CS 457/557: Functional Languages

Lecture 1: Introduction

Mark P Jones
Portland State University

What is Functional Programming?

What is Functional Programming?

- An alternative to dysfunctional programming?
- Programming with functions?
- Programming without side-effects?

What is Functional Programming?

- Functional programming is a style of programming that emphasizes the evaluation of expressions, rather than execution of commands

- Expressions are formed by using functions to combine basic values

- A functional language is a language that supports and encourages programming in a functional style

Functions:

In a pure functional language:

- The result of a function depends *only* on the values of its inputs:
  - Like functions in mathematics
  - No global variables / side-effects

- Functions are first-class values:
  - They can be stored in data structures
  - They can be passed as arguments or returned as results of other functions

Functional Languages:

- Pure, lazy evaluation, strong typing:
  - Haskell, Miranda, Orwell, …
- Impure, strict evaluation, strong typing:
  - Standard ML (SML), Objective CAML (OCaml), F#, …
- Impure, strict evaluation, dynamic typing:
  - Lisp, Scheme, Erlang, …
- Pure, strict evaluation, strong typing:
  - Relatively unexplored (Timber, Habit, …)

Good News, Bad News:

- Good News: You can write Functional Programs in almost any language
- Bad News: You can write “C code” in a functional language …

Example:

- Write a program to add up the numbers from 1 to 10

In C, C++, Java, C#, … :

initialization

initialization

iteration

```
int tot = 0;
for (int i=1; i<11; i++)
   tot = tot + i;
```

update

update

implicit result returned in the variable `tot`

In ML:

accumulating parameter

```
let fun sum i tot
      = if i>10
          then tot
          else sum (i+1) (tot+i)
in sum 1 0
end
```

(tail) recursion

initialization

result is the value of this expression

In Haskell:

```
sum [1..10]
```

combining function

the list of numbers to add

result is the value of this expression

Reflections:

- I’ve tried to use “idiomatic” solutions in each language
- This example makes Haskell look good
- But it wouldn’t be too difficult to adapt any one solution to any of the other languages
- An imperative version of the Haskell solution would require linked list code that is built-in to Haskell
- An objective comparison between languages should account for library code as well as the main program

Reflections (continued):

- What makes a good program?
  - correctness
  - clarity
  - conciseness (none of my solutions are optimally concise!)
  - Performance (not really an issue here)

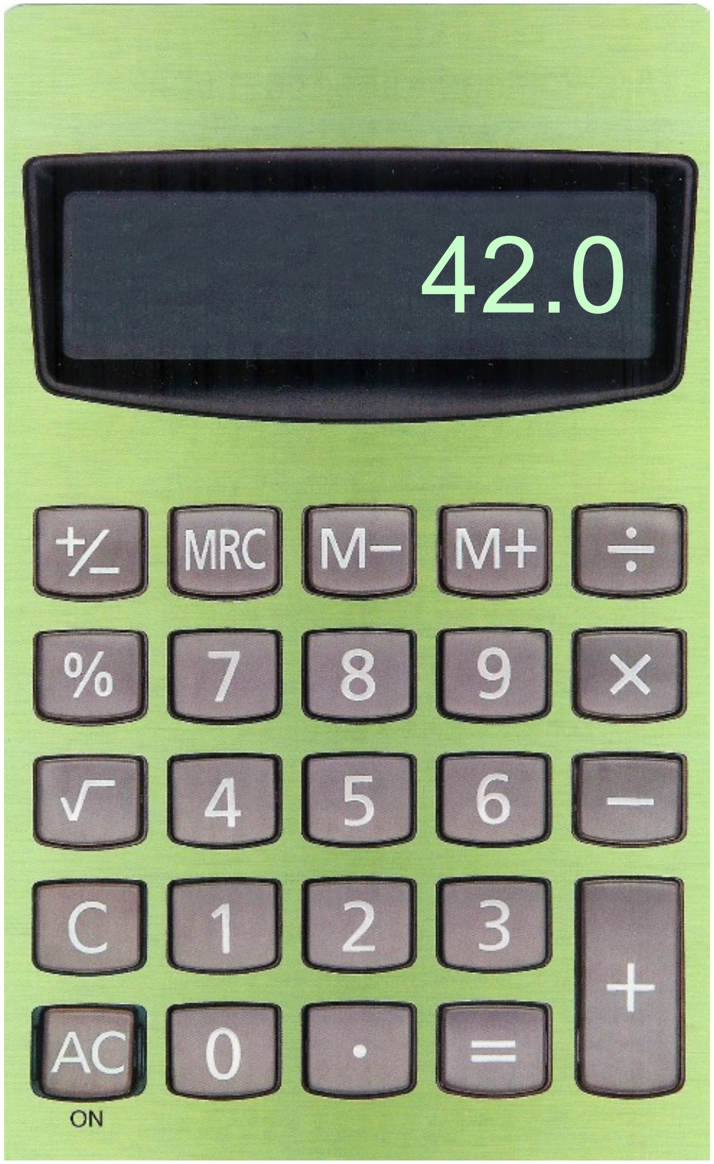
Raising the Level of Abstraction:

"If you want to reduce [design time], you have to stop thinking about something you used to have to think about." (Joe Stoy, quoted on the Haskell mailing list)

- Example: memory allocation
- Example: data representation
- Example: order of evaluation
- Example: (restrictive) type annotations

Computing by Calculating:

- Calculators are a great tool for manipulating numbers
- Buttons for:
  - entering digits
  - combining values
  - using stored values
- Not so good for manipulating large quantities of data
- Not good for manipulating other types of data



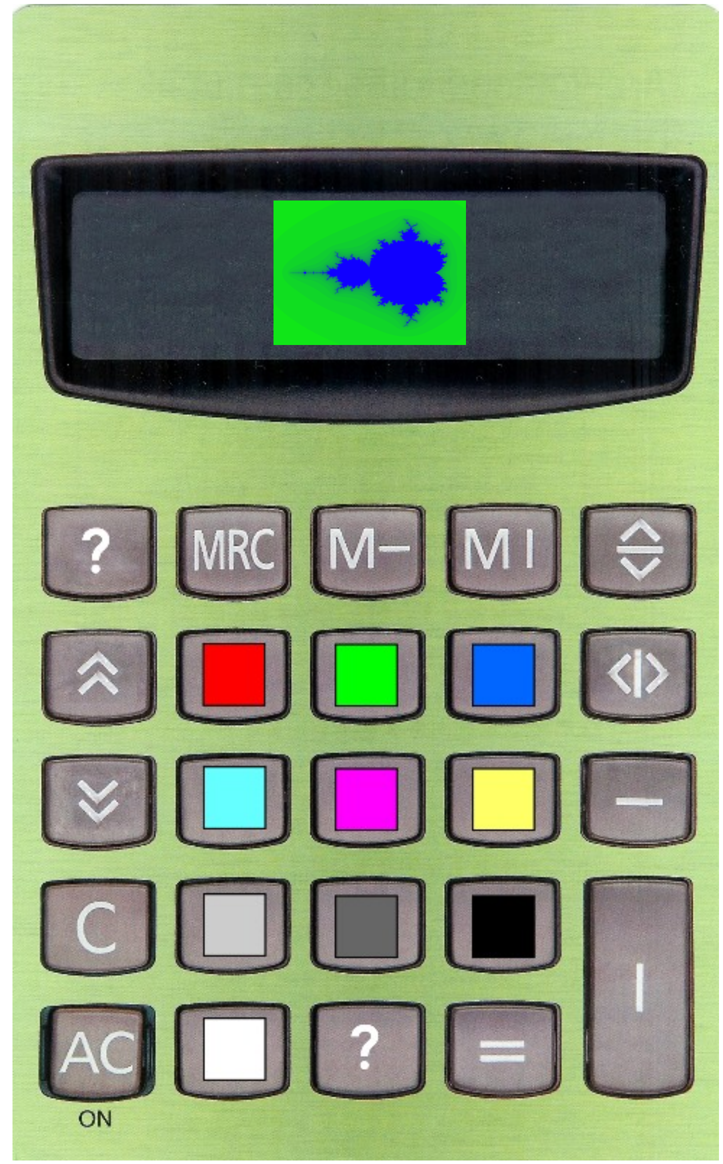
Computing by Calculating:

- What if we could "calculate" with other types of value?

- Buttons for:
  - entering pixels
  - combining pictures
  - using stored pictures

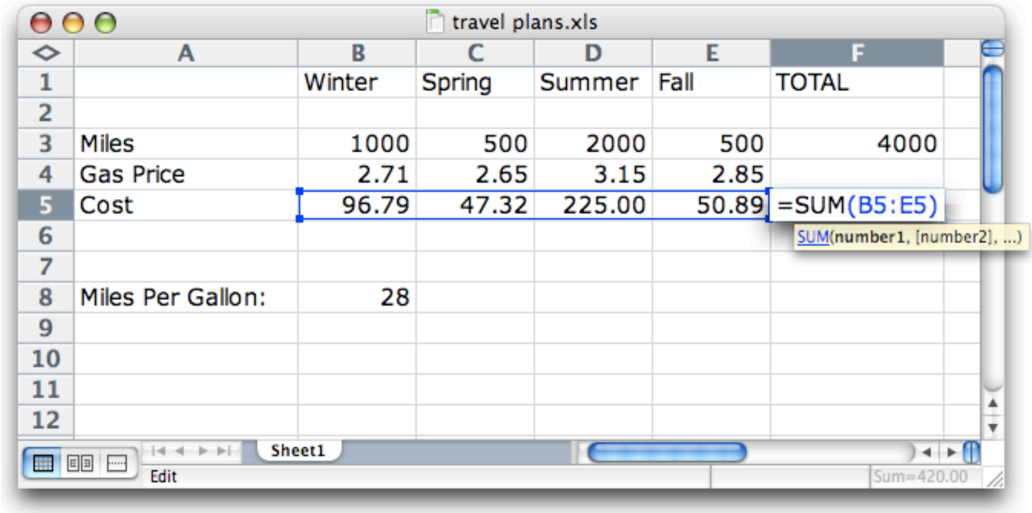
- I wouldn't want to calculate a whole picture this way!

