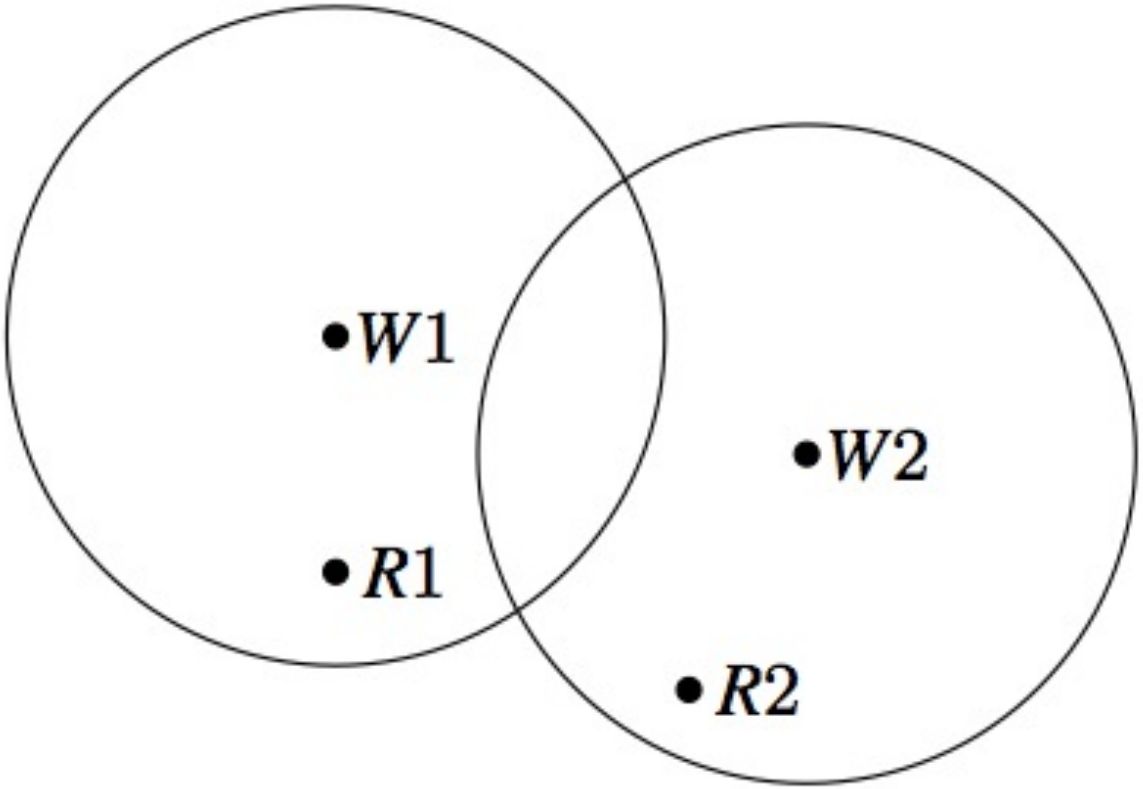
Relativistic Red Black Trees

Relativistic Programming

- Concurrent reading and writing improves performance and scalability
  - concurrent readers may disagree on the order of concurrent updates
  - orders may be non-linearizable
  - is this OK?
- Relativistic programming provides tools for enforcing the order that is necessary for correctness
  - linearizability is not always necessary!

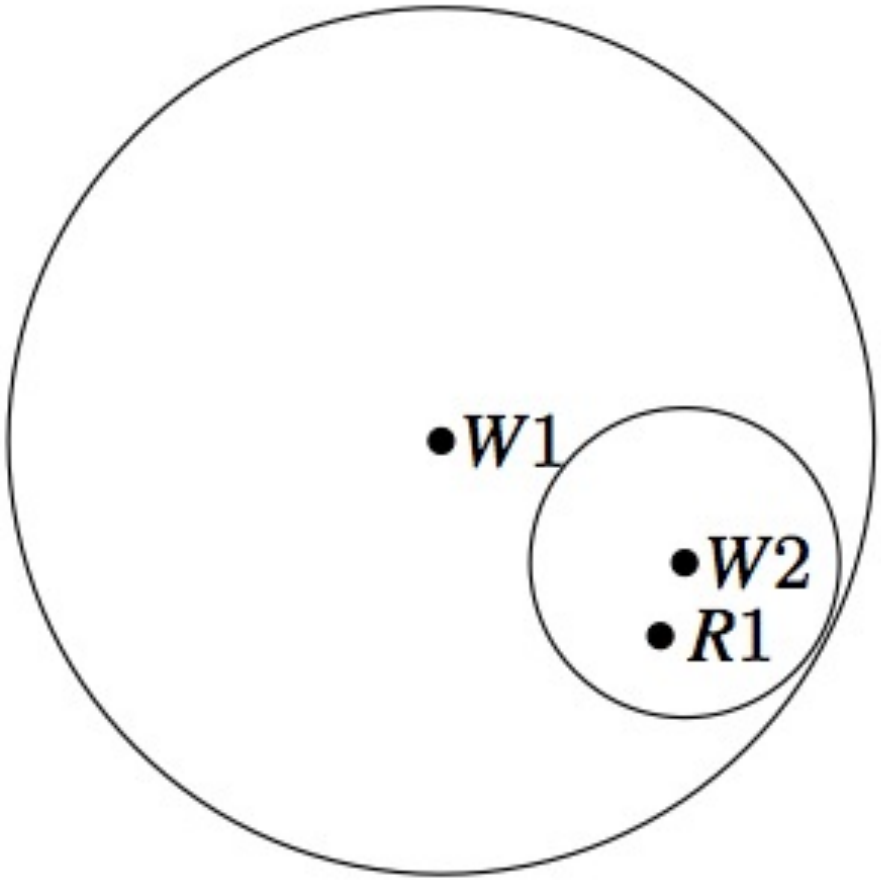
The Natural World is Relativistic



Implications for Parallel Computing

- Communication over distance takes time
  - recipients at different distances will receive information/ results at different times
  - potential to receive information out of order
- Forcing all recipients to agree on the order can only be done by delaying receipt
  - i.e. nobody gets it until the slowest has received it
  - delays slow down computation
  - the approach is inherently non-scalable

The Natural World is also Causal

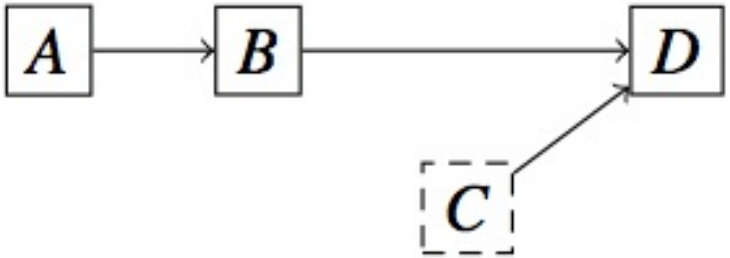


Implications for Parallel Computing

- Scalability is all about allowing local computation to proceed unhindered
  - but violations of causality are confusing

- Delays can be introduced to preserve causality
  - i.e., if two updates are causally related, we can ensure that all readers see them in their correct order (the order the programmer specified)
  - if they are unrelated, then don't force all to agree on an order: its unnecessary and expensive

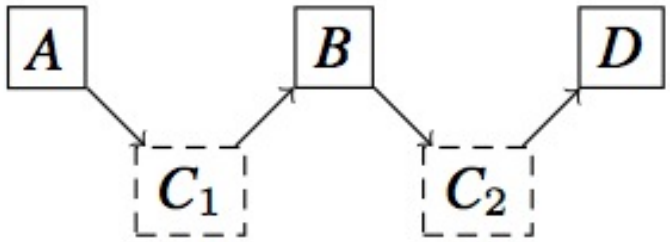
Simple (atomic) Operations



List delete operation

- readers either see it or they don't
- no ordering problems
- no incoherency

Complex (non-atomic) Operations



List move operation

- readers might see before state, after state, or two during states

Dealing with Complex Operations

- What options do we have for list move?
  - do we add new before removing old?
  - do we remove old before adding new?
- What can concurrent readers observe?
  - both old and new?
  - neither old nor new?
  - old but not new?
  - new but not old?
- Which options are OK, and how can we enforce them?

