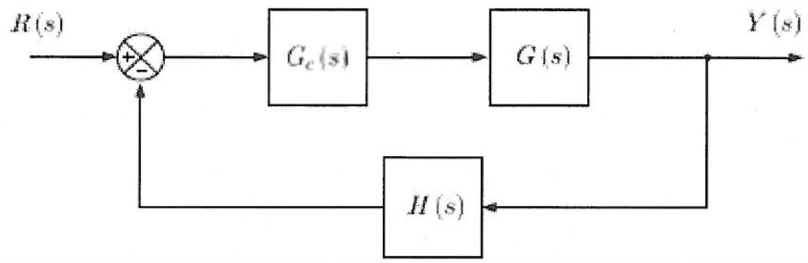
ECE311

Problem:

Consider a feedback system



where,

$$G(s) = \frac{G_o}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}$$

$$H(s) = k$$

where $G_o = 500,\ \omega_1 = 2\pi(10),\ \omega_2 = 2\pi(100),\ \omega_3 = 2\pi(300)$. and $k = 0.5$ and $G_c(s) = 1$.

Determine the loop gain of the system $T(s)$, and for this loop gain using asymptotic approximations only,

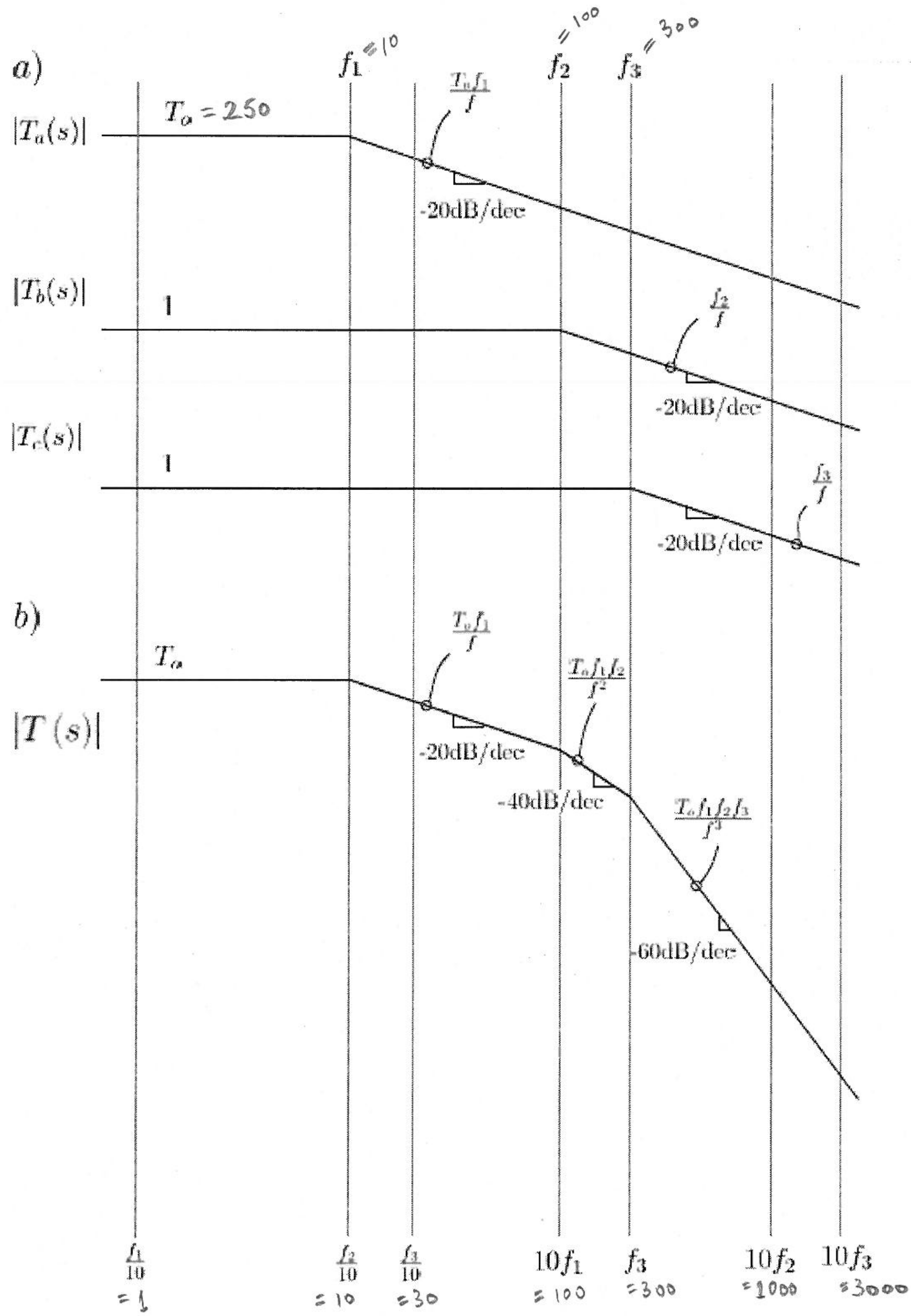
i. Sketch the three components of the Bode magnitude plots individually and subsequently obtain the total magnitude response. Annotate the plots.
ii. Sketch the three components of the Bode phase plots individually and subsequently obtain the total phase response. Annotate the plots.
iii. Using your results from (i) and (ii), sketch the Bode magnitude and phase plots in standard Bode plot form where the phase response is shown below the magnitude response. Label all break frequencies, slopes of sloping lines, gains of sloping lines and phase levels on zero slope lines.
iv. Using the plots from (iii), determine the phase margin and the associated crossover frequency.
v. Using the plots from (iii), determine the gain margin and the associated crossover frequency.
vi. Determine whether the closed loop system is stable.

