrow \frac{d}{dt}(2s^4 + 48s^2 - 50) = 0 \Rightarrow 8s^3 + 96s = 0 \qquad \text{(P5.1.5)}$$

At the first column of Routh's tabulation, there is a change of sign; therefore, the characteristic equation has one root in the right-half s-plane. It can be computed by solving the auxiliary equation $2s^4 + 48s^2 - 50 = 0$.

We have $s_{1,2}^2 = 1$ and $s_{3,4}^2 = -25$. Thus,

$$\Rightarrow s_{1,2} = \pm 1 \quad \text{and} \quad s_{3,4} = \pm j5 \qquad \text{(P5.1.6)}$$

Hence, the initial equation of relationship (P5.1.4) is written as

$$(s+1)(s-1)(s+j5)(s-j5)(s+2) = 0$$

Notice that the root $s = 1$ is at the right-half s-plane.