Hiding Incorrect States

- If its OK to see either old, or new, or both, then we must hide the “neither” state
  - any reader that fails to find the old must find the new
  - is it enough to insert the new before removing the old?
  - if the new appears earlier in the list than the old then we need to wait for readers before we delete the old!
- If we must only see the before or after state then we need atomic transactions (later)

Relativistic Programming Primitives

- Generalization of RCU primitives
  - write-lock, write-unlock
    - for synchronization among writers
  - start-read, end-read
    - to delimit read sections
  - wait-for-readers
    - to wait for all pre-existing readers
  - deferred-free
    - to safely reclaim memory
  - publish, read
    - to remove reordering problems

Rules for Relativistic Programming

- Writers must keep data in a continually consistent state
  - a reader must be able to safely traverse a data structure at any time
  - individual updates must take effect at a well-defined point with respect to a concurrent reader
    - this is like linearizability
    - but it does not necessarily imply that all readers must agree on the order of unrelated updates!

Rules for Relativistic Programming

- When updates must be seen in order, writers must insert the appropriate delay
  - wait-for-readers
  - sometimes rp-read is enough
- Readers must delimit their read sections and not hold references between read sections
  - just like conventional locking rules
- To simplify CPU and compiler reordering issues
  - readers must use the rp-read to access shared data
  - writers update shared data using the rp-publish

Rules for Relativistic Programming

- Sometimes writers can reason about read traversal order and avoid using wait-for-readers
  - readers will naturally see updates in order even without it
  - example: moving an element from earlier to later in a singly linked list, by copying then removing, guarantees that all readers will see one or the other or both

But How Generally Useful is This?

- We have some primitives and a few simple rules
- They work well for simple list operations
- They also work well for hash-tables
- But is this enough for more complex data structures?

Red Black Trees

- RB-trees store sorted <key,value> pairs
- They guarantee O(log(N)) performance for inserts, deletes, and lookups
  - they limit the depth of the tree (partially balanced)
  - updates require restructuring of the tree
- They are very difficult to parallelize
  - difficult to avoid deadlock with per-node locking
  - most implementations use a single global lock
  - they are a stiff challenge for Relativistic Programming!

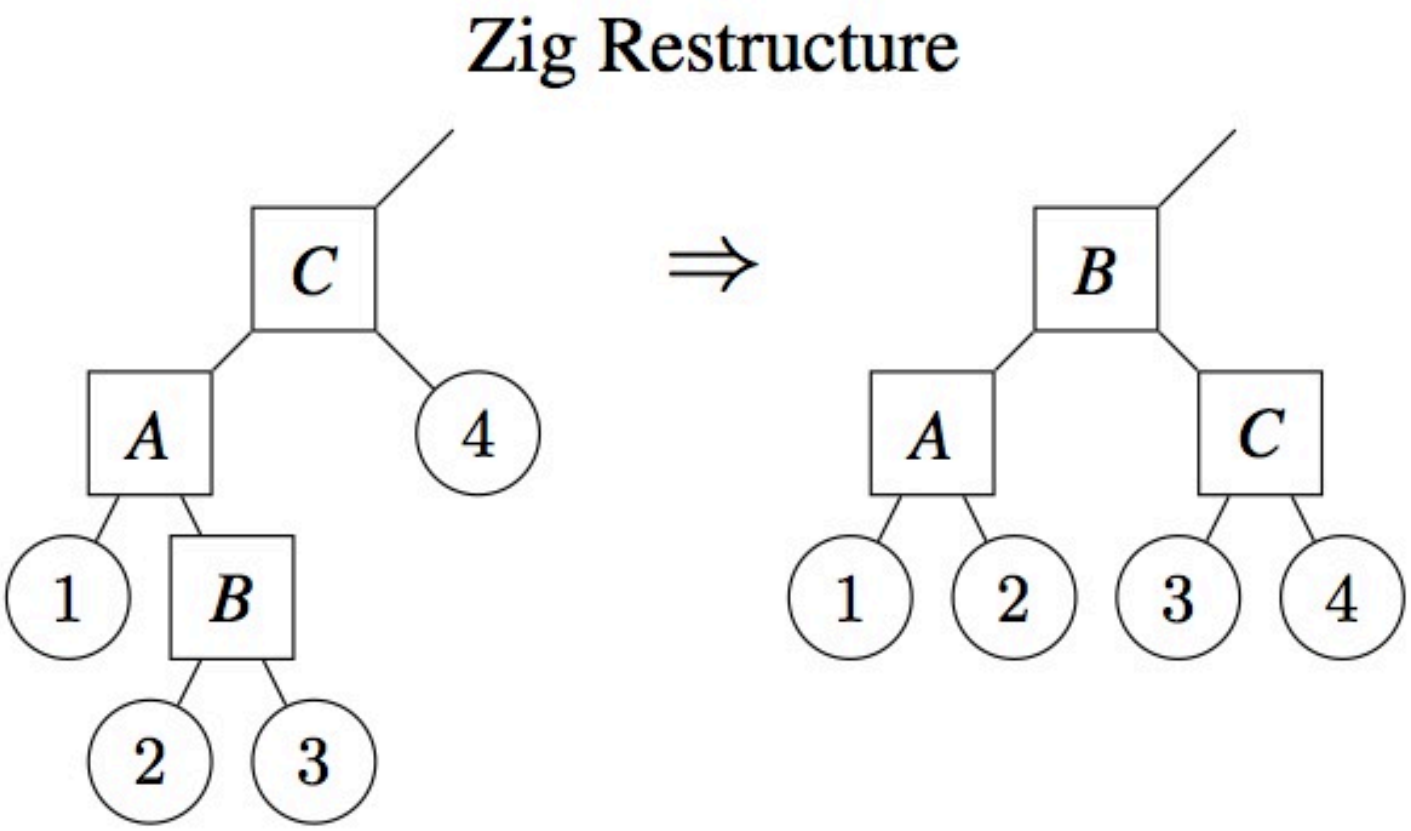
Red Black Tree Properties

- All nodes on the left branch of a subtree have a key less than that of the root of the subtree
- All nodes on the right branch of a subtree have a key greater or equal to that of the root
- Each node has a color (red or black)
- Both children of a red node are black
- The black depth of every leaf is the same
  - this is the balance property

Rebalancing

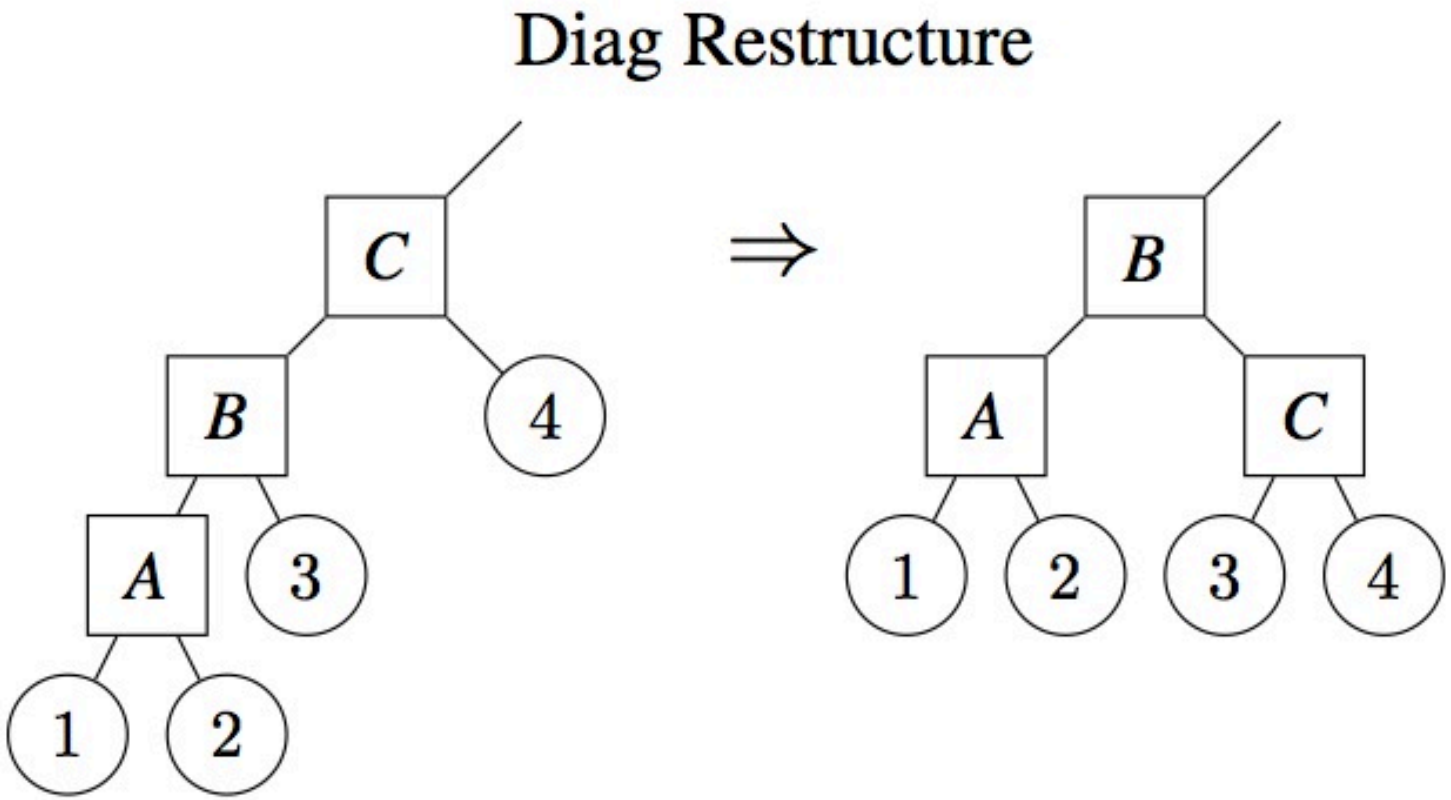
- Updates potentially require the tree to be restructured to maintain its balance properties
  - changes can affect any node between the update and root
  - locking requires a lock for each affected node
    - acquiring locks on the way up can deadlock with readers that are descending
    - acquiring all possible locks ahead of time degenerates to coarse grain locking
  - conventional approaches use a single lock for the tree
    - no concurrency

