A Spectral Analysis of the Sunspot Time Series Using the Periodogram

Felipe E. Olvera, Jr., *Student Member, IEEE*

Abstract— **The sunspot number time series was used to examine different methods of the periodogram as a spectrum estimator. The periodogram was used as the benchmark to compare the results of the sunspot cycle period estimate to the modified periodogram, the Blackman-Tukey method of periodogram smoothing, the Welch-Bartlett method of periodogram averaging and the Multitaper method. The Welch-Bartlett method resulted in the closest estimate to the modern sunspot cycle period estimate of 10.4883 years/cycle.**

Index Terms— **Autocorrelation Function, Blackman-Tukey Method, Modified Periodogram, Multitaper Method, Periodogram, Power Spectral Density, Sunspot Number, Welch-Bartlett Method.**

I. INTRODUCTION

SOLAR magnetic disturbances manifest as dark spots on the surface of the sun which are visible from Earth. These disturbances, commonly named sunspots, have been observed since ancient times but the spectral study of this phenomenon began only within the last two centuries.

Schwabe collected 17 years of sunspot observations while searching for intramercurial planets. His observations revealed a 10-year periodicity in the number of visible sunspots. [1] Schuster studies of the sunspot numbers collected by Wolf and Wolfer led him to develop the periodogram, [2] a form of spectrum estimation, to determine if there was any hidden periodicity to the sunspot cycle.

The investigation of the sunspot time series has led to advances in spectrum estimation. As an example, Yule [3] introduced the concept of a finite parameter model for a stationary random process with special reference to Wolfer's sunspot numbers. The periodogram was the only numerical method of spectrum estimation available until Yule extended the analysis by developing this autoregressive spectrum estimation method. [4] Many papers are still being written investigating this important time series. [5]

Determining the sunspot cycle period was and is important in order to compare the period estimate with disruptions to radio and satellite communications and with weather cycles. There are strong indications that the cooling and warming of the Earth might be due to the changes in the number of observed sunspots. [6] During the Maunder Minimum (1645 to 1715), few sunspots were observed and this time coincided with the Little Ice Age during which Europe and North America were subjected to bitterly cold winters. [7]

Originally the daily sunspot number calculated by Wolf and Wolfer was obtained by multiplying the number of groups they observed by ten and then adding this product to the total count of individual spots. The daily sunspot number today is a more relative number and not the actual observed number of spots on the surface of the sun. The sunspot number is a *smoothed* number based on the weighted average of measurements made from a network of observatories. This ensures that the differences in observations due to location, weather, observer, etc., do not adversely affect the daily sunspot number.

From the historical data available to Wolf and Wolfer and from Wolf's observations, Wolf concluded that the sunspot cycle period was about 11.1 years/cycle. When Schuster analyzed the same data using the periodogram to find the hidden periodicity, he determined that the sunspot cycle period was 11.125 years/cycle confirming Wolf's original results. The currently accepted estimate for the sunspot cycle period is 10.4883 years/cycle.

Unfortunately, the periodogram is a poor estimator [8] of the power spectrum. Even though it is a poor estimator, if there are enough observations, the period and amplitude of the underlying sinusoid can be approximated. By using the modified periodogram, the method of periodogram averaging, and the multitaper method, a better estimate of the power spectrum was obtained and the sunspot cycle period was estimated more accurately.

II. METHODOLOGY

A. Experimental Data

The daily sunspot number data were obtained from the National Geophysical Data Center. [9] There are two different types of sunspot numbers, the Boulder Sunspot Number and the International Sunspot Number. The difference between the two sunspot numbers is scaling so the decision was made to use the International version because of the greater length of the available data. Two sets of these International Sunspot Numbers are available: 1) daily sunspot numbers from 1818 to 2005 and 2) yearly mean sunspot numbers from 1700 to 2004. The yearly mean sunspot numbers were used to generate the spectral analysis results.

