

CS581 – Theory of Computation – HW1

Tuesday, April 2, 2013
due in class Tuesday, April 9, 2013

Answer each question below. You will turn this homework in using D2L. In addition, you may also turn in a paper copy in class. In this case the TA will mark up your homework with comments and return the comments to you.

You may format your answers using some document processing software, or you may write it up with pencil and paper and scan it. In either case submit a pdf document. Be sure your submission is clearly identified as Homework 1, and contains your name and your email on the first line. The first line should look like:

CS581 HW #1

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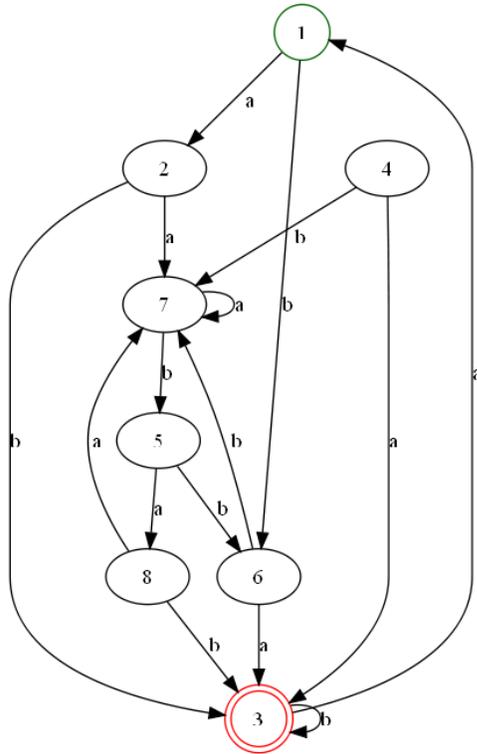
When doing a proof, set up the structure of the proof first, then carry out the steps. Use the formatting discussed in the class notes.

- Facts. List all the facts, including function definitions, and any facts about arithmetic you might use.
- State clearly the proposition you are proving.
- List each step. Justify each step with one of the facts.

Be sure and state what kind of proof: Proof by induction (state all the cases, what the induction variable is, and what the inductive hypotheses are), Proof by Contradiction (state what is to be proved, state what contradiction you reach), etc. and then format the proof (using numbering, indentation, boxes, or other lexicographic conventions) so that the structure is evident in the proof steps.

1. The sum of the first n integers ($1 + 2 + 3 + \dots + N$) can be computed by the formula $(1 + N) * (N/2)$. Prove by induction that this formula is correct. Format your proof as discussed in the class notes and in the instructions above.
2. If an alphabet A contains k elements, how many elements does A^n contain? Use induction to prove your answer. Format your proof as discussed in the class notes above.

3. Construct DFAs for the following languages over $\{a, b\}^*$. Your answer should be in the form of a state-diagram (a picture or graph representing the DFA).
- Strings with an even number of a 's
 - Strings with an even number of b 's
 - Strings that contain the substring aa
 - Strings that contain the substring abb
 - The empty language
 - The language consisting of the empty string
4. Consider the state-diagram below. Construct a formal description from the diagram. Recall a DFA is a 5 tuple (States,Alphabet,Transition function,start state,final states).



5. Recall the definition of acceptance by DFA (page 40 of the text). Prove that the DFA from question 4 above either accepts or rejects the strings:
- $aaaaabba$
 - $baaa$
 - $babbbaab$

If it accepts the string, exhibit the path, and argue that it meets all 3 conditions. If it rejects, find a path and show what condition it fails to meet.