Restructuring is also a concern for RP



A thread searching for B could fail to find it

Restructuring is also a concern for RP



A thread searching for B could fail to find it

Lookups

- Readers ignore color and do not access parent pointers
- Readers stop searching when they find a key that matches
- Updates can change colors and place multiple copies in the tree so long as they appear in valid sort positions, without affecting readers
- Relativistic lookups are essentially sequential code!

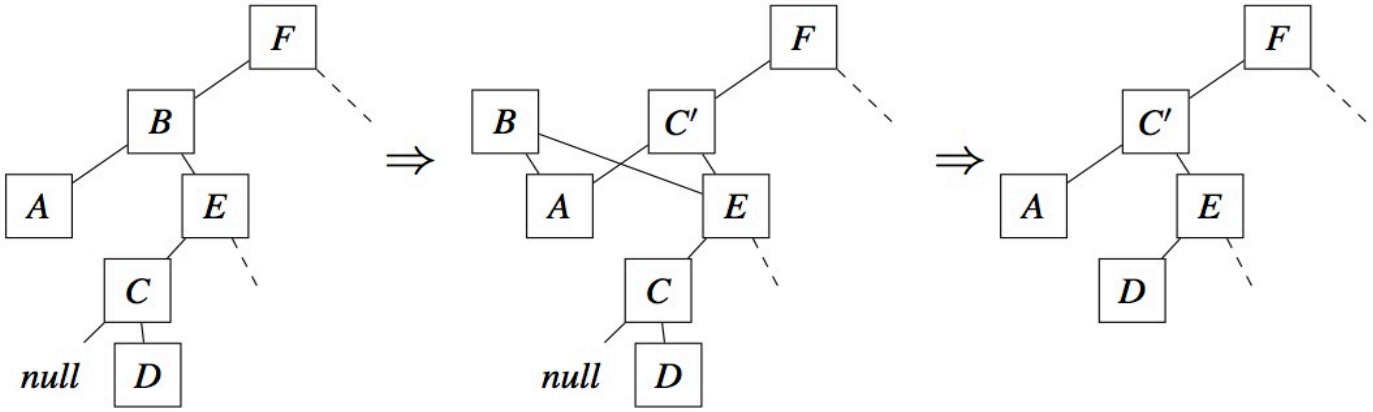
Inserts

- A new node is always inserted as a leaf
- Concurrent readers will see it if they dereference its pointer before the update publishes the pointer
- If the insert breaks the tree’s color or balance properties it must be recolored or rebalanced
  - that's the! tricky part

Deletes

- Nodes only deleted from the bottom of the tree
  - this may require some restructuring (called a swap)
- Readers will either see the deleted node, or they will not, depending on when they dereference its pointer
- The node's memory must not be reclaimed while readers are still accessing it
  - use rp-free
- If the delete leaves the tree unbalanced it must be restructured

Swap and Deletion of Node B



Before During After

Move B's next node (C) to B's position
- create new copy of C (C')
- assign B's children to C'
- give C' B's parent (B is now unreachable)
- defer free of B's memory

Defer deletion of C
- wait for readers, then reassign C's children to C's parent (E)

Defer free of C's memory

Code for Swap

```
1  C = next(B);
   C_prime = C.copy();

   C_prime.color = B.color;
5
   C_prime.left = B.left;
   C_prime.left.parent = C_prime;

   C_prime.right = B.right;
10 C_prime.right.parent = C_prime;

   F = B.parent;
   C_prime.parent = F;

15 if (F.left == B)
     rp-publish(F.left, C_prime);
   else
     rp-publish(F.right, C_prime);

20 rp-free(B);

   wait-for-readers();

   E = C.parent;
25 rp-publish(E.left, C.right);
   C.right.parent = E;

   rp-free(C);
```

Listing 1: Code for swap

Special Case Swap

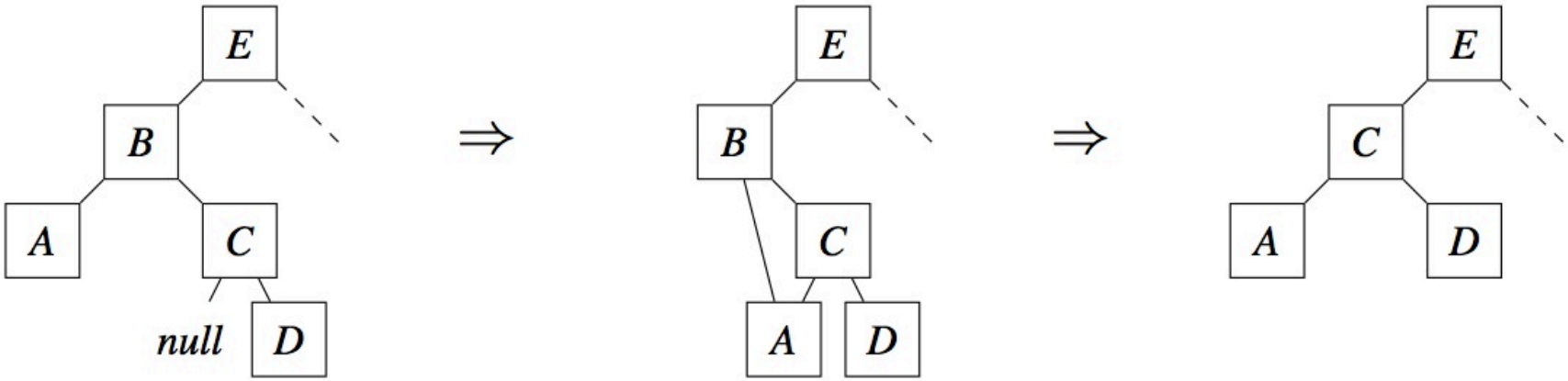


Figure 5. Tree before and after deletion of node *B* including one intermediate step

No copy is necessary
C adopts B's left child (A)
A appears twice in the tree (its temporarily a DAG)
- this does not affect readers
Defer free of B

Code for Special Case Swap

```
1  C = next(B);

   C.color = B.color;
   C.left = B.left;
5  C.left.parent = C;

   E = B.parent;
   if (E.left == B)
     rp-publish(E.left, C);
10 else
     rp-publish(E.right, C);

   rp-free(B);
```