B. Procedure

In the following spectral analysis, the sunspot cycle was first derived from sunspot data to establish the periodicity that is evident in the data. The mean was removed from the sunspot data and then the periodograms were generated. The difference in each periodogram method affected the sunspot cycle period

This work was completed as part of a course project for *Statistical Signal Processing* at Portland State University during the Fall term of 2005.

F. Olvera is a graduate student at the Portland State University Maseeh College of Engineering and Computer Science.

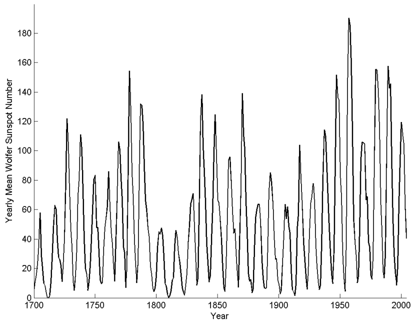


Fig. 1. The yearly mean of observed sunspots from 1700 to 2004.

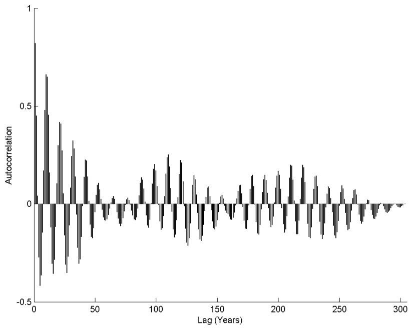


Fig. 2. The autocorrelation function of the yearly mean sunspot number.

estimate. The sunspot cycle period estimate was then derived from each of the periodograms.

The autocorrelation function shown in Fig. 2 was generated for stationary analysis of the sunspot number time series and for observations regarding this series' periodicity.

III. RESULTS

The autocorrelation function for the sunspot number data clearly demonstrates periodicity with small cycles inside of larger cycles. Looking closer at the period of the smaller cycles as shown in Fig. 3, the 11-year period can be clearly seen in the autocorrelation and yearly mean sunspot number plots. The autocorrelation estimate does not decay significantly for large lag values so the *memory* of the series does not disappear quickly.

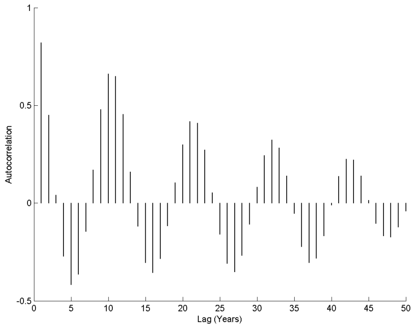


Fig. 3. Magnified autocorrelation function to demonstrate the 11-year period.

The periodogram for the yearly mean sunspot number and the sunspot cycle period estimate were generated and are shown in Fig. 4. Zero-padding was not used in order to replicate Schuster's work. The periodogram resulted in the sunspot cycle period estimate that was close in value to Schuster's original estimate.

The modified periodogram using the Blackman window achieved good results for the sunspot cycle period estimate as shown in Fig. 5. Surprisingly, the window choice did not make as much of a difference in the sunspot cycle period estimate as the choice of window length. Window lengths above 50% of the data length resulted in higher period estimates.

The Blackman-Tukey method of smoothing the periodogram as shown in Fig. 6 resulted in a lower sunspot cycle period estimate than the periodogram but worse than the modified periodogram. Various windows and window lengths were tried but the period estimate became worse when the window length exceeded 20% of the data length.

Smooth periodograms were achieved with the Welch-Bartlett method of periodogram averaging as shown in Fig. 7 but the period estimate was the same as the Bartlett method alone. The Blackman windows resulted in the smoothest periodograms and the rectangular window resulted in the periodogram as shown in Fig. 4. The percentage of overlap changed the period estimate slightly but the best results were with overlaps of 40% to 50%. The period estimate was sensitive to segment length and segment length values of about ten percent of the data length resulted in the best period estimate. As the segment length increased, the Welch-Bartlett method returned results similar to the periodogram.

