Errata and Notes for

Statistical and Adaptive Signal Processing

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Errata are denoted with a $*$ and my notes and comments are denoted with a •. This list was a joint effort of me and the students who were enrolled in my course, *Statistical Signal Processing*, during winter term of 2003 at Portland State University.

Chapter 2

$*$ **Page 39:** The second to last sentence is phrased badly. It should end with "…by an impulse train and a continuous spectrum in the frequency domain."

$*$ **Page 44:** The poles of $X(z)$ are the values that make $|X(z)| = \infty$, not just $X(z) = \pm\infty$, as stated in the second paragraph.

• **Page 51:** The first sentence begins, "If $d_k = 0$ for $k = 1, 2, \ldots, Q$, the system has only poles..." This sentence would be more clearly (but equivalently) stated if it began, "If $Q = 0$, the system has only poles..."

$*$ **Page 53:** The first equality in (2.3.27) should be $r_y(\ell) = h^*(-\ell) * r_{yx}(\ell)$.

$*$ **Page 54:** In equation (2.3.35) the last term in the sum should be $h^*(n-\ell)$, not $h^*(n-1)$.

$*$ **Page 56:** A maximum-phase system is one in which both the system and its inverse are *anticausal* and stable.

• **Page 61:** Section 2.4.4 assumes $x(n)$ is an all-pass signal (see pg. 54).

$*$ **Page 63:** The limits of the integral of the Paley-Wiener theorem should be $-\pi$ and π, not $-\infty$ and ∞ as listed in the text. See section 10.1 of Priestley for a more thorough discussion of this topic.

$*$ **Page 71:** In problem 2.7, part (c) the last expression should be $\angle[H(e^{j0}] - 2\pi P$. Note that it is an upper-case P representing the number of poles, not the lower-case p representing the pole itself.

* **Page 71:** In problem 2.8, part (a), it should say, "Consider the following stable minimum-, maximum-, and mixed phase systems." A system cannot be stable, causal, and mixed or maximum phase as the problem statement implies. You should assume that the systems are stable, not causal.

* **Page 73:** In problem 2.13, part (c), the last sentence should state, "If not, then construct a *maximum-phase* system ..."

Chapter 3

* **Page 76:** In the last sentence of the first paragraph, the book states "...,events and outcomes are now numbers,..." The outcomes, denoted ζ, and events, defined as sets of possible outcomes $\{\zeta_1, \zeta_2, \ldots, \zeta_k\}$, are still not numbers. However, the random variables are. The sentence should be replaced with "Since random variables as numbers, they can be added, substracted, or manipulated otherwise."

* **Page 76:** In paragraph 3 just under the illustration the last sentence should have three subscript zeros, rather than superscripts.

* **Page 78:** In the second to last line, it states $\gamma_m^{(0)} = 1$. The subscript should be x, not m. It should state $\gamma_x^{(0)} = 1$.

* **Page 79:** In (3.1.16) the exponent should be $r_x^{(m-k)}$, not $r_x^{(n-k)}$ as given.

* **Page 81:** The superscipt of the first moment in (3.1.28) should be 1, not x.

- **Page 83:** The sentence following (3.1.43) states that the moment generating function does not exist because $E\{x^2\}$ doesn't exist. This should have been qualified with "in general." Clearly the moment generating function does exist if, for example, $s = 0$. A discussion of the ROC should have been included here.

* **Page 85:** The fragment immediately following (3.2.12) lists "while its *marginal* pdf...." It should state "while its *joint* pdf...."

* **Page 85:** The joint pdf listed in (3.2.13) is wrong. It should be

$$f_{\boldsymbol{x}}(\boldsymbol{x}) = \lim_{\substack{\triangle x_{\text{R}1} \to 0 \\ \vdots \\ \triangle x_{\text{I}M} \to 0}} \frac{\Pr\{\boldsymbol{x}_{\text{R}} < \boldsymbol{x}_{\text{R}}(\zeta) \leq \boldsymbol{x}_{\text{R}} + \triangle \boldsymbol{x}_{\text{R}}, \boldsymbol{x}_{\text{I}} < \boldsymbol{x}_{\text{I}}(\zeta) \leq \boldsymbol{x}_{\text{I}} + \triangle \boldsymbol{x}_{\text{I}}\}}{\triangle x_{\text{R}1} \triangle x_{\text{I}1} \ldots \triangle x_{\text{R}M} \triangle x_{\text{I}M}}$$

  which is analogous to the real case listed in (3.2.4).