Listing 2: Code for special case Swap

Diagonal Restructure

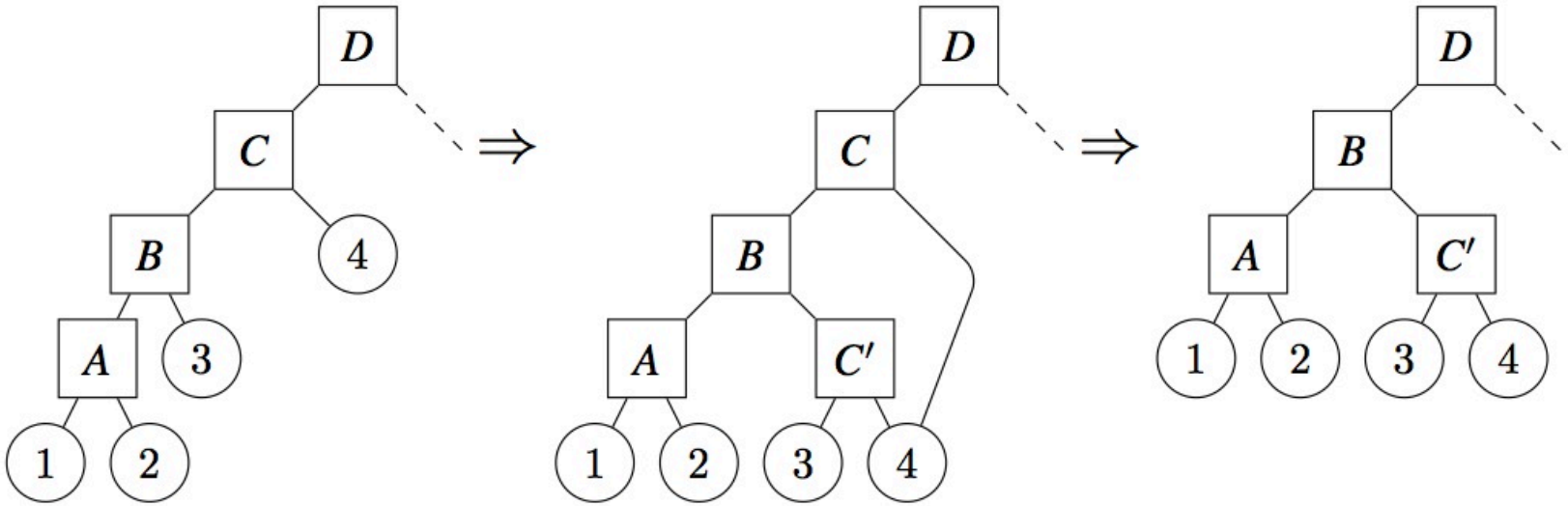


Figure 6. Arrangement of nodes before and after a diag restructure including one intermediate step.

Code for Diagonal Restructure

```
1  C_prime = C.copy();
   C_prime.left = B.right;
   C_prime.left.parent = C_prime;

5  rp-publish(B.right, C_prime);
   C_prime.parent = B;

   D = C.parent;

10 if (D.left == C)
     rp-publish(D.left, B);
   else
     rp-publish(D.right, B);

15 B.parent = D;

   rp-free(C);
```

Listing 3: Code for diag left restructure

Zig Restructure

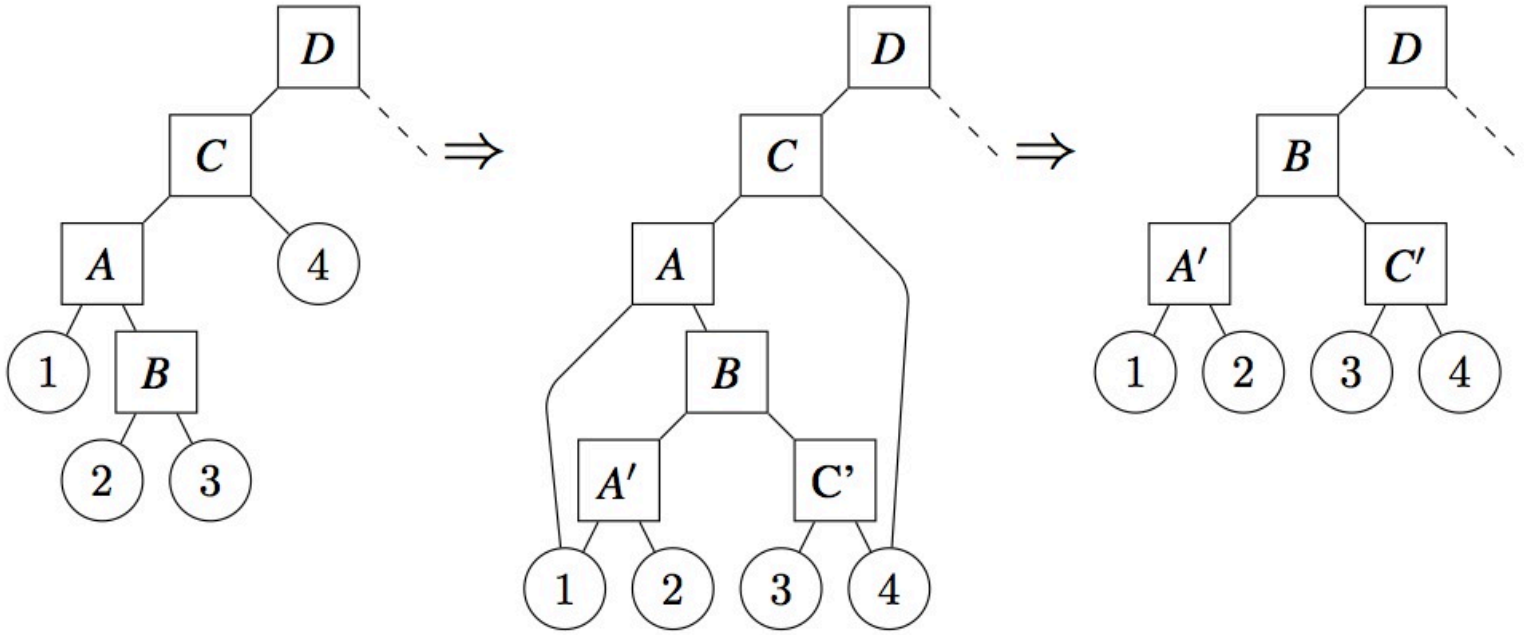


Figure 7. Arrangement of nodes before and after a zig restructure including one intermediate step.

Code for Zig Restructure

```
1  A_prime = A.copy();
   A_prime.right = B.left;
   A_prime.right.parent = A_prime;

5  rp-publish(B.left, A_prime);
   A_prime.parent = B;

   C_prime = C.copy();
   C_prime.left = B.right;
10 C_prime.left.parent = C_prime;

   rp-publish(B.right, C_prime);
   C_prime.parent = B;

15 D = C.parent;
   if (D.left == C)
     rp-publish(D.left, B);
   else
     rp-publish(D.right, B);
20
   rp-free(A);
   rp-free(C);
```

Listing 4: Code for zig left restructure

Read (lookup) Performance

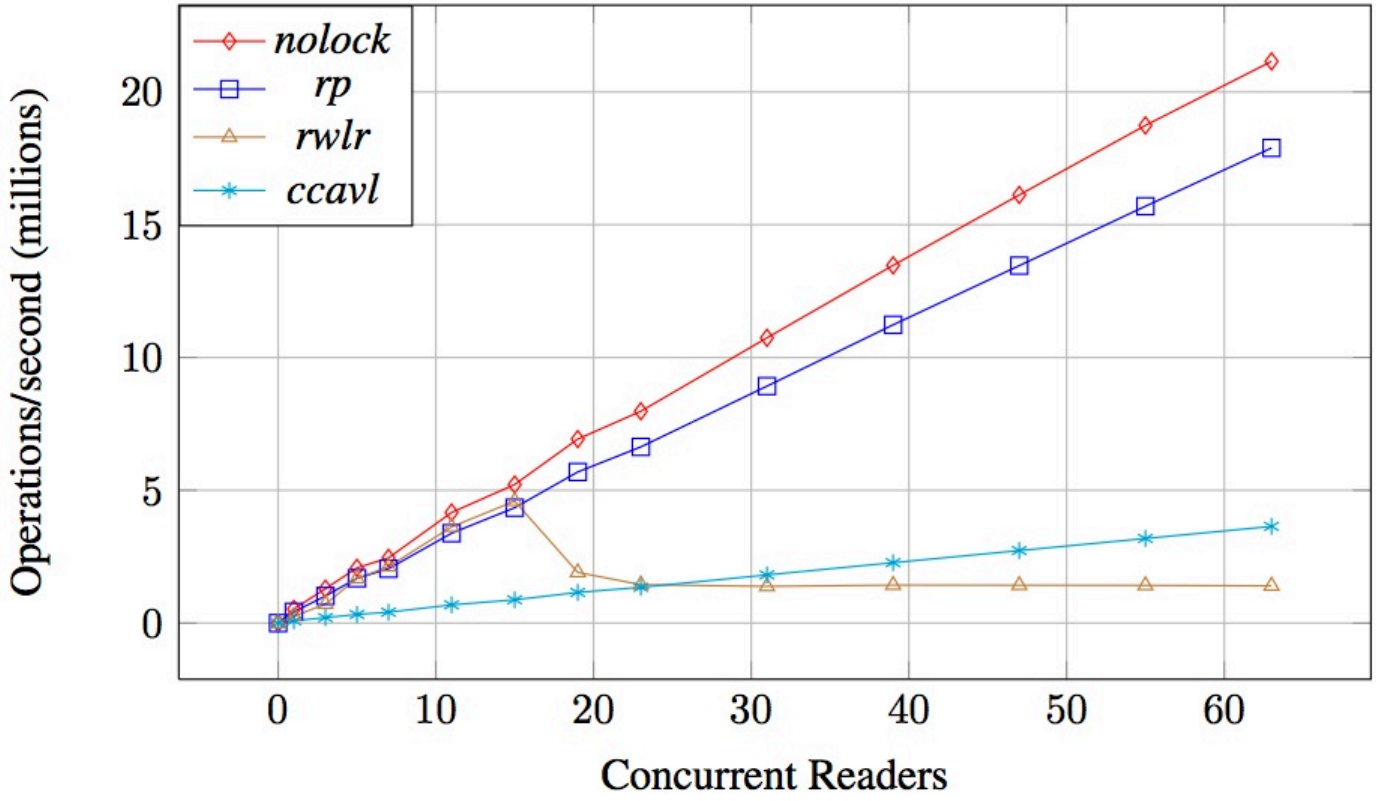


Figure 8. Read performance of 64K node red-black trees using a variety of synchronization techniques.

