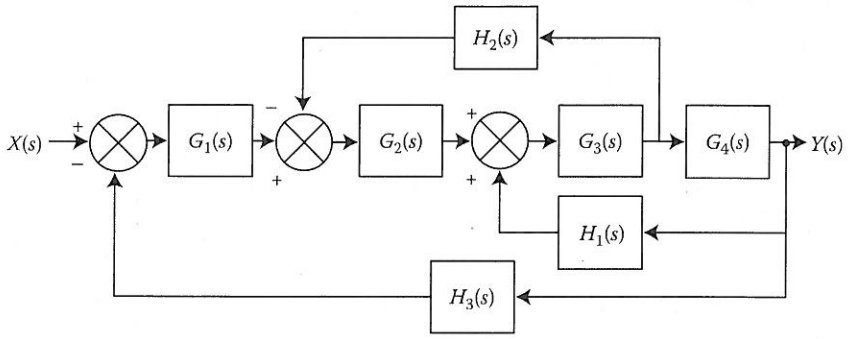
HW 1 SOLUTION

Problems

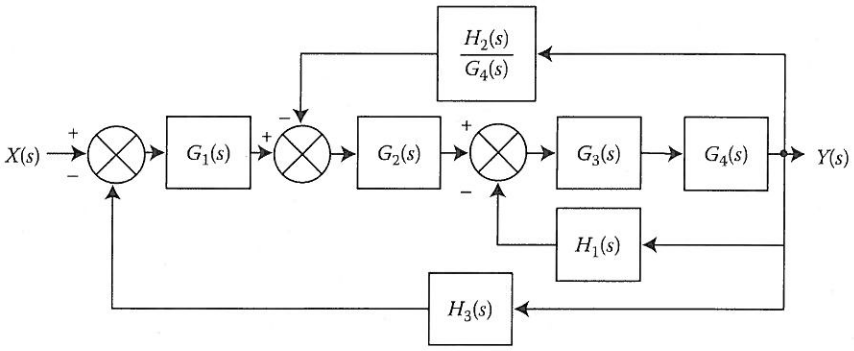
2.1 Compute the transfer function of the depicted block diagram

a. By reduction

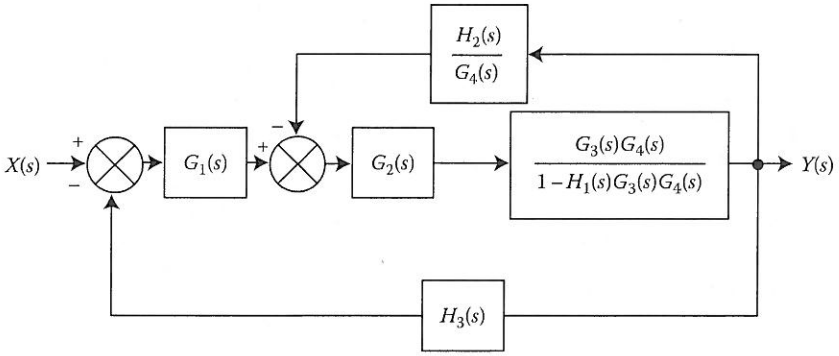


Solution SEE BELOW

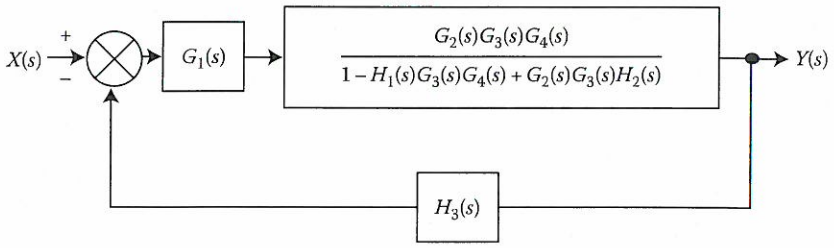
a. By applying transformation 7 (Table F2.1), the branch point at the left of the block with transfer function $G_4(s)$ is moved at the right of $G_4(s)$. The equivalent block diagram is:



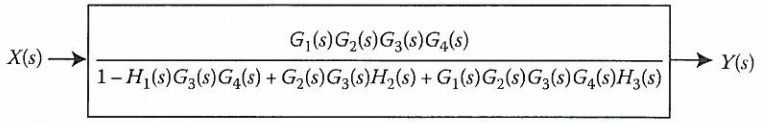
The next block diagram emerges when transformations 1 and 3 are applied to the loop that contains the blocks with transfer functions $G_3(s)$, $G_4(s)$, and $H_1(s)$.



Next we apply transformations 1 and 3 to the loop that contains the transfer function $H_2(s)/G_4(s)$ as feedback and get the following block diagram:



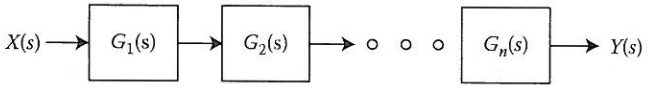
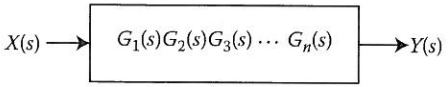
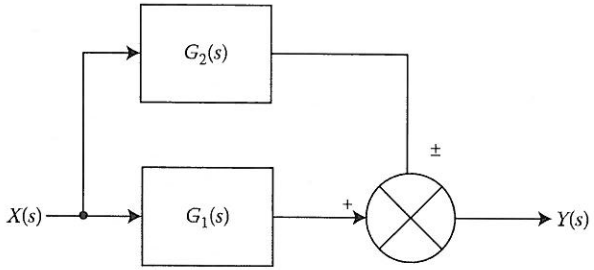
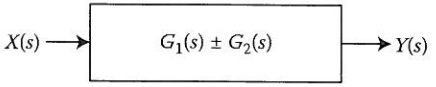
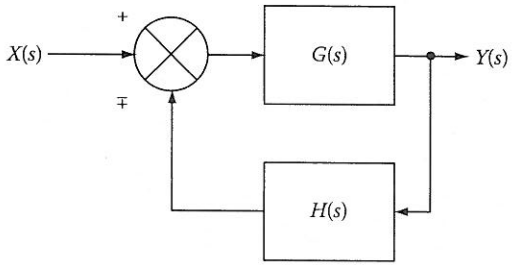
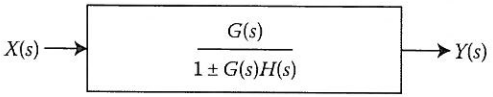
Similarly, by applying transforms 1 and 3 we obtain the simplified block diagram that represents the system's transfer function.



Formulas

TABLE F2.1

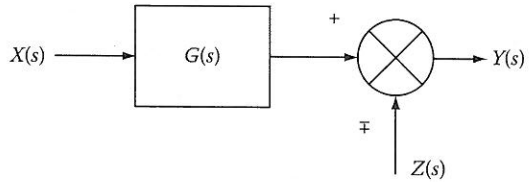
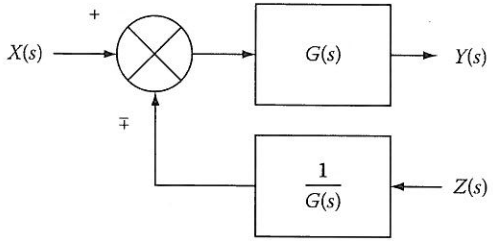
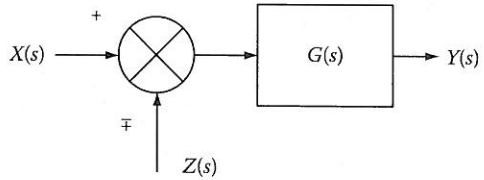
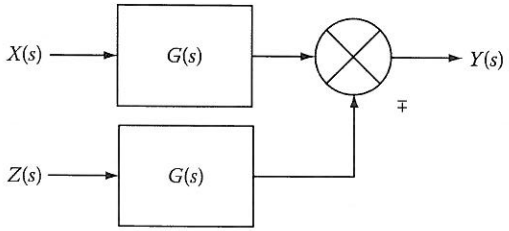
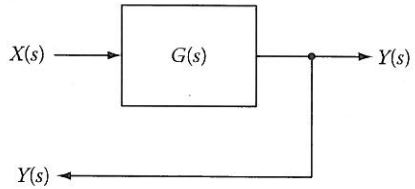
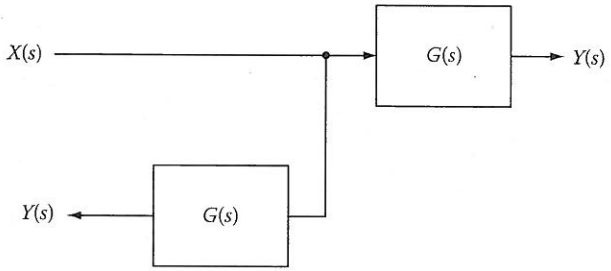
Block Diagram Transformations