Solution:

$$T(s) = \underbrace{\frac{T_o}{\left(1+\frac{s}{\omega_1}\right)}}_{T_a(s)}\ \underbrace{\frac{1}{\left(1+\frac{s}{\omega_2}\right)}}_{T_b(s)}\ \underbrace{\frac{1}{\left(1+\frac{s}{\omega_3}\right)}}_{T_c(s)}$$

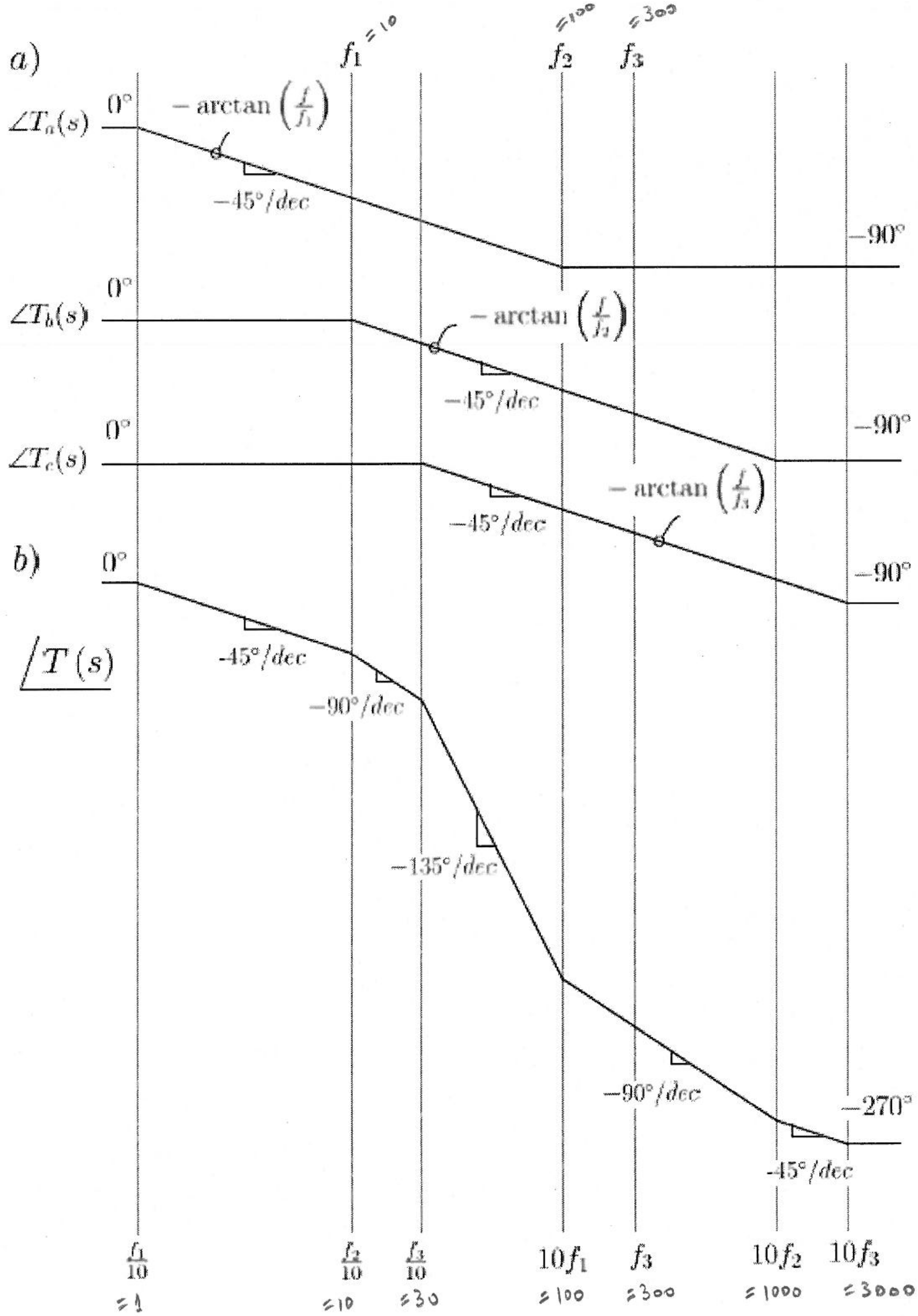
where $T(s) = G_c(s)G(s)H(s)$, where $T_o = k\,G_o = 0.5 \cdot 500 = 250$.