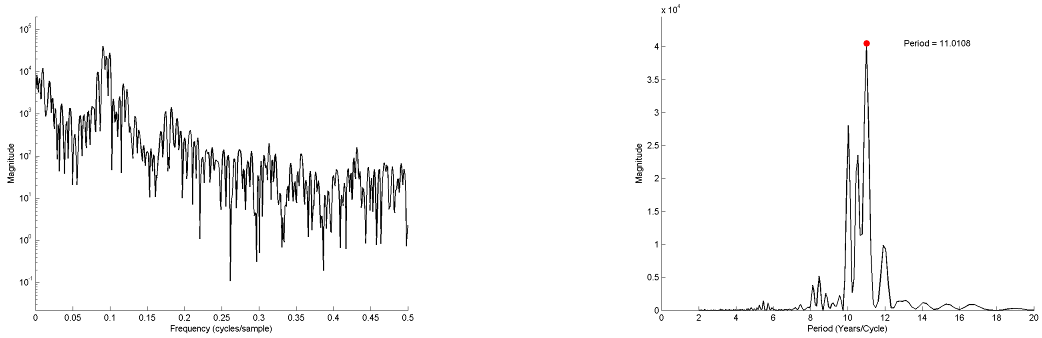
The final periodogram method used was the multitaper method. Various Slepian sequences were tried and the 5/2 sequence resulted in the best period estimate as shown in Fig. 8. The periodogram showed a lot of *roughness* as expected from this method.

IV. DISCUSSION

Can these spectral analysis methods be used to estimate the sunspot number power spectrum? In order to use these methods, second-order stationarity must be assumed. The question of stationarity was rarely addressed in the literature concerning this time series. The available data while seemingly large are but an infinitesimal fraction of a process that follows cosmological time.

Is this process ergodic? There is no method available currently that allows many realizations of sunspot numbers to be evaluated. Once sunspot data are available for other stars, then the question of general ergodicity might by answered. However, since the statistical properties can be measured using the time averages of this time series, then this process can be considered ergodic.

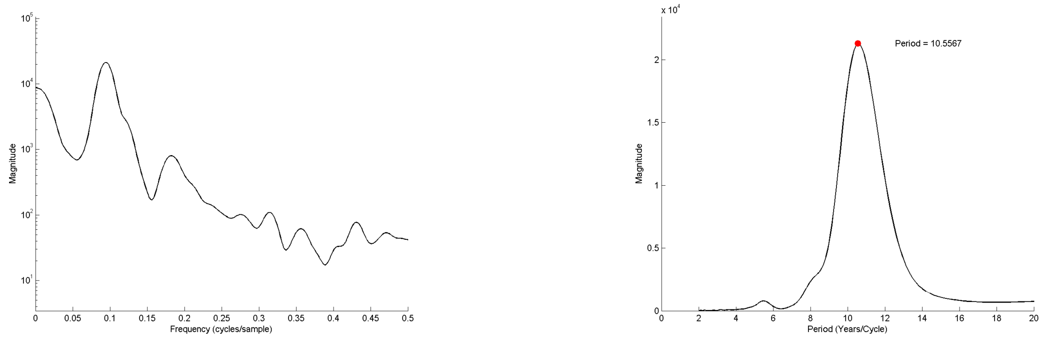
Is this process stationary? Since the data are evenly spaced, regular and there are no long-term trends, this time series can be considered stationary. All that can be assumed for this time series is that it is stationary to at least order two. Because



(a) Yearly mean sunspot number periodogram.

(b) The period estimate is 11.01018 years/cycle.

