NFA Closure Properties

Sipser pages  pages 58-63
NFAs also have closure properties

• We have given constructions for showing that DFAs are closed under
  1. Complement
  2. Intersection
  3. Difference
  4. Union

• We will now establish that NFAs are closed under
  1. Reversal
  2. Kleene star
  3. Concatenation
Reversal of $\varepsilon$-NFAs

• Closure under reversal is easy using $\varepsilon$-NFAs. If you take such an automaton for $L$, you need to make the following changes to transform it into an automaton for $L^{\text{Rev}}$:

1. Reverse all arcs

2. The old start state becomes the only new final state.

3. Add a new start state, and an $\varepsilon$-arc from it to all old final states.
1. Reverse all arcs

2. The old start state becomes the only new final state.

3. Add a new start state, and an ε-arc from it to all old final states.
Concatenation

• \( L \circ R = \{ x \circ y \mid x \text{ in } L \text{ and } y \text{ in } R \} \)

• To form a new \( \varepsilon \)-NFA that recognizes the concatenation of two other \( \varepsilon \)-NFAs with the same alphabet do the following
  – Union the states (you might have to rename them)
  – Add an \( \varepsilon \)-transition from each final state of the first to the start state of the second.
Formally

- Let
  \[ L = (Q_L, \Lambda, T_L, s_L, F_L) \]
  \[ R = (Q_R, \Lambda, T_R, s_R, F_R) \]
- \[ L \cdot R = = (Q_L \cup Q_R, \Lambda, T, s_L, F_R) \]

Where

\[
T \; s \; \epsilon \; | \; s \in F_L = S_R \cup T_L \quad s \; \epsilon \\
T \; s \; c \; | \; s \in Q_L = T_L \quad s \; c \\
T \; s \; c \; | \; s \in Q_R = T_R \quad s \; c
\]
Kleene - Star

• If a language $L$ is recognized by an NFA then so is the language $L^*$

• Add a new state.
• Make it the start state in the new NFA.
• Add an $\epsilon$-arc from this state to the old start state.
• Add $\epsilon$-arcs from every final state to this new state.
Example

- Add a new state.
- Make it the start state in the new NFA, and an accepting state.
- Add an $\varepsilon$-arc from this state to the old start state.
- Add $\varepsilon$-arcs from every final state to this new state.