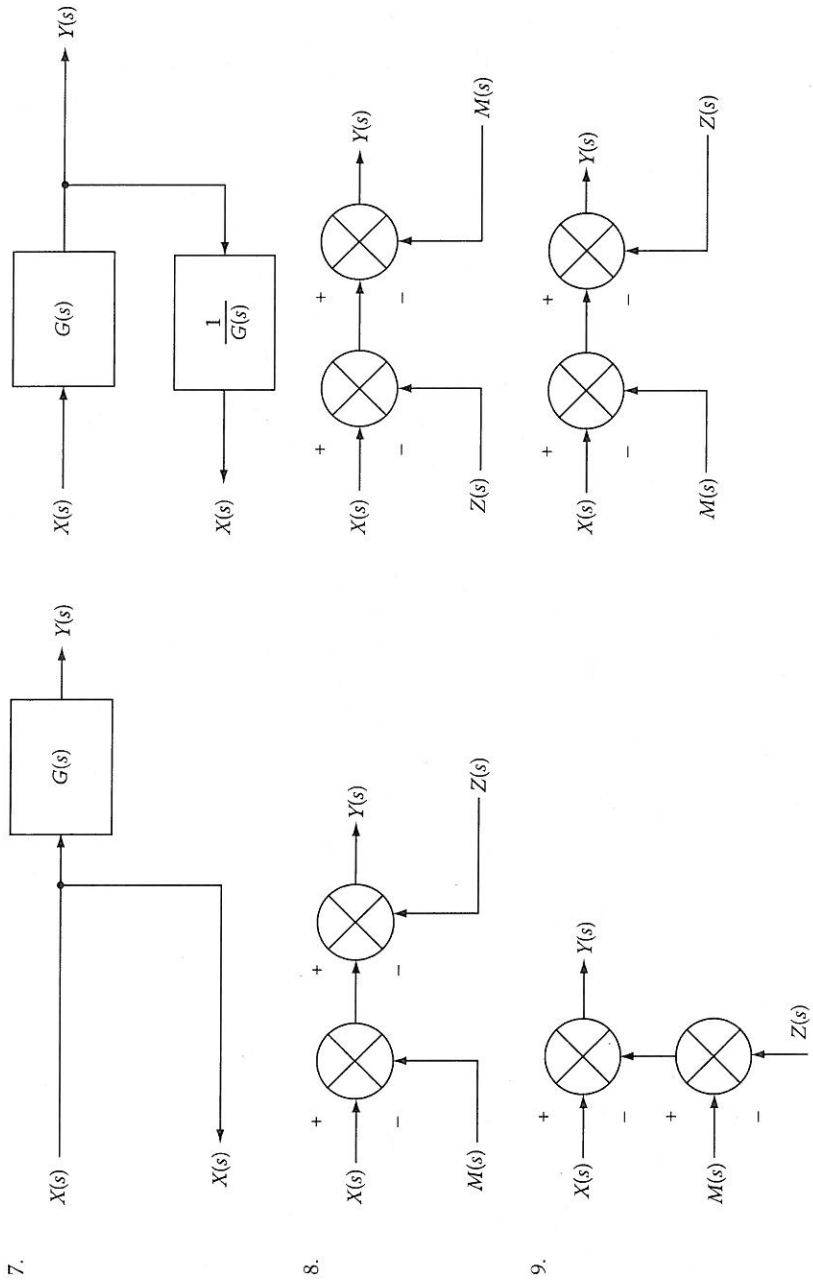
S/N	Initial Diagram	Equivalent Diagram
1.		
2.		
3.		