- I probably want to deal with *several different types* of data *at the same time*



Computing by Calculating:

- Spreadsheets are better suited for dealing with larger quantities of data



- Values can be named (but not operations)
- Calculations (i.e., programs) are recorded so that they can be repeated, inspected, modified
- Good if data fits an “array”
- Not so good for multiple types of data

Functional Languages:

- Multiple types of data
  - Primitive types, lists, functions, …
  - Flexible user defined types …
- Operations for combining values to build new values (combinators)
- Ability to name values and operations (abstraction)
- Scale to arbitrary size and shape data
- "Algebra of programming" supports reasoning

Quick Introductions

Starting Hugs:

```
user$ hugs

__   __ __  __  ____   ___      _________________________________________
||   || ||  || ||  || ||__      Hugs 98: Based on the Haskell 98 standard
||___|| ||__|| ||__||  __||     Copyright (c) 1994-2005
||---||         ___||           World Wide Web: http://haskell.org/hugs
||   ||                         Bugs: http://hackage.haskell.org/trac/hugs
||   || Version: September 2006 _________________________________________

Haskell 98 mode: Restart with command line option -98 to enable extensions

Type :? for help
Hugs>
```

The most important commands:

- :q quit
- :l file load file
- :e file edit file
- expr evaluate expression

The read-eval-print loop:

1. Enter expression at the prompt
2. Hit return
3. *The expression is read, checked, and evaluated*
4. *Result is displayed*
5. Repeat at Step 1

Simple Expressions:

Expressions can be constructed using:

- The usual arithmetic operations:

  1 + 2 * 3

- Comparisons:

  1 == 2   'a' < 'z'

- Boolean operators:

  True && False   not False

- Built-in primitives:

  odd 2   sin 0.5

- Parentheses:

  odd (2 + 1)   (1 + 2) * 3

- Etc …

Expressions Have Types:

- The *type* of an expression tells you what kind of value you might expect to see if you evaluate that expression

- In Haskell, read "::" as "has type"

- Examples:
  - 1 :: Int, 'a' :: Char, True :: Bool, 1.2 :: Float, …

- You can even ask Hugs for the type of an expression: :t expr

Type Errors:

```
Hugs> 'a' && True
ERROR - Type error in application
*** Expression       : 'a' && True
*** Term             : 'a'
*** Type             : Char
*** Does not match : Bool

Hugs> odd 1 + 2
ERROR - Cannot infer instance
*** Instance    : Num Bool
*** Expression : odd 1 + 2

Hugs>
```

Pairs:

- A pair packages two values into one

  (1, 2) ('a', 'z') (True, False)

- Components can have different types

  (1, 'z') ('a', False) (True, 2)

- The type of a pair whose first component is of type A and second component is of type B is written (A,B)
- What are the types of the pairs above?

Operating on Pairs:

- There are built-in functions for extracting the first and second component of a pair:
  - fst (True, 2) = True
  - snd (0, 7)   = 7

- Is the following property true?

  For any pair p, (fst p, snd p) = p

Lists:

- Lists can be used to store zero or more elements, in sequence, in a single value:
  [] [1, 2, 3] ['a', 'z'] [True, True, False]

- All of the elements in a list must have the same type
- The type of a list whose elements are of type A is written as [A]
- What are the types of the lists above?

Operating on Lists:

- There are built-in functions for extracting the head and the tail components of a list:
  - head [1,2,3,4] = 1
  - tail [1,2,3,4] = [2,3,4]

- Conversely, we can build a list from a given head and tail using the “cons” operator:
  - 1 : [2, 3, 4] = [1, 2, 3, 4]

- Is the following property true?