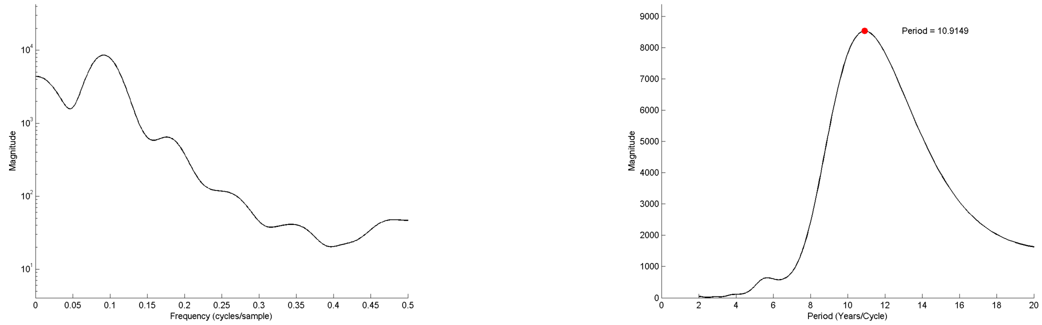
Fig. 4. Periodogram results for the yearly mean sunspot number.



(a) Yearly mean sunspot number modified periodogram.

(b) The period estimate is 10.5567 years/cycle.

Fig. 5. Modified periodogram using the Blackman window.



(a) Yearly mean sunspot number periodogram.

(b) The period estimate is 10.9149 years/cycle.

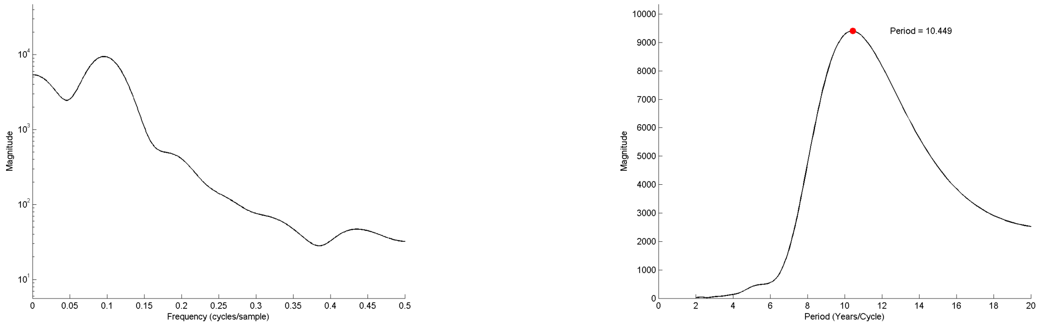
Fig. 6. Periodogram using the Blackman-Tukey method with a Hanning window.

of this assumption, the methods used to estimate the power spectral density can be considered valid.

The periodogram method developed by Schuster had the expected performance. The modified periodogram and the periodogram averaging methods returned better results but care was needed in order to control the period estimate. Different windows were used in order to determine which window resulted in the best estimate of the power spectral density.

The multitaper method was the easiest to use since only the choice of tapers was necessary and more importantly, returned results consistent with the periodogram averaging method. The choice of tapers was limited to five so the selection for the best results was easy but had the period estimate not been known ahead of time, the selection of the best period estimate would have been more difficult.

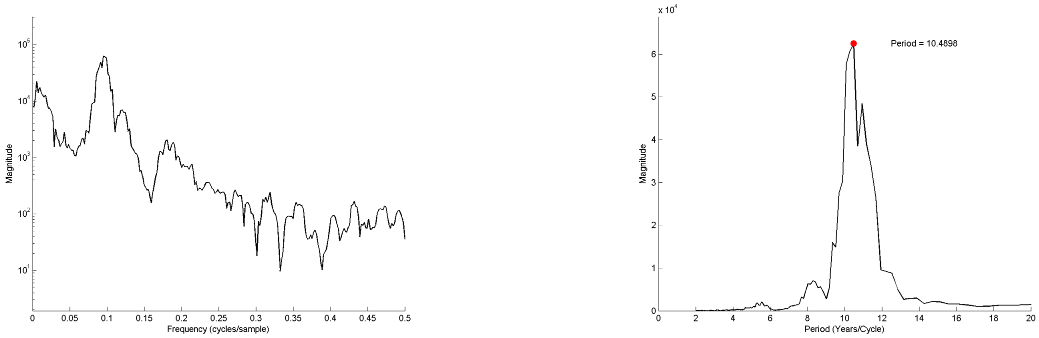
The main surprise was the poor performance of the Blackman-Tukey method of periodogram smoothing in analyzing this time series. Various windows of different lengths were tried along with different FFT lengths. Small window lengths resulted in smooth periodograms but the period estimate never went below 10.9 years/cycle. There was no satisfactory explanation for this result.