(continued)

TABLE F2.1 (continued)

Block Diagram Transformations

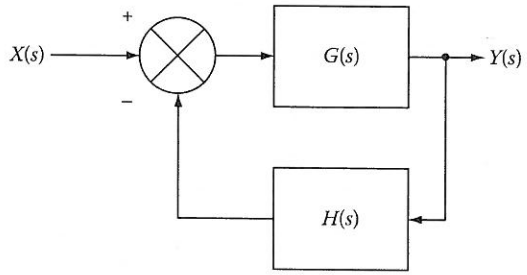
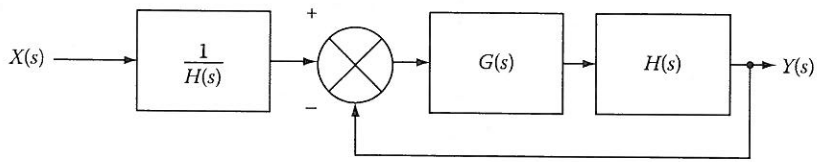
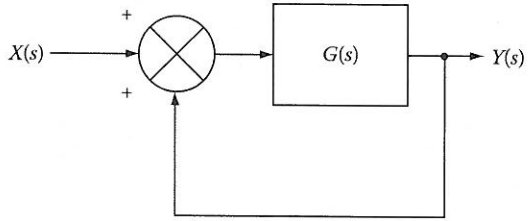
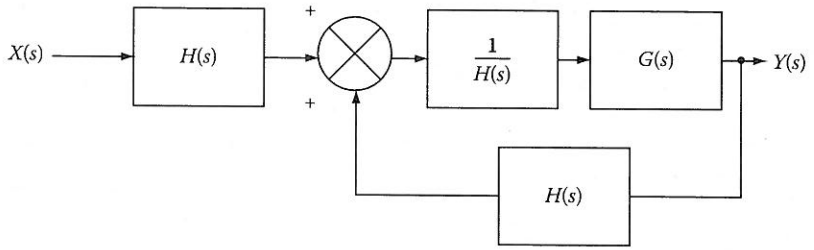
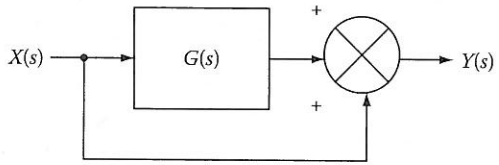
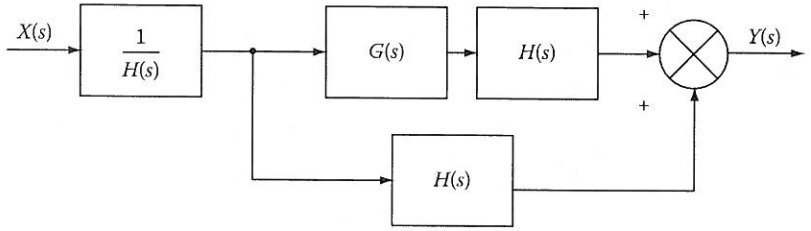
S/N	Initial Diagram	Equivalent Diagram
4.		
5.		
6.		