* **Page 87:** The term in the bottom right part of the Jacobian matrix in (3.2.35) should be $\frac{\partial y_M}{\partial x_M}$, not $\frac{\partial y_M}{\partial y_M}$.

* **Page 91:** In (3.2.63) the superscripts and subscripts are reversed.

- **Page 95:** After (3.2.89) the text states that "...only by the *location* parameter." It seems to me σ is also affected. That is, the standard deviation of $F_M(x)$ is different than that of $F_x(x)$.

- ∗ **Page 97:** It appears that the distributions shown in Figure 3.6 have $\beta = 0$ since they are symmetric, not $\beta = 1$.
- ∗ **Page 100:** The subscript of the term x_k in the numerator of (3.3.2) should be lower case.
- ∗ **Page 101:** Two random processes $x(n)$ and $y(n)$ are uncorrelated and orthogonal for the conditions listed in (3.3.19) and (3.3.20) even when $n_1 = n_2$.
- ∗ **Page 102:** In (3.3.21) the term on the right side should be $f_x(x_1, \ldots, x_N; n_1 + k, \ldots, n_N + k)$. The k in the time indices should not be a subscript.
- ∗ **Page 102:** The sentence after (3.3.24) states, "From (3.3.22), (3.3.24), and (3.3.5) it. . . " Equation (3.3.5) should be (3.3.6).
- ∗ **Page 103:** In Example 3.3.2 the probabilities of heads and tails should both be 0.5, not 1.0 for both as listed.
- ∗ **Page 103:** Equation (3.3.26) is clearly wrong since the left side does not equal the right side, $r_{xy}(\ell) \neq r_{xy}(\ell) - \mu_x \mu_y^*$. This should be broken into two equivalent equations,

$$\begin{aligned} r_{xy}(n_1, n_2) &= r_{xy}(n_1 - n_2) = r_{xy}(\ell) = E\{x(n_1) y^*(n_2)\} \\ \gamma_{xy}(n_1, n_2) &= \gamma_{xy}(n_1 - n_2) = \gamma_{xy}(\ell) = r_{xy}(\ell) - \mu_x \mu_y^* \end{aligned}$$

- • **Page 104:** Property 3.3.3 would be more clear if the condition was "and any vector $\alpha \in \mathbb{R}^M$".
- ∗ **Page 105:** In the equation for $\mu_x(n)$, the last asymptotic equality should be $\mu_w/(1 - \alpha)$, not $\mu_x/(1 - \alpha)$
- ∗ **Page 106:** Not all random processes are power signals, as stated in the last sentence of the second paragraph. Perhaps all stationary processes are, but certainly not all random processes.
- ∗ **Page 110:** Between (3.3.46) and (3.3.47), the equality should be $\mathrm{E}\{w(n)\} = \mu_w$, not $\mathrm{E}\{w(n)\} = \mu_x$ as stated
- ∗ **Page 111:** The upper limit of the sum in (3.3.52) should be M, not N.
- • **Page 111:** The line spectrum of a harmonic process has $2M$ lines over the range $[-\pi, \pi]$. The PSD is periodic, so if it contains one impulse (line), it contains an infinite number for all $-\infty \leq \omega \leq \infty$. This should be clarified in the second sentence of the second paragraph and in (3.3.53).
- ∗ **Page 115:** The sentence preceding Theorem 3.3 states "...restrictive condition of finite power ...." It should state "...restrictive condition of finite energy ...."
- ∗ **Page 117:** In the sentence above (3.4.16), the sentence should begin, "From (3.4.5), (3.4.6), and (3.4.9) we obtain" The last equation reference in the text was incorrectly listed as (3.4.7).