i.

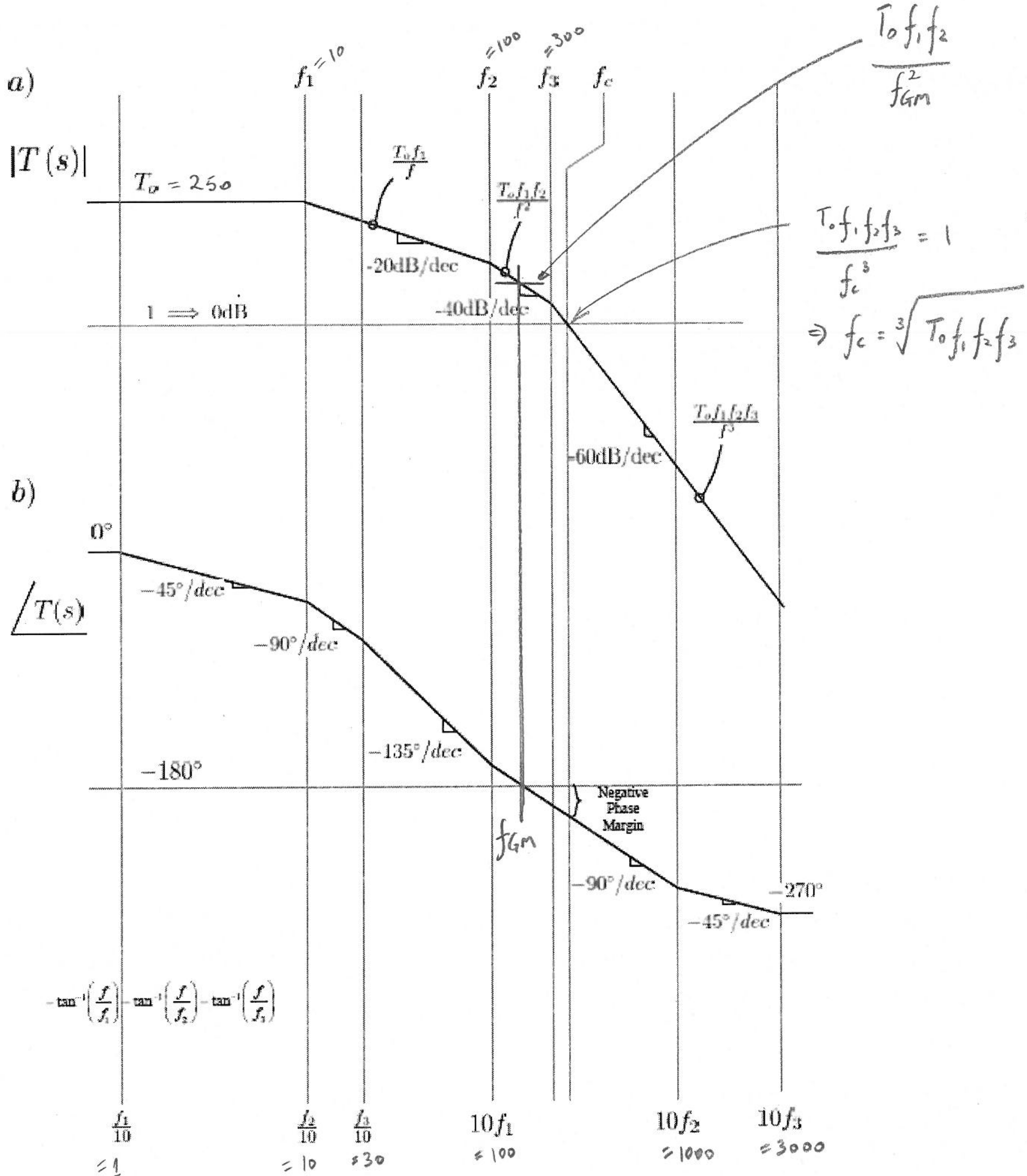


a)
f_1 = 10
f_2 = 100
f_3 = 300
$|T_a(s)|$
T_o = 250
$\frac{T_o f_1}{f}$
-20dB/dec
$|T_b(s)|$
1
$\frac{f_2}{f}$
-20dB/dec
$|T_c(s)|$
1
$\frac{f_3}{f}$
-20dB/dec
b)
T_o
$|T(s)|$
$\frac{T_o f_1}{f}$
-20dB/dec
$\frac{T_o f_1 f_2}{f^2}$
-40dB/dec
$\frac{T_o f_1 f_2 f_3}{f^3}$
-60dB/dec
$\frac{f_1}{10}$ = 1
$\frac{f_2}{10}$ = 10
$\frac{f_3}{10}$ = 30
$10f_1$ = 100
f_3 = 300
$10f_2$ = 1000
$10f_3$ = 3000

ii.


a)
b)
f_1 ≈10
f_2 ≈100
f_3 ≈300
$\angle T_a(s)$
$\angle T_b(s)$
$\angle T_c(s)$
0°
0°
0°
0°
$-\arctan\left(\frac{f}{f_1}\right)$
$-\arctan\left(\frac{f}{f_2}\right)$
$-\arctan\left(\frac{f}{f_3}\right)$
−45°/dec
−45°/dec
−45°/dec
−90°
−90°
−90°
$\angle T(s)$
-45°/dec
−90°/dec
−135°/dec
−90°/dec
-45°/dec
−270°
$\frac{f_1}{10}$ ≈1
$\frac{f_2}{10}$ ≈10
$\frac{f_3}{10}$ ≈30
$10f_1$ ≈100
f_3 ≈300
$10f_2$ ≈1000
$10f_3$ ≈3000

iii.



a)
$f_1 = 10$
$f_2 = 100$
$f_3 = 300$
f_c
$\frac{T_o f_1 f_2}{f_{GM}^2}$
$|T(s)|$
$T_o = 250$
$\frac{T_o f_1}{f}$
$\frac{T_o f_1 f_2}{f^2}$
$\frac{T_o f_1 f_2 f_3}{f_c^3} = 1$
$\Rightarrow f_c = \sqrt[3]{T_o f_1 f_2 f_3}$
-20dB/dec