  For any list xs, head xs : tail xs = xs

More Operations on Lists:

- Finding the length of a list:

  length [1,2,3,4,5] = 5

- Finding the sum of a list:

  sum [1,2,3,4,5] = 15

- Finding the product of a list:

  product [1,2,3,4,5] = 120

- Applying a function to the elements of a list:

  map odd [1,2,3,4] = [True, False, True, False]

Continued …

- Selecting an element (by position):

  [1,2,3,4,5] !! 3 = 4

- Taking an initial prefix (by number):

  take 3 [1,2,3,4,5] = [1,2,3]

- Taking an initial prefix (by property):

  takeWhile odd [1,2,3,4,5] = [1]

- Checking for an empty list:

  null [1,2,3,4,5] = False

More ways to Construct Lists:

- Concatenation:

  [1,2,3] ++ [4,5] = [1,2,3,4,5]

- Arithmetic sequences:

  [1..10] = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

  [1,3..10] = [1, 3, 5, 7, 9]

- Comprehensions:

  [2 * x | x <- [1,2,3,4,5]] = [2, 4, 6, 8, 10]

  [y | y <- [1,2,3,4], odd y] = [1, 3]

Strings are Lists:

- A String is just a list of Characters

  ['w', 'o', 'w', '!'] = "wow!"

  ['a'..'j'] = "abcdefghij"

  "hello, world" !! 7 = 'w'

  length "abcdef" = 6

  "hello, " ++ "world" = "hello, world"

  take 3 "functional" = "fun"

Functions:

- The type of a function that maps values of type A to values of type B is written A -> B

- Examples:
  - odd :: Int -> Bool
  - fst :: (a, b) -> a   (a,b are type variables)
  - length :: [a] -> Int

Operations on Functions:

- Function Application. If f :: A -> B and x :: A, then f x :: B

- Notice that function application associates more tightly than any infix operator:

  f x + y = (f x) + y

- In types, arrows associate to the right:

  A -> B -> C = A -> (B -> C)

  Example: take :: Int -> [a] -> [a]

  take 2 [1,2,3,4] = (take 2) [1,2,3,4]

Sections:

- If ⊕ is a binary op of type A -> B -> C, then we can use "sections":
  - (⊕) :: A -> B -> C
  - (expr ⊕) :: B -> C (assuming expr::A)
  - (⊕ expr) :: A -> C (assuming expr::B)
- Examples:
  - (1+), (2*), (1/), (<10), ...

Higher-order Functions:

- map :: (a -> b) -> [a] -> [b]
  - map (1+) [1..5] = [2,3,4,5,6]
- takeWhile :: (a -> Bool) -> [a] -> [a]
  - takeWhile (<5) [1..10] = [1,2,3,4]
- (.) :: (a -> b) -> (c -> a) -> c -> b
  - (odd . (1+)) 2 = True

"composition"

Definitions:

- So far, we've been focusing on expressions that we might want to evaluate.

- What if we wanted to:
  - Define a new constant (i.e., Give a name to the result of an expression)?
  - Define a new function?

- Definitions are placed in files with a .hs suffix that can be loaded into the interpreter

Simple Definitions:

Put the following text in a file "defs.hs":

```
greet name = "hello " ++ name

square x = x * x

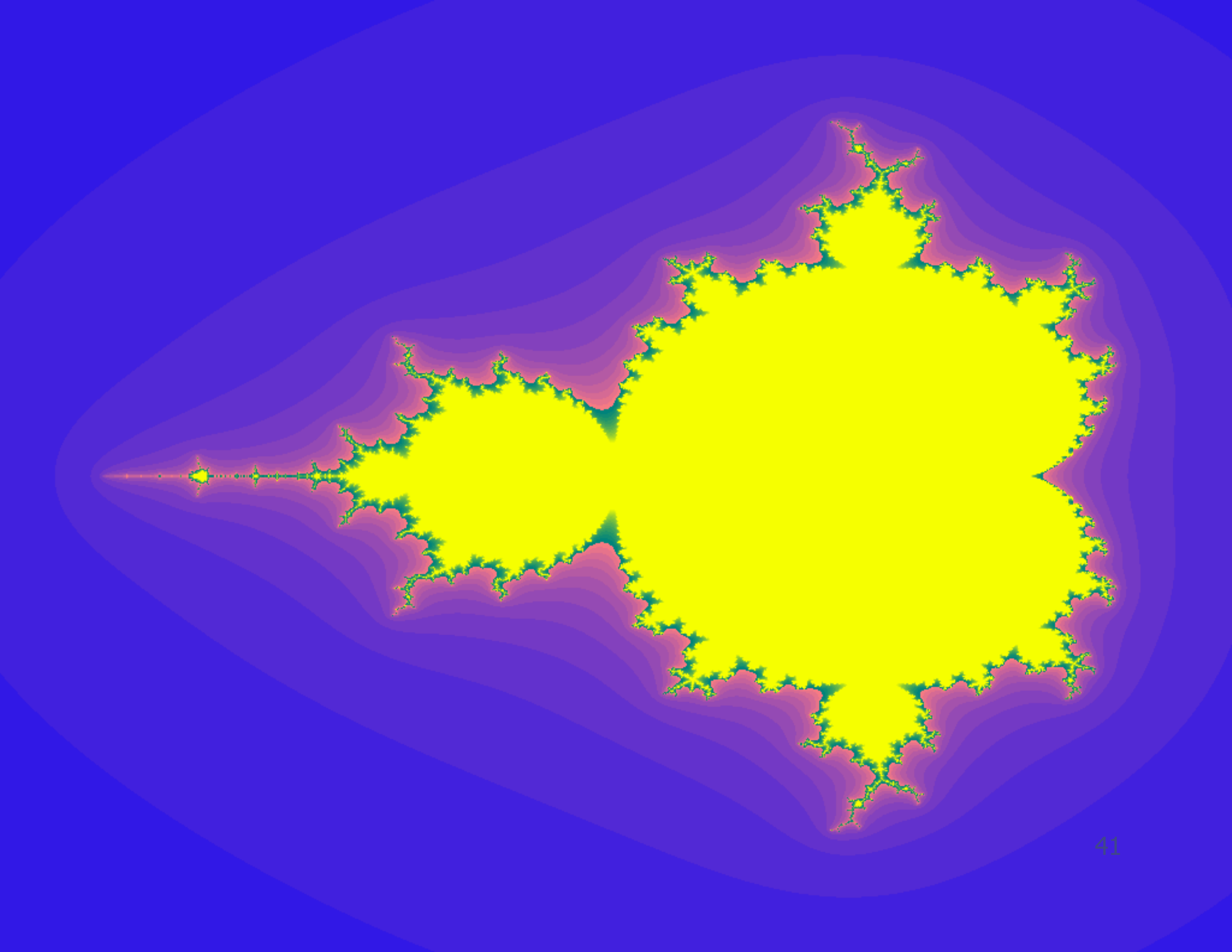
fact n = product [1..n]
```