* **Page 118:** The upper limit in (3.4.24) should be ∞, not l.
* **Page 118:** Equation (3.4.26) is wrong because it divides by 2π but the limits of integration only include positive frequencies. Further, the approximation should be

$$\approx \frac{\triangle\omega}{\pi} R_x\left(\mathrm{e}^{j\omega_c}\right)$$

if we assume that $R_x(\mathrm{e}^{j\omega_c})$ can be treated as a constant over the range of integration.
* **Page 118:** The units of frequency listed after (3.4.27) should be *radians per sample*, not *cycles per second*.
* **Page 118:** In Table 3.2, the middle column and last entry has the wrong subscript on the left side. It should be $R_y(e^{j\omega})$, not $R_x(e^{j\omega})$ as listed.
* **Page 119:** The phrase "...what amplitudes *takes the signal*, how ..." should read "...what amplitudes *the signal takes*, how ..."
* **Page 121:** The second sentence of the proof of Property 3.4.4 is more clearly stated as follows: "If there exists M not-all-zero scalars $\{\alpha_i\}|_{i=1}^{M}$, such that

$$\sum_{i=1}^{M} \alpha_i \boldsymbol{q}_i = 0,$$

then ...."
* **Page 123:** In (3.4.55) they use two different notations, Γ and Γ_x, to denote the same matrix.
* **Page 122:** The sentence below (3.4.45) "...the Hermitian property(3.4.35) of R, ..." should refer to (3.4.28).
* **Page 124:** The denominator of (3.4.52) should be $\boldsymbol{q}_i^{\mathrm{H}}\boldsymbol{q}_i$, not $\boldsymbol{q}_i^{\mathrm{T}}\boldsymbol{q}_i$ as stated. The same error is made in (3.4.65).
* **Page 125:** I think the distance measure would be more clearly stated as

$$d^2(\boldsymbol{x}_0, \boldsymbol{x}_1) \triangleq (\boldsymbol{x}_0 - \boldsymbol{x}_1)^H \Gamma_x^{-1} (\boldsymbol{x}_0 - \boldsymbol{x}_1)$$

* **Page 134:** The left side of (3.6.2) should be $\hat{\sigma}_x^2$ since it is an estimate, not the true variance.
* **Page 134:** The bias term, B, in (3.6.7) should be written as $B(\hat{\theta})$.
* **Page 137:** The lower limit of the sum in (3.6.26) should be 1. With this change, it is no longer clear that the sum is greater than zero and that increased correlation reduces the variance of the estimate.
* **Page 137:** In the sentence above (3.6.27) it should state "...then from (3.6.21) and (3.6.21), it can ..."
* **Page 138:** In Example 3.6.1, the confidence interval for v=100 should be $(\hat{\mu}_x \pm 1.96)$.

∗ **Page 144:** In problem 3.9b, the sum should have an upper limit of M instead of N, and the coefficient $|c_k|$ should be $|A_k|$, to be consistent with (3.3.50), which the problem refers to.

∗ **Page 145:** In problem 3.23 there is an unnecessary right bracket,], at the end of the equation that shouldn't be there.

∗ **Page 147:** In problem 3.31 part b, it should ask you to show that above result reduces to $E\{\hat{\sigma}^2\} = (N-1)\sigma_x^2/N$, not the variance of the estimated mean.

Chapter 4

• **Page 151:** It is not clear to me why such a complicated expression such as that listed in (4.1.8) is necessary to get across the idea that $w(n+1)$ carries all the *new* information necessary to determine $x(n+1)$. It seems to me the same idea could be more concisely explained by simply substituting $n+1$ for n in (4.1.6) and changing the limit of the summation to 0 to ∞. Equation (4.1.6) then becomes

$$\begin{aligned}
x(n) &= -\sum_{k=1}^{\infty} h_I(k)x(n+1-k) + w(n) \\
x(n+1) &= -\sum_{k=1}^{\infty} h_I(k)x(n+1-k) + w(n+1) \\
x(n+1) &= -\sum_{k=0}^{\infty} h_I(k+1)x(n-k) + w(n+1) \\
x(n+1) &= -\sum_{j=-\infty}^{n} h_I(n-j+1)x(j) + w(n+1)
\end{aligned}$$

The last equation clearly demonstrates the same idea, but only uses a single summation rather than the double summation in (4.1.8).

∗ **Page 153:** In the sentence following (4.1.21) it should be, "where $x(n)$ and $w(n)$ are the input and output signals, respectively." The text had these references reversed.

• **Page 154:** In the sentence preceding (4.1.27), it should refer to section 3.4.3 as well as 3.4.2.