-40dB/dec
$1 \Longrightarrow 0\text{dB}$
$\frac{T_o f_1 f_2 f_3}{f^3}$
-60dB/dec
b)
0°
$-45°/dec$
$\angle T(s)$
$-90°/dec$
$-135°/dec$
$-180°$
Negative Phase Margin
f_{GM}
$-90°/dec$
$-270°$
$-45°/dec$
$-\tan^{-1}\left(\frac{f}{f_1}\right)-\tan^{-1}\left(\frac{f}{f_2}\right)-\tan^{-1}\left(\frac{f}{f_3}\right)$
$\frac{f_1}{10} = 1$
$\frac{f_2}{10} = 10$
$\frac{f_3}{10} = 30$
$10 f_1 = 100$
$10 f_2 = 1000$
$10 f_3 = 3000$

iv. f_c, the unity gain crossover frequency, and PM, the phase margin:

$$\frac{T_o f_1 f_2 f_3}{f_c^3} = 1 \implies f_c = \sqrt[3]{T_o f_1 f_2 f_3}$$

$$PM = 180 - \arctan\left(\frac{f_c}{f_1}\right) - \arctan\left(\frac{f_c}{f_2}\right) - \arctan\left(\frac{f_c}{f_3}\right)$$

result in $f_c = 422$ Hz and $PM = -40°$.

v. f_{GM}, the frequency at which the phase reaches $-180°$, and subsequently the gain margin:

$$-180 = -\arctan\left(\frac{f_{GM}}{f_1}\right) - \arctan\left(\frac{f_{GM}}{f_2}\right) - \arctan\left(\frac{f_{GM}}{f_3}\right)$$

$$GM = -20\log\left(\frac{T_o f_1 f_2}{f_{GM}^2}\right)$$

results in $f_{GM} = 184$ Hz and $GM = -17.3$ dB

vi. $PM = -40° < 0 \rightarrow$ Unstable