CS581 – Theory of Computation – HW6

Tuesday, May 14, 2013
due in class Tuesday, May 21, 2013

Answer each question below. You will turn this homework in using D2L. In addition, you may also turn in a paper copy in class. In this case the TA will mark up your homework with comments and return the comments to you.

You may format your answers using some document processing software, or you may write it up with pencil and paper and scan it. In either case submit a pdf document. Be sure your submission is clearly identified as Homework 6, and contains your name and your email on the first line. The first line should look like:

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1. **Proof by induction.** Consider the definition and of Trees, and some equations that describe two functions (size and flip) over trees.

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1) If n is an integer, then (Tip n) is a tree.
2) It x and y are trees, then (Fork x y) is a tree.
3) Nothing else is a tree.
    An example tree:  (Fork (Tip 3) (Fork (Tip 1) (Tip 67)))

4) size (Tip n) = 0
5) size (Fork x y) = 1 + max (size x) (size y)

6) flip (Tip n) = (Tip n)
7) flip (Fork x y) = (Fork (flip y) (flip x))
```

   Prove by induction that `size(flip x) = size x`. Do the following:

   - Write down what you are to prove.
   - Make a formula (or function) parameterized by the induction variable.
   - You will need to prove several cases. One for each way a tree can be constructed. Express each case using your formula from above.

  - A case might need an induction hypothesis. Use the formula as well to state what you can assume.
  - Then work through the steps for each case.
  - You will need a property of the max function. Clearly state what property you are assuming.

2. Explain why the following is not a descrition of a legitimate Turing machine.

   - M_{Bad} = The input is a polynomial p over variables $x_1, x_2, ..., x_k$.
     (a) Try all possible settings of $x_1, x_2, ..., x_k$ to integer values
     (b) Evaluate p on all of these settings.
     (c) If any of these setting evaluate to 0, *accept*, otherwise *reject*.

3. Let A be the language containing only the single string s, where $s =$

   (a) 0 if life will never be found on Mars.
   (b) 1 if life will be found on Mars someday.

   Is A decideable? Why or why not? For the purposes of this problem, assume that the question of whether life will be found on Mars has an unambiguous Yes or No answer.

4. Let $T = \{(i, j, k) \mid i, j, k \in \text{Nat}\}$. Show that T is countable.

5. Let $A = \{< M > \mid \text{M is a DFA which doesn't accept any string containing an odd number of 1's}\}$. Show that A is decideable.