• **Page 154:** It is bad form to break a line in the middle of ARMA(P,Q).

∗ **Page 158:** Equation (4.2.17) contains the wrong expression for the autocorrelation in the sum. It should be either $r^*(k)$ or $r(-k)$.

∗ **Page 158:** The lower limit in the sum in the denominator of (4.2.13) should be 1, not 0.

∗ **Page 159:** The sentence following (4.2.25) should assign the pole $p = -1/a$ to the *anticausal* sequence, not just the noncausal sequence.

- ∗ **Page 159:** In the last sentence they refer to the AP(2) model in Section 4.1.2. They probably mean Section 4.2.4, though it is odd that they would refer to a section that the reader has presumably not yet read.
- ∗ **Page 160:** In the sentence preceding (4.2.29) they state that they are multiplying both sides by $\tilde{h}(n-\ell)$. They actually multiply by $\tilde{h}^*(n-\ell)$.
- ∗ **Page 160:** The autocorrelation matrix in (4.2.30) should be transposed to match (4.2.15), given that $r_h(-k) = r_h^*(k)$. If the system impulse response is real, $r_h(-k) = r_h(k)$ and this error is of no consequence. This error was distributed through many of the equations that followed including (4.2.34) and (4.2.43).
- ∗ **Page 160:** In the sentence following (4.2.30) there should be a comma between Hermitian and Toeplitz.
- ∗ **Page 162:** Equation (4.2.41) should be $R(z) = H(z)H^*(\frac{1}{z^*})$.
- • **Page 174:** In the first equality of (4.3.12) they assumed that the impulse response was real so that they could use $R_h(z) = H(z)H(z^{-1})$, rather than the more general $R_h(z) = H(z)H^*(\frac{1}{z^*})$ for systems with a complex-valued impulse response. They should have noted this assumption.
- ∗ **Page 177:** Equation (4.4.4) should be

$$R(z) = H(z)H^*(\frac{1}{z^*}) = \frac{D(z)D^*(\frac{1}{z^*})}{A(z)A^*(\frac{1}{z^*})}$$

- ∗ **Page 178:** Again, equation (4.4.5) should be $A(z)R_h(z) = D(z)H^*(\frac{1}{z^*})$ and $H(1/z^*)$ in the sentence that follows should be $H^*(1/z^*)$.
- ∗ **Page 178:** In the sentence above (4.4.12) they should have stated, "…is the convolution of $r_a(\ell)$ with $r_h(\ell)$, given …".
- ∗ **Page 179:** The last equality in the equation for $R_d(z)$ in Example 4.4.1 is wrong. The first term in the parentheses should be $\left(1 + \frac{z^{-1}}{2}\right)$. The same error propagates to the sentence that follows.
- ∗ **Page 190:** In problem 4.3 part (b), the hint, "[see Section (1.2.1)]." should be "[see Equation (1.2.1)]."
- ∗ **Page 190:** In problem 4.3 part (c), it should refer to $x(n)$, not $y(n)$.
- ∗ **Page 190:** In problem 4.4, the last term of equations (a) and (b) should be $w(n-2)$, not $w(n-1)$.
- ∗ **Page 190:** In problem 4.4, the subscript for the signal in part (b) should be 2 such that the equation begins $x_2(n) = \ldots$. This enables you to then contrast this signal with that in part (a).

Chapter 5

- ∗ **Page 211:** In the last sentence, the reference to (3.6.33) should be (5.2.1).
- ∗ **Page 213:** The last equality of (5.3.8) should be

  $$= \frac{1}{N} \left| \sum_{m=0}^{N-1} x(N-1-m) \mathrm{e}^{j\omega_k (m+1)} \right|$$

