Analysis of the Instantaneous Estimate of Autocorrelation

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Overview
• Introduction & Background
• The Instantaneous Estimate of Autocorrelation defined
• The Data
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• Conclusion

Introduction: The Problem
• Many signals are not stationary of the second order (their Autocorrelation Function (ACF) is time-varying)
• The ACF is used to develop filters and predictors (e.g. for signal compression)
• This suggests that we need filters and predictors that are time-varying
• A common approach has been to assume local stationarity:
  → We can estimate filters for the signal by using ‘frame blocking’
  → Frame blocking does not necessarily identify appropriate frame boundaries

Introduction: A Potential Solution
• It would be better if we could design filters which changed along with the autocorrelation of the underlying data
• The Instantaneous Estimate of Autocorrelation (IEAC) is a time-varying estimate of the autocorrelation of a signal
• Thus, we should vary our filters according to variation in the IEAC
• We concentrate on analysis of the IEAC, and leave the development of filters based on this signal to a forthcoming paper

The IEAC Defined
• The IEAC of a first order signal \( x \) is a ‘second order signal’, which is defined as:
  \[
  x_c(n) = \{ x(n) \} (x(n-1) \ldots x(n-M))
  \]
• The first several lagged values are defined to be zero, since we don’t know \( x(n) \) for \( n<0 \)
  → This will create bias in our estimate
• The vector space of the IEAC should be viewed as the ‘correlation space’

IEAC: The First Moment
• The ‘natural’ estimator of the mean of the IEAC is equivalent to a vector composed of the (biased) estimates of autocorrelation:
  \[
  \hat{r}_k(t) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)x(n-k)
  \]
  \[
  \mu_{x_c} = [\hat{r}_0(0) \hat{r}_x(1) \ldots \hat{r}_x(M)]
  \]
• In general, we do not want to reduce this signal to a point estimate
• On the other hand, we need a time varying estimate with less variance
• We used a sliding window to average each element of the IEAC over a short duration
IEAC: The Second Moment
- Since we are dealing with the variance of a vector-valued signal, we may estimate the (scalar) variance projected onto an arbitrary vector.
- In other words, we are not only finding the variance, but the direction of variance.
- We find it useful to select vectors that maximize the variance.
- These vectors are called the principal components of the signal.

IEAC: Spectral Interpretations
- If we assume that the ACF is time-varying, then we have a Power Spectral Density (PSD) that changes with time.
- We define the time-varying PSD as the Short Time Fourier Transform of the (windowed) IEAC.
- We reduced the variance of the PSD estimate via the Blackman-Tukey method, since the Welsh-Bartlett method requires a longer time domain extent.

The Data
- We will analyse the following signals:
  - A stationary signal
  - A non-stationary signal

\[ x(n) = \begin{cases} 
  1.0e(n) + 0.4e(n-1) - 0.4e(n-2) & 0 \leq n < \frac{N}{2} \\
  1.0e(n) + 0.4e(n-1) + 0.4e(n-2) & \frac{N}{2} \leq n < N 
\end{cases} \]

✓ If we blocked all N samples of the non-stationary signal to compute the ACF, we would expect to obtain:

\[ [1.0 0.4 0.0] \]

Results: A Time-Varying PSD
(stationary signal)

Results: Average of Time-Varying PSD
(stationary signal)

Results: A Plot of the IEAC
(non-stationary signal)
Results: Correlation Space (non-stationary signal)

Results: Filtered Correlation Space (non-stationary signal)

Results: Projection onto PC(1) (non-stationary signal)

Discussion
• In the case where the data is stationary, we have not lost anything (we can average the IEAC)
• We can see (if we assume that the ACF is piecewise constant) where to introduce frame boundaries
• For more complicated signals, we will have to parameterise the trajectory of the IEAC in the correlation space

Conclusion
• The approach seems like a good one, but it is computationally expensive
  → The IEAC is a huge signal if we consider many lags
• The extension of the IEAC to cross correlation (IECC) is straightforward
• We need to be able to parameterize the variation of the IEAC in order to
  → block the signal appropriately
  → develop time-varying filters