(*continued*)

TABLE F2.1 (continued)

Block Diagram Transformations

S/N	Initial Diagram	Equivalent Diagram
10.		
11.		
12.		

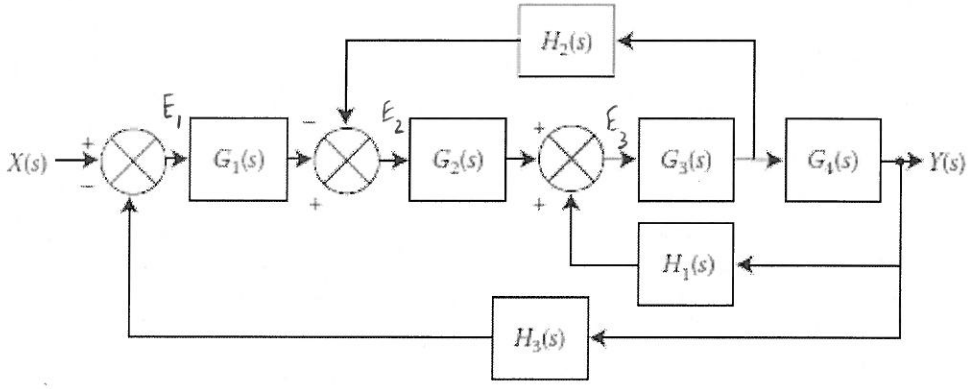
ALTERNATIVE SOLUTION

ECE311

Homework 1

Problem:

Compute the transfer function of the depicted block diagram.



ADDED SIGNAL NAMES E_1, E_2 AND E_3 TO ABOVE DIAGRAM $\Rightarrow$

$E_1 = X - H_3 Y$ (1)

$E_2 = G_1 E_1 - H_2 G_3 E_3$ (2)

$E_3 = G_2 E_2 + H_1 Y$ (3)

$Y = G_3 G_4 E_3$ (4)

(2) $\rightarrow$ (3) $\Rightarrow$

$$E_3 = G_2\left(G_1 E_1 - H_2 G_3 E_3\right) + H_1 Y$$

$$= G_2 G_1 E_1 - G_2 H_2 G_3 E_3 + H_1 Y \quad (5)$$

(1) $\rightarrow$ (5) $\Rightarrow$

$$E_3 = G_2 G_1 (X - H_3 Y) - G_2 H_2 G_3 E_3 + H_1 Y$$

$$= G_2 G_1 X - G_2 G_1 H_3 Y - G_2 H_2 G_3 E_3 + H_1 Y$$

$$\Rightarrow E_3 (1 + G_2 H_2 G_3) = G_2 G_1 X - (G_2 G_1 H_3 - H_1) Y$$

$$\Rightarrow E_3 = \frac{G_2 G_1 X}{1 + G_2 H_2 G_3} - \frac{(G_2 G_1 H_3 - H_1)}{1 + G_2 H_2 G_3} Y \quad (6)$$

(6) $\rightarrow$ (4) $\Rightarrow$

$$Y = \frac{G_3 G_4 G_2 G_1 X}{1 + G_2 H_2 G_3} - \frac{G_3 G_4 (G_2 G_1 H_3 - H_1)}{1 + G_2 H_2 G_3} Y$$

$$\Rightarrow Y\left(1 + \frac{G_3 G_4 (G_2 G_1 H_3 - H_1)}{1 + G_2 H_2 G_3}\right) = \frac{G_1 G_2 G_3 G_4 X}{1 + G_2 H_2 G_3}$$

$$\Rightarrow Y\left[\frac{1 + G_2 H_2 G_3 + G_1 G_2 G_3 G_4 H_3 - G_3 G_4 H_1}{1 + G_2 H_2 G_3}\right] = \frac{G_1 G_2 G_3 G_4 X}{1 + G_2 H_2 G_3}$$

$$\Rightarrow \frac{Y}{X} = \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3 - G_3 G_4 H_1}$$