  where they have used $m = N - 1 - n$. This error propagates through the equations on the following page. However the ideas are still correct. Only the phase of $h_k(n)$ is affected.
- • **Page 214:** To understand where (5.3.13) came from, it is helpful to refer to (3.3.45) and (3.4.22).
- ∗ **Page 216:** The values listed in Table 5.2 appear to be sample estimates, not the true mean, variance, and mean-square error. The mean (and possibly variance) also appear to be estimated incorrectly. The (ensemble) mean should be $\sigma_w^2 = 1$ by Parseval's theorem, as listed in (3.3.48).
- ∗ **Page 217:** The expectation operator in (5.3.17) is missing the right curly brace, }.
- • **Page 217:** Equation (5.3.20) came from (5.3.10).
- • **Page 217:** I think the conditions listed at the bottom of this page are loose and approximate. For example, condition 2 assumes that a mainlobe of the windows spectrum exists. More precise conditions are given in Brockwell & Davis.
- ∗ **Page 221:** For the equation listed in Example 5.3.4, the phases for each sinusoidal term should be ϕ_1, ϕ_2, and ϕ_3. They are currently all listed as ϕ_1.
- • **Page 224:** It would be more accurate if the practice side of the diagram in Figure 5.17 showed the estimated autocorrelation for both positive and negative lags.
- • **Page 225:** Following equation (5.3.40), they state that the only condition necessary for $W_a(\mathrm{e}^{j\omega})$ and $W_B(\mathrm{e}^{j\omega})$ to become periodic impulse trains as N and L approach ∞, is $w_a(0) = 0$. It seems to me there must be other conditions for this to be true. Surely not all possible windows $w_a(\ell)$ that meet this single condition have Fourier transforms that become impulse trains. Perhaps with the added condition that $w_a(\ell)$ be positive definite, this statement might then be true.
- • **Page 230:** In the text around (5.3.59) they fail to state the assumption that the segments are independent. We know this isn't true, except for white noise processes.
- ∗ **Page 226:** The variable of summation in (5.3.48) should be ℓ, not k.
- ∗ **Page 232:** The last two sentences in Example 5.3.7 list "$L = 8$". It should be $K = 8$.
- • **Page 232:** I think the discussion of aliasing of $\hat{r}_x(\ell)$ due to using sampled estimates of the continuous spectrum is misleading. This "aliasing" in the "lag domain" only occurs if insufficient zero-padding is used in generating the spectral estimate. This is easily implemented and the aliasing can be completely eliminated. Thus, I don't think it warrants so much discussion.

- **Page 233:** In equation (5.3.73) they are using the biased estimate defined in (5.2.1), not the estimate of the windowed segment $\{w(n)x(n)\}_0^{N-1}$ discussed at the top of the page.

∗ **Page 239:** The citation in the bottom of the first paragraph, "Carter (1987)", is not in the bibliography.

∗ **Page 242:** At the end of the first paragraph, they state that the coherence should be greater than zero. It should state that $R_x(\mathrm{e}^{j\omega})$ and $|H(\mathrm{e}^{j\omega})|$ should be greater than zero to avoid a divide-by-zero error.

∗ **Page 251:** In the third sentence of Example 5.5.2 they refer to the *plots* shown in Figure 5.34 and the sentence that follows refers to the upper graph. However, there is only one graph (or plot) shown in Figure 5.34.

- **Page 252:** The 95% confidence intervals shown in Figure 5.34 seem to be too variable (as a function of frequency). They should probably note that these are the 95% confidence intervals for a fixed, given frequency. It is wrong to interpret these as meaning in 95% of separate realizations the true PSD would fall inside the intervals at all frequencies.

Chapter 6

∗ **Page 273:** In Figure 6.9 the slope of the diagonal dashed line is too negative. It should be parallel to the diagonal line from the origin to $\hat{y}_o$ on the $x_1 - -x_2$ plane.

- **Page 273:** I think the illustration in Figure 6.9 is misleading. The orthogonality is in expectation only. I don't think it has a clear geometric representation since it applies to random vectors.

∗ **Page 275:** Equation (6.3.9) and the equations that follow are wrong. $LDk = d$ does not imply that $Lk = D^{-1}d$, which was used to solve for this equation.

- **Page 277:** They used the fact that $\det(AB) = \det(A)\det(B)$ to obtain (6.3.15).

- **Page 281:** To be consistent with earlier notation, the autocorrelation in (6.4.16) and the preceding sentence should be written as $r_x(\ell)$, not $r(\ell)$ without the x subscript.

- **Page 284:** There are some odd characteristics of the filtered signal in Figure 6.13. There appears to be a phase shift in $\hat{y}(n)$. If the filter was causal, why is there no transient due to the lack of samples of $x(n)$ for the first few samples? Why is $\hat{y}(n)$ not shown for the last few points in the record? Was a non-causal filter used? Needs more explanation.