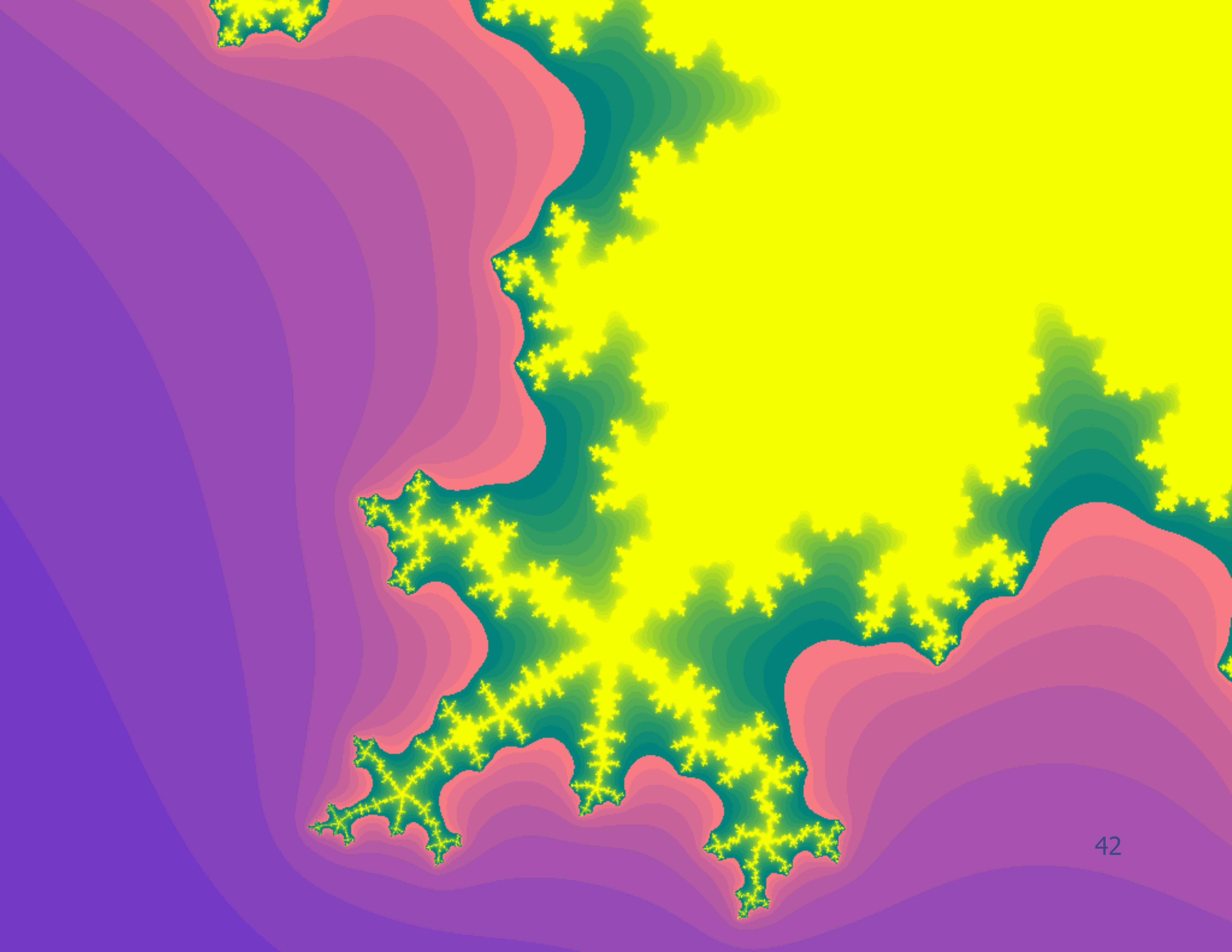
Loading Defined Values:

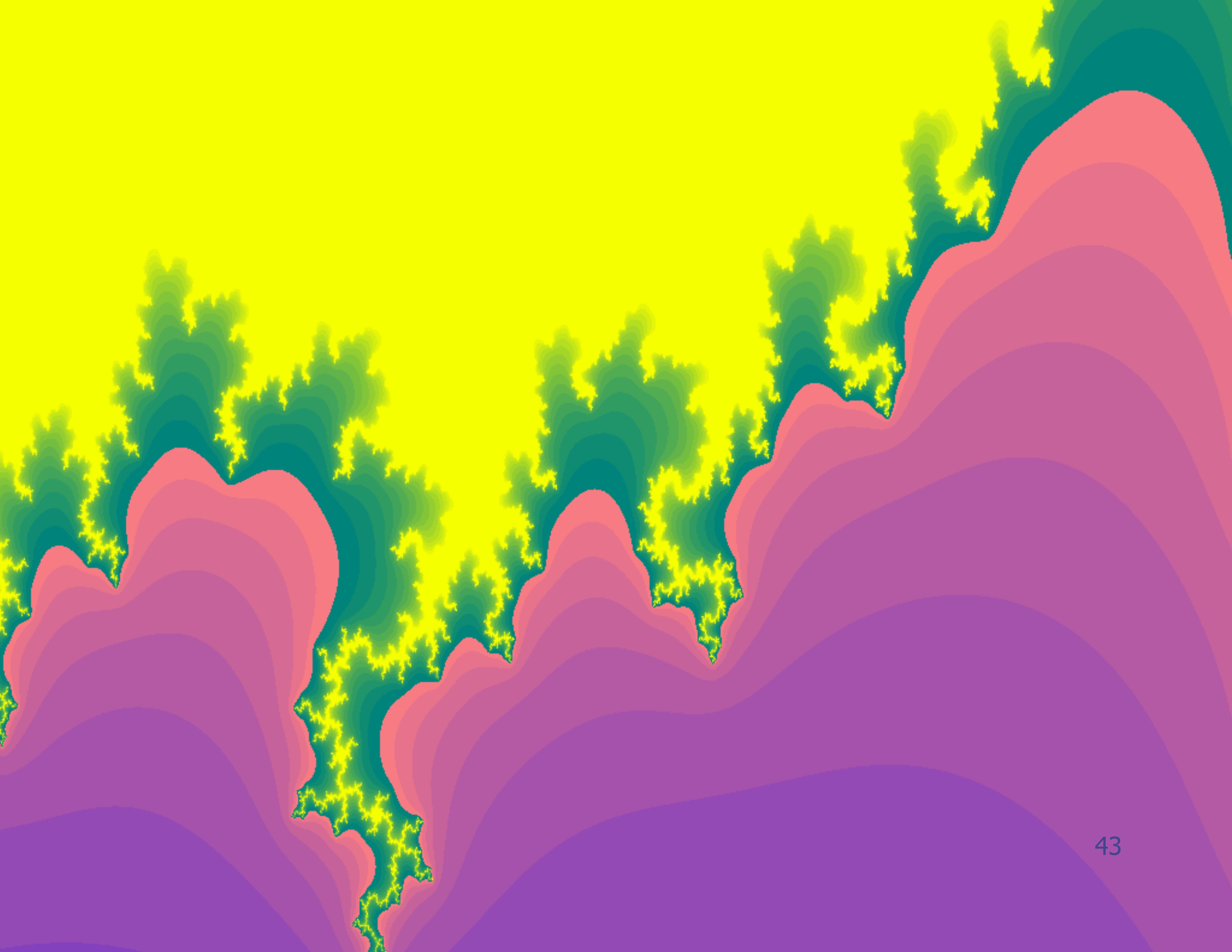
Pass the filename as a command line argument to Hugs, or use the :l command from inside Hugs:

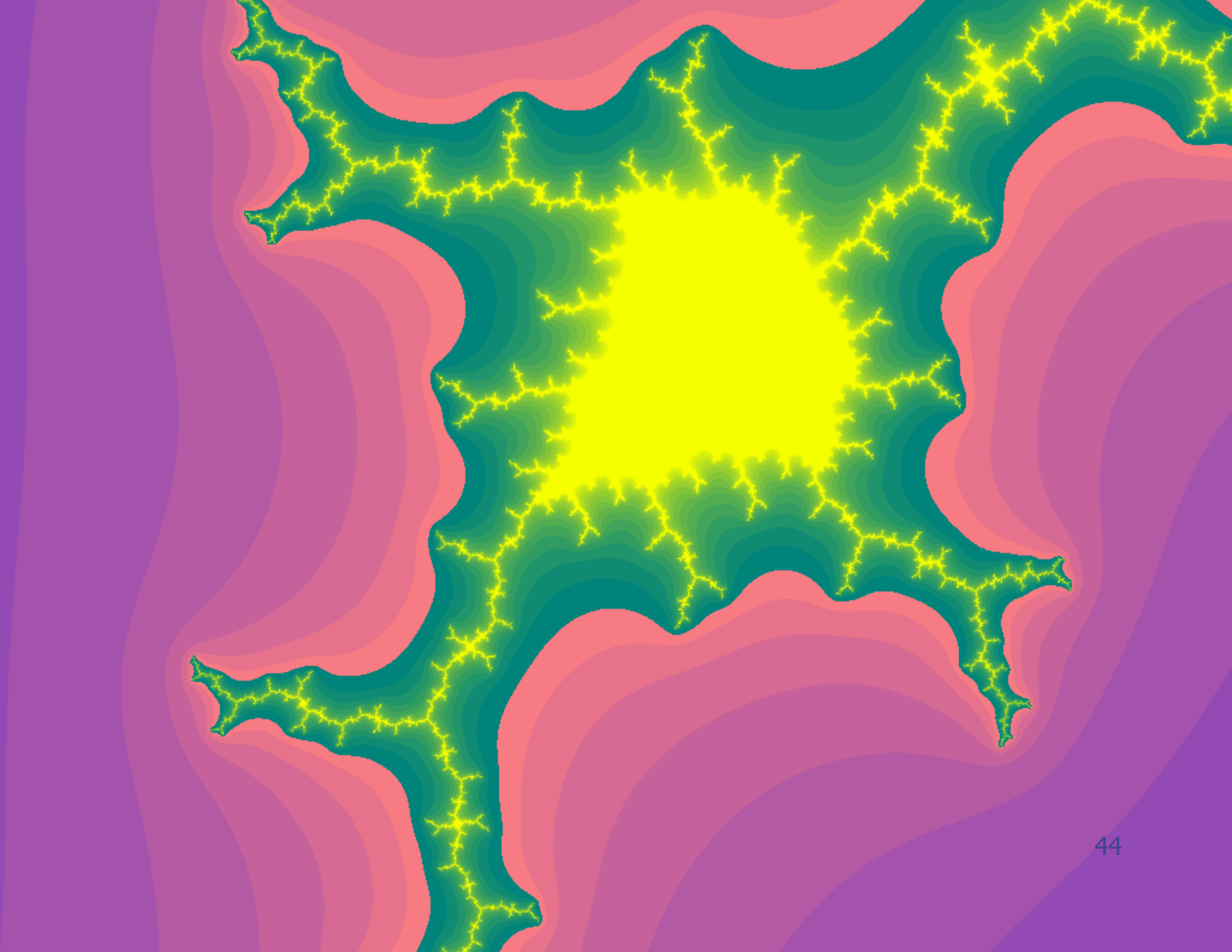
```
Main> :l defs
Main> greet "everybody"
"hello everybody"
Main> square 12
144
Main> fact 32
263130836933693530167218012160000000
Main>
```