Sequential Write (insert/delete) Performance

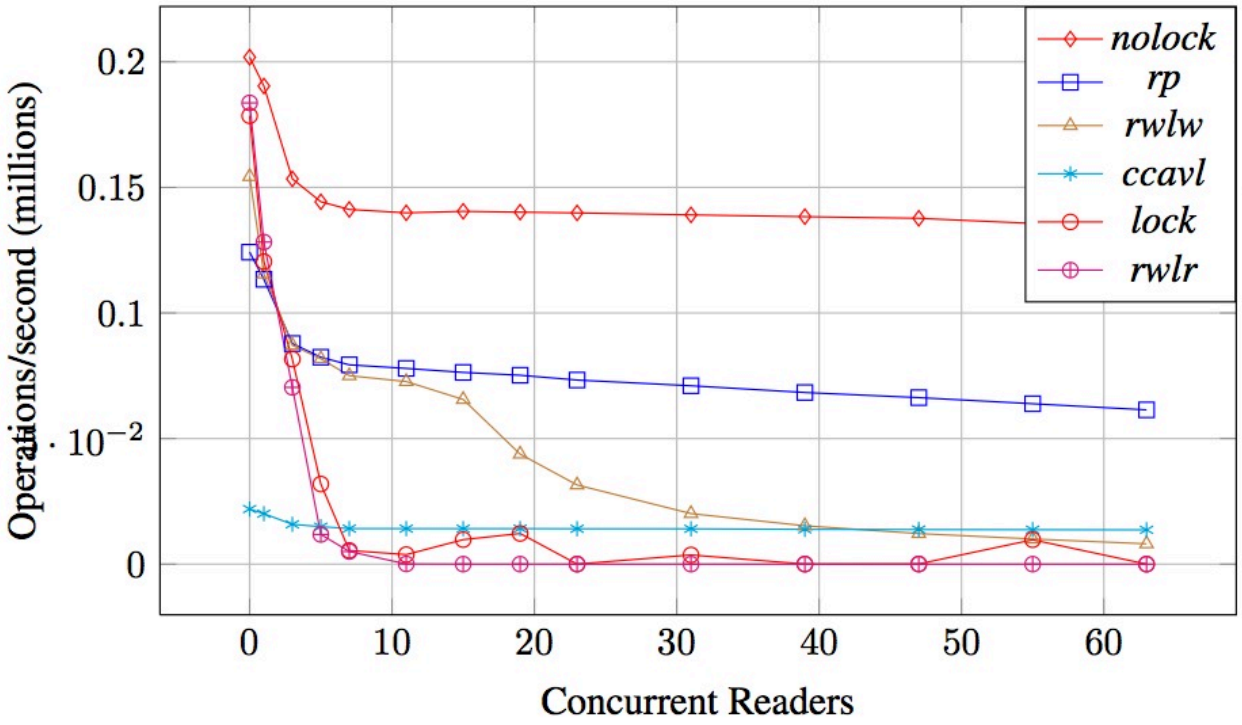


Figure 9. Update performance of 64K node red-black trees with a single updater and multiple readers. The left-most data point shows uncontended write performance. The remainder of the data points show the update performance with a variable number of concurrent readers.

Write-side Synchronization

- Global locking
  - no concurrent writes
- Fine-grain locking
  - deadlock
- CCAVL
  - concurrent, but different data structure
- STM - transactional memory
  - disjoint access parallelism (to be discussed later)

Concurrent Write (insert/delete) Performance

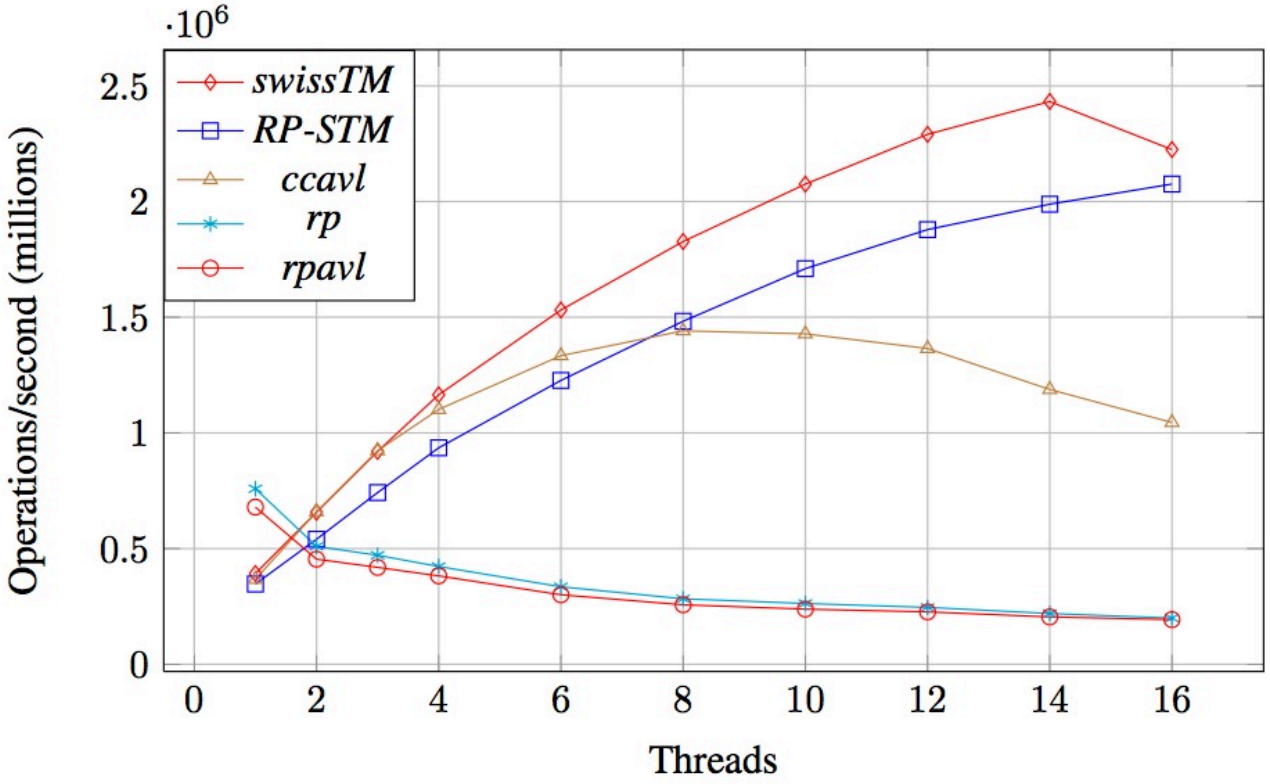
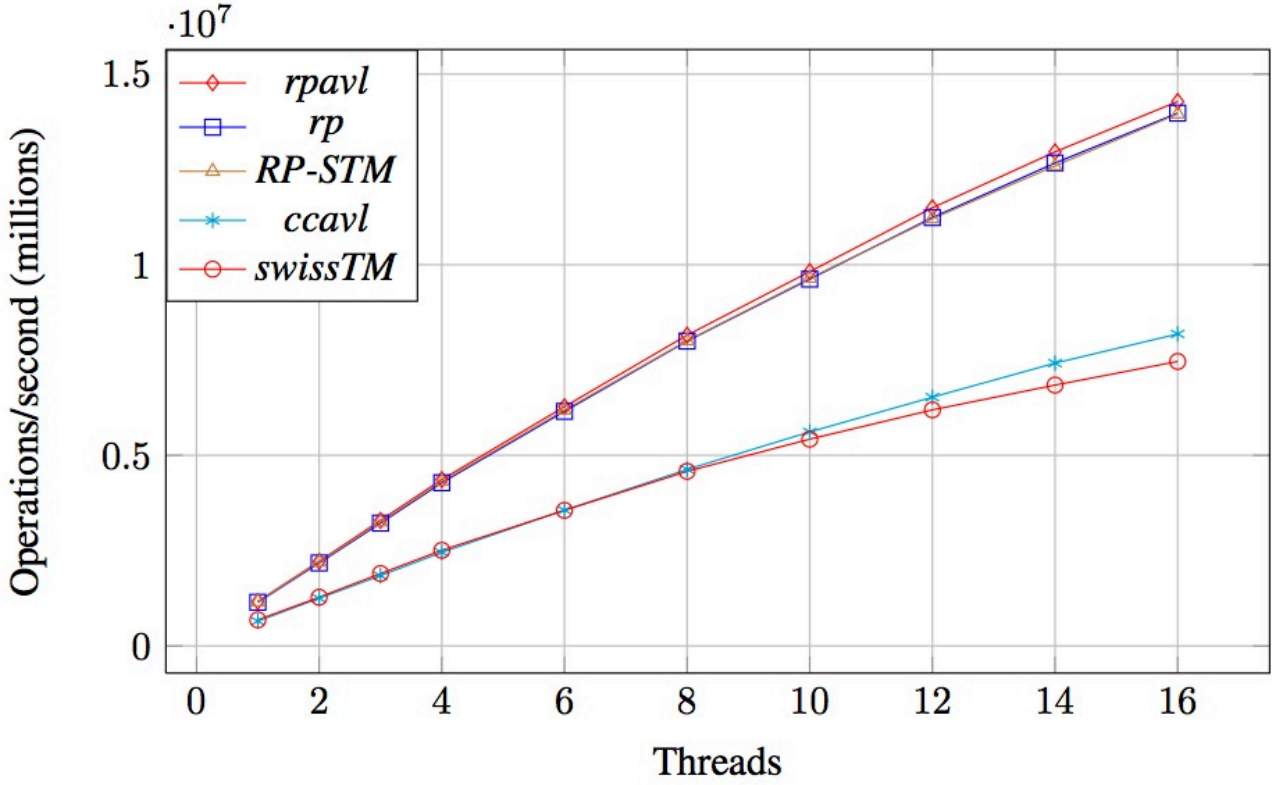