- **Page 285:** The horizontal lines in Figure 6.15 were not defined. Presumably one is the theoretical MMSE and the other is the sample average over all 100 runs.

- **Page 291:** Note that $JJ^{\mathrm{H}} = J^{\mathrm{H}}J = I$ and $J = J^{\mathrm{H}} = J^{\mathrm{T}}$.

- **Page 292:** It is useful to remember that $r(0)$ is real, even if the signal is complex.

* **Page 292:** Towards the bottom of the page after the fragment "We note that" the equation $a_1^{(1)} = b_1^{(1)}$ should be $a_1^{(1)} = b_0^{(1)}$.

• **Page 298:** It is useful to recall that $r_{yx}(\ell) = r_{yw}(\ell) * h_x^*(-\ell)$ to understand where (6.6.18) came from.

• **Page 300:** Equation (6.6.31) only holds if $R_y(z) = R_y^*(\frac{1}{z^*})$. This is true if $r_y(\ell)$ is real.

* **Page 317:** There is an unexplained arrow under the first zero in (6.8.35).

* **Page 319:** In the second paragraph of Section 6.9, the phrase "...plus an additive noise plus interference signal..." should be "...plus an additive noise or interference signal...".

* **Page 321:** The vertical axis should be labelled $c_0(n)$, not $c(n)$ as shown.

* **Page 324:** Towards the bottom of Example 6.9.3 they state that binomial filters are widely used in radar systems to eliminate *stationary (i.e., nonmoving)* clutter. However, they are describing a highpass filter that actually eliminates low-frequency drift that is considered a non-stationary effect in short records. This either needs further explanation or is an error and they actually mean "... *nonstationary (i.e., moving)* clutter."

* **Page 324:** The vertical axis of Figure 6.37 should have negative values.

Chapter 7

* **Page 345:** Observation 4 should refer to Sections 6.4 and 6.6.

• **Page 365:** Chapter 7 uses a slightly different notation than Chapter 2 for lattice filters. In Chapter 7 the coefficients range from k_0 to k_{M-1} whereas in Chapter 2 the coefficients range from k_1 to k_P. This leads to some confusion because in places Chapter 7 refers to Chapter 2 where the subscripts have a different range. As an example, see the sentence preceding (7.5.26).

• **Page 383:** They are inconsistent in their use of "Kalman filter." In some cases "filter" is capitalized (e.g. last paragraph of 383), but not others (e.g. page 384). "Filter" should not be capitalized in these cases.

* **Page 384:** $\boldsymbol{H}(n$ in Table 7.5, 1b is missing its right parenthesis.

Chapter 8

* **Page 398:** "Snapshop" in Figure 8.1 should be "Snapshot."

• **Page 399:** Note that the time-average operator listed in the second paragraph is $\sum_{n=0}^{N-1}(\cdot)$ instead of the expected $\frac{1}{N}\sum_{n=0}^{N-1}(\cdot)$. As listed, it is missing the constant $\frac{1}{N}$. I believe this is intentional because this factor cancels in all of the least-squares equations, but the text would be more clear if this omission was explained explicitly.

- ∗ **Page 401:** Figure 8.5 contains the same error as Figure 6.9 on pg. 273 described above.
- **Page 404:** In the proof of Property 8.2.3 they used a more general property of the trace operator than that listed in the appendix. Specifically, $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ even if A and B are not square matrices. However, $n_A = m_B$ and $m_A = n_B$ must hold, where n and m represent the number of rows and columns of the corresponding matrices, respectively.
- **Page 405:** In Equation (8.2.46) they ignored assumption 2 listed on the previous page.
- **Page 408:** Near the top of the page they describe a fast method of computing $\hat{R}$. In the first step they suggest filling in the first row using M dot products. For signals this may be done much more efficiently using the FFT to calculate the estimated autocorrelation.
- ∗ **Page 410:** The second equality in (8.3.15) is wrong. It should be $1 - g^{\mathrm{H}}(i)c_{\mathrm{ls}}^{(i)}$. where $g^{\mathrm{H}}(i)$ is the ith row of G, as in (8.2.7).
- **Page 423:** The hermitian superscript on $\mathbf{0}$ in (8.6.19) is unnecessary.