(a) Yearly mean sunspot number periodogram.

(b) The period estimate is 10.449 years/cycle.

Fig. 7. Periodogram using the Welch-Bartlett method. The Blackman window was used with a 50% overlap.



(a) Yearly mean sunspot number periodogram.

(b) The period estimate is 10.4898 years/cycle.

Fig. 8. Periodogram using the multitaper method.

The autocorrelation plots demonstrated the different cycles in the sunspot number time series. The 11-year cycle is clearly visible when the autocorrelation plot is magnified as shown in Fig. 3. By observing each section of the autocorrelation plot, the number of sunspots for the next year can be estimated. More importantly, the autocorrelation function of this time series demonstrates how strongly periodic it is.

An interesting observation of the sunspot time series is the upward trend of the number of observed sunspots as shown in Fig. 1. The sunspot number seems to be increasing from cycle to cycle (currently we are in Cycle 23) but that analysis is beyond the scope of this report.

V. CONCLUSION

The periodogram performed as expected in estimating the sunspot cycle period. The modified periodogram performed better in estimating the period as did the multitaper method but surprisingly, the Blackman-Tukey method of periodogram smoothing had the worst performance in estimating the period. The method of periodogram averaging had the best period estimates and these period estimates were always within a consistent range.

The smoothing effect of the newer periodogram methods made a positive difference on the estimate of the sunspot cycle period as long as care was taken with the selection of the window and segment length. This was an expected condition due to the fundamental tradeoff between the variance of the power spectral density and its frequency resolution. By the use of these periodogram methods, Schwabe's and Wolf's original sunspot cycle period estimates have been refined.

REFERENCES

[1] S. H. Schwabe, "Solar observations during 1843," *Astronomische Nachrichten*, vol. 20, no. 495, pp. 234–235, 1843.

[2] A. Schuster, "On the Periodicities of Sunspots," *Philosophical Transactions of the Royal Society*, vol. 206, pp. 69–100, 1906.

[3] G. U. Yule, "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers," *Philosophical Transactions of the Royal Society*, vol. 226, pp. 267–298, 1927.

[4] E. A. Robinson, "A Historical Perspective of Spectrum Estimation," *Proceedings of the IEEE*, vol. 70, pp. 885–907, 1982.

[5] G.-M. Le and J.-L. Wang, "Wavelet Analysis of Several Important Periodic Properties in the Relative Sunspot Numbers," *Chinese Journal of Astronomy and Astrophysics*, vol. 3, pp. 391–394, 2003.

[6] J. L. Ballester and R. Oliver, "Discovery of the Near 158 Day Periodicity in Group Sunspot Numbers During the Eighteenth Century," *The Astrophysical Journal*, vol. 522, pp. 153–156, 1999.

[7] D. T. Shindell, G. A. Schmidt, M. E. Mann, D. Rind, and A. Waple, "Solar Forcing of Regional Climate Change During the Maunder Minimum," *Science*, vol. 294, no. 5549, pp. 2149–2152, 2001. [Online]. Available: http://www.sciencemag.org/cgi/content/reprint/294/5549/2149.pdf

[8] D. G. Manolakis, V. K. Ingle, and S. M. Kogan, *Statistical and Adaptive Signal Processing*. Artech House, 2005.

[9] [Online]. Available: http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html