Apply what you’ve learned to rewrite this proof. Due at the beginning of class, Tuesday, January 31

The Largest Prime

Suppose there were a largest prime number $p_i$. Then consider the product $\prod_{j=0}^{p_i-1} p_i - j$. Then $\left( \prod_{j=0}^{p_i-1} p_i - j \right) + 1$ cannot be divided evenly by any of the numbers up to $p_i$, 2, 3, 4, \ldots, $p_i$ because each of these divides the left factor evenly, but not the right factor, hence not their sum. (Recall that if $a_1$ divides $a_2$ and $a_2 = a_3 + a_4$ then if $a_1$ divides $a_3$, it will also divide $a_4$.) Since we are assuming $p_i$ is the largest prime, $\left( \prod_{j=0}^{p_i-1} p_i - j \right) + 1$ can have no prime factors greater than $p_i$, hence $\left( \prod_{j=0}^{p_i-1} p_i - j \right) + 1$ is a prime, and it is greater than $p_i$, since $\prod_{j=0}^{p_i-1} p_i - j \geq p_i$. This contradicts the maximality of $p_i$. Hence the assumption that $p_i$ is the largest prime must be false, and so there is no largest prime.