Figure 10. Concurrent update performance of 64K node trees.

Concurrent Read (lookup) Performance



Concurrent read performance of 64K node trees. Note that the *rp* and *RP-STM* lines are on top of each other.

Linearizability

- Lookup, insert and delete operations are linearizable
  - there is a well defined point at which they take effect
  - they are primitive operations with only one update (can't be seen out of order!)

Linearizability

- Lookups
  - last rp-read is the linearization point
- Inserts
  - rp-publish is the linearization point
- Deletes
  - store is the linearization point
- Traversals are not necessarily linearizable

Traversals

- Traversals visit the nodes in order
  - they make use of the next operation
- Traversals are challenging for RP because they read more than one location
  - restructures might cause nodes to be missed or duplicated

Traversals

- Three approaches explored:
  - treat a traversal as a single indivisible operation
    - acquire a lock and hold it for the duration
    - replace the write lock with a reader-writer lock
  - O (N log (N)) relativistic traversals
    - can't use parent pointers
    - use relativistic lookups to construct the traversal (traversal take O (N log (N)))
    - updates only wait for lookups
    - traversal is non-linearizable

Traversals

- Third approach: O(N) relativistic traversal
  - requires modification of update operations to allow readers to use parent pointers
  - requires additional node copies to preserve the parent pointers

Traversals

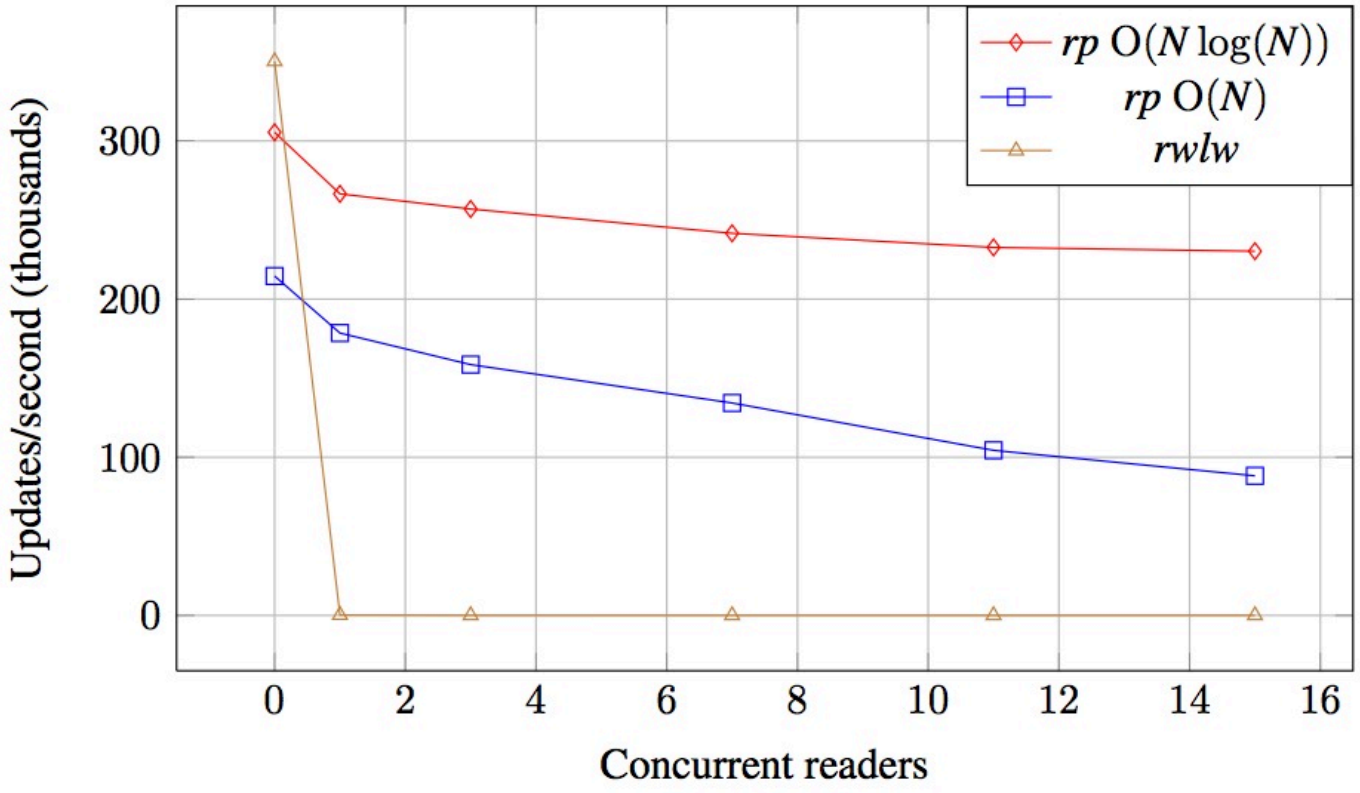


Figure 12. Update performance in the presence of concurrent readers performing traversals

Traversals

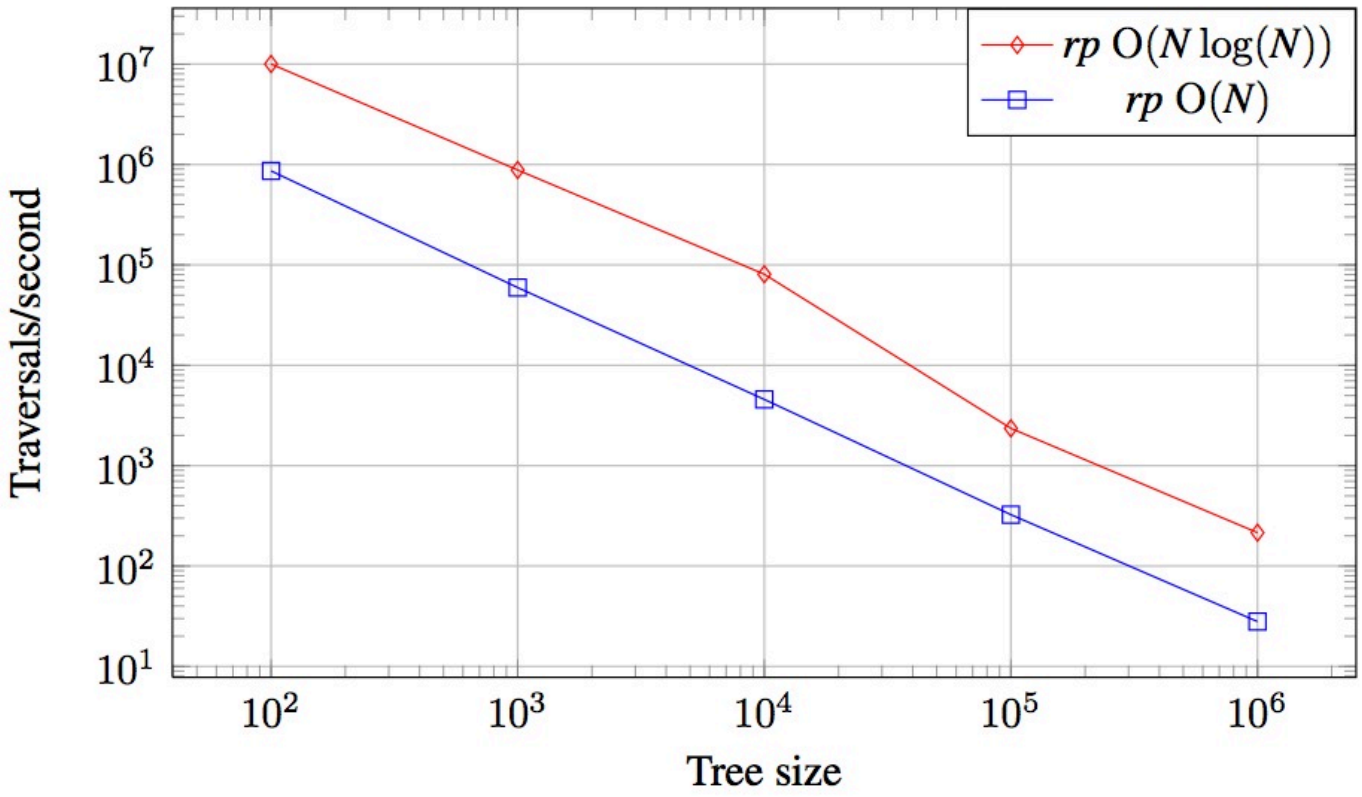


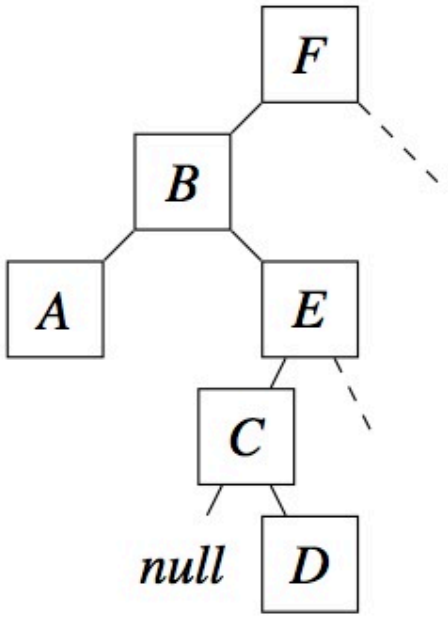
Figure 13. Read performance

Conclusions

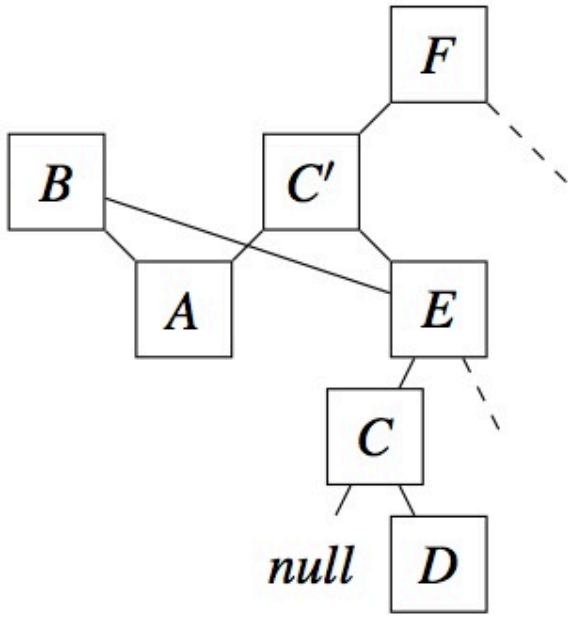
- Relativistic programming can be applied to a data structure as complex as a Red-Black Tree with excellent performance and scalability results

Spare Slides

Reader-visible States in Swap

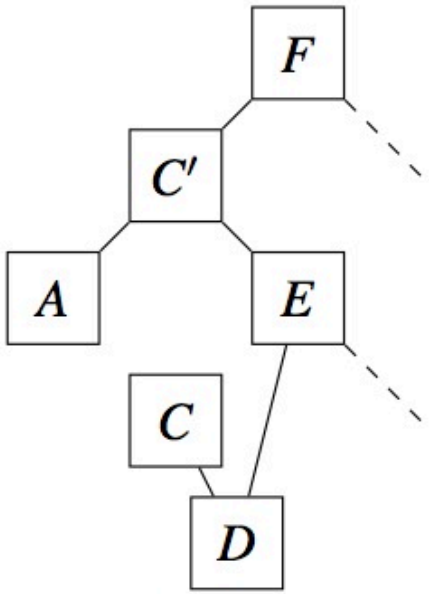


a) Initial subtree

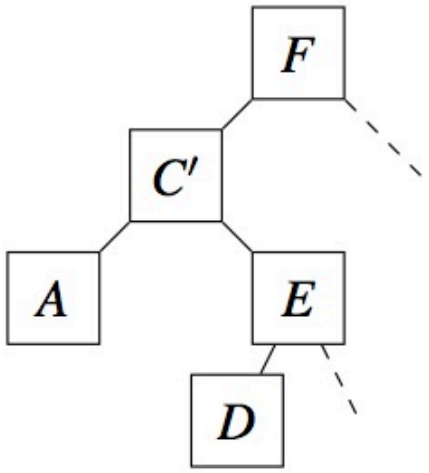


b) After insertion of C'

Reader-visible States in Swap

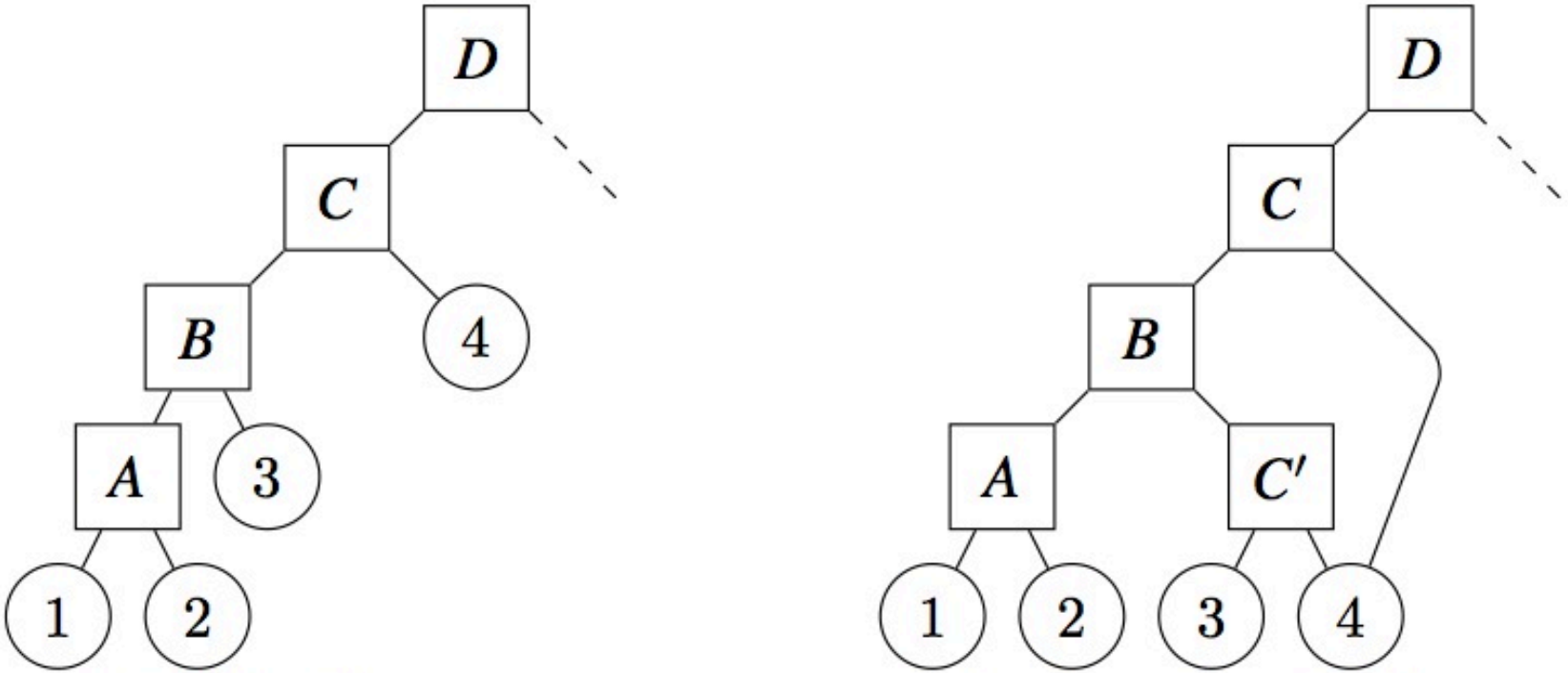


c) After removal of *C*



d) Final subtree

Reader-visible States in Diagonal Restructure



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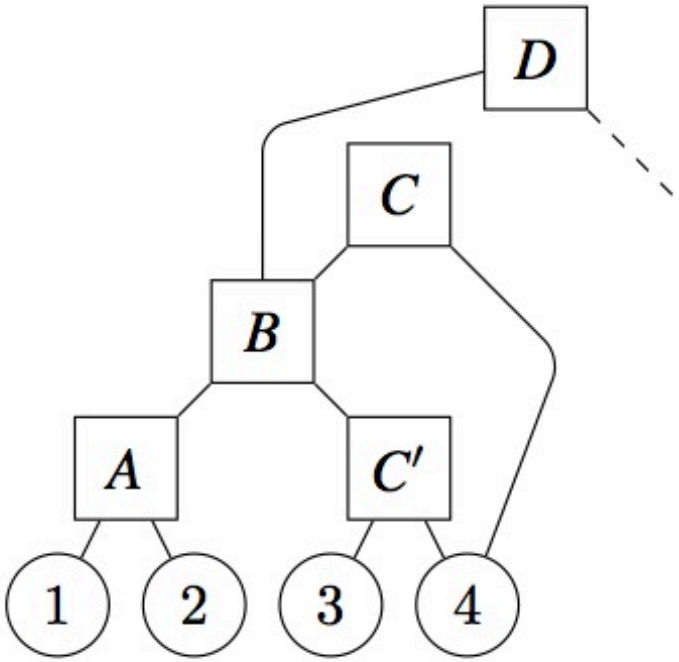
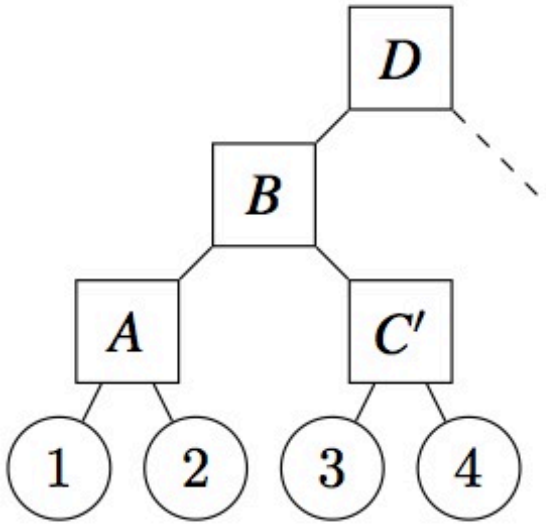
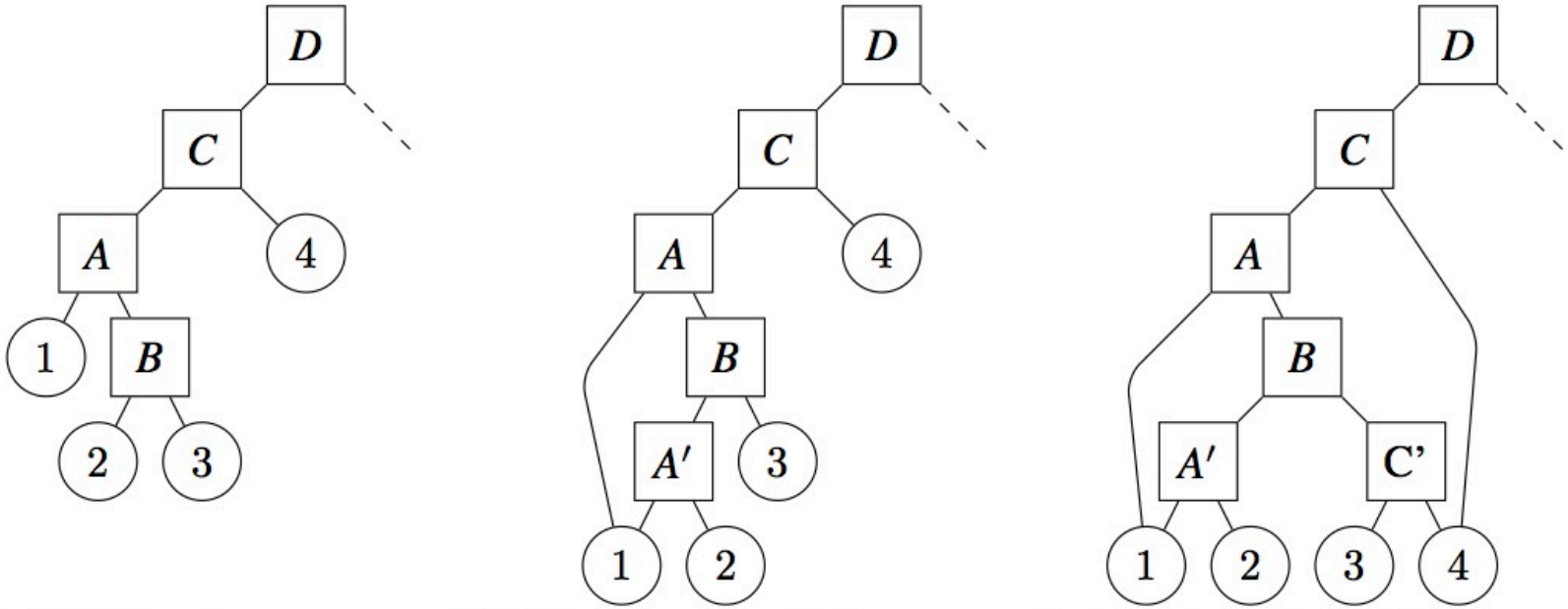


c) After removal of C



d) Final subtree

Reader-visible States in Zig Restructure

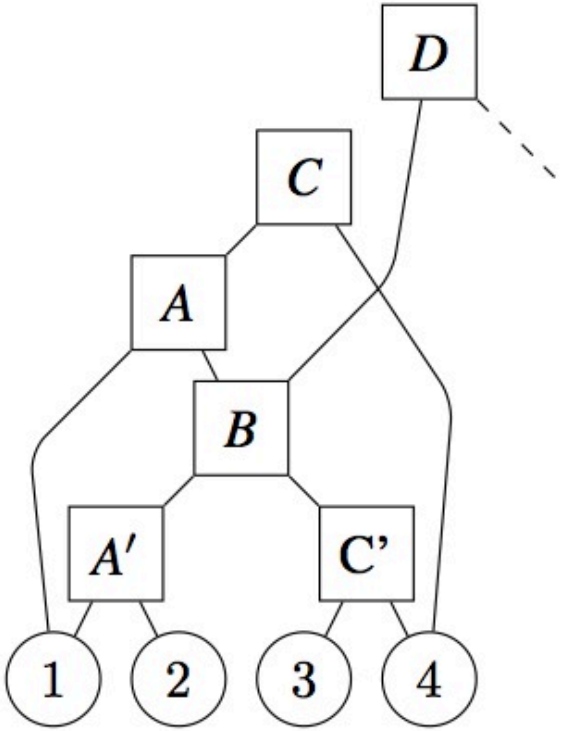


a) Initial subtree

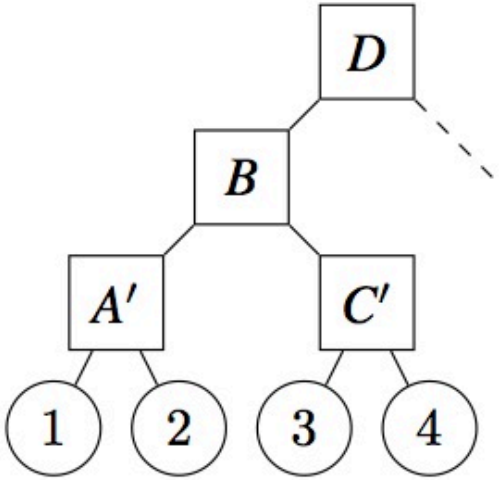
b) After insertion of A'

c) After insertion of C'

Reader-visible States in Zig Restructure



d) After removal of A and C



e) Final subtree