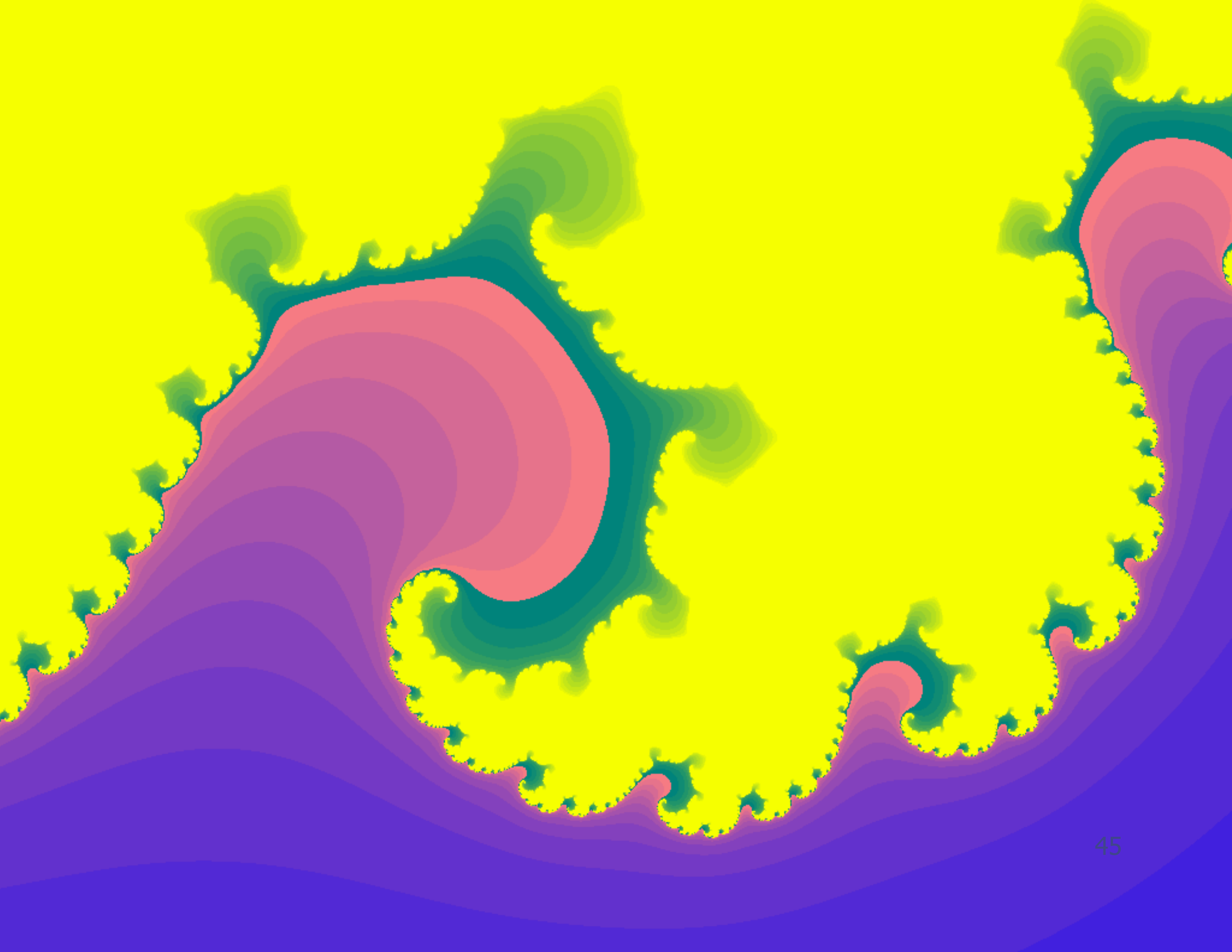
Example: Calculating Fractals











Calculating Fractals:

- Based on my article “Composing Fractals” that was published as a “functional pearl” in the Journal of functional Programming
- Flexible programs for drawing Mandelbrot and Julia set fractals in different ways
- No claim to be the best/fastest fractal drawing program ever created!
- Illustrates key features of functional programming in an elegant and “calculational” style
- As it happens, no recursion!

Mandelbrot Sequences:

```
type Point = (Float, Float)

next              :: Point -> Point -> Point
next (u,v) (x,y) = (x*x-y*y+u, 2*x*y+v)
```

The source of all that beauty & complexity!

```
mandelbrot    :: Point -> [Point]
mandelbrot p  = iterate (next p) (0,0)
```

Apply function repeatedly, producing as many elements as we like …

Converge or Diverge?

```
Fractals> mandelbrot (0,0)
 [(0.0,0.0),(0.0,0.0),(0.0,0.0),(0.0,0.0),(0.0,0.0),(0.0,0.0),
   (0.0,0.0),^C{Interrupted}

Fractals> mandelbrot (0.1,0)
[(0.0,0.0),(0.1,0.0),(0.11,0.0),(0.1121,0.0),(0.1125664,0.0),
   (0.1126712,0.0),(0.1126948,0.0) ^C{Interrupted}

Fractals> mandelbrot (0.5,0)
[(0.0,0.0),(0.5,0.0),(0.75,0.0),(1.0625,0.0),(1.628906,0.0),
   (3.153336,0.0),(10.44353,0.0) ^C{Interrupted}

Fractals> mandelbrot (1,0)
[(0.0,0.0),(1.0,0.0),(2.0,0.0),(5.0,0.0),(26.0,0.0),(677.0,0.0),
   (458330.0,0.0) ^C{Interrupted}

Fractals>
```

The Mandelbrot Set:

- The Mandelbrot Set is the set of all points for which the corresponding Mandelbrot sequence converges
- How can we test for this?
- How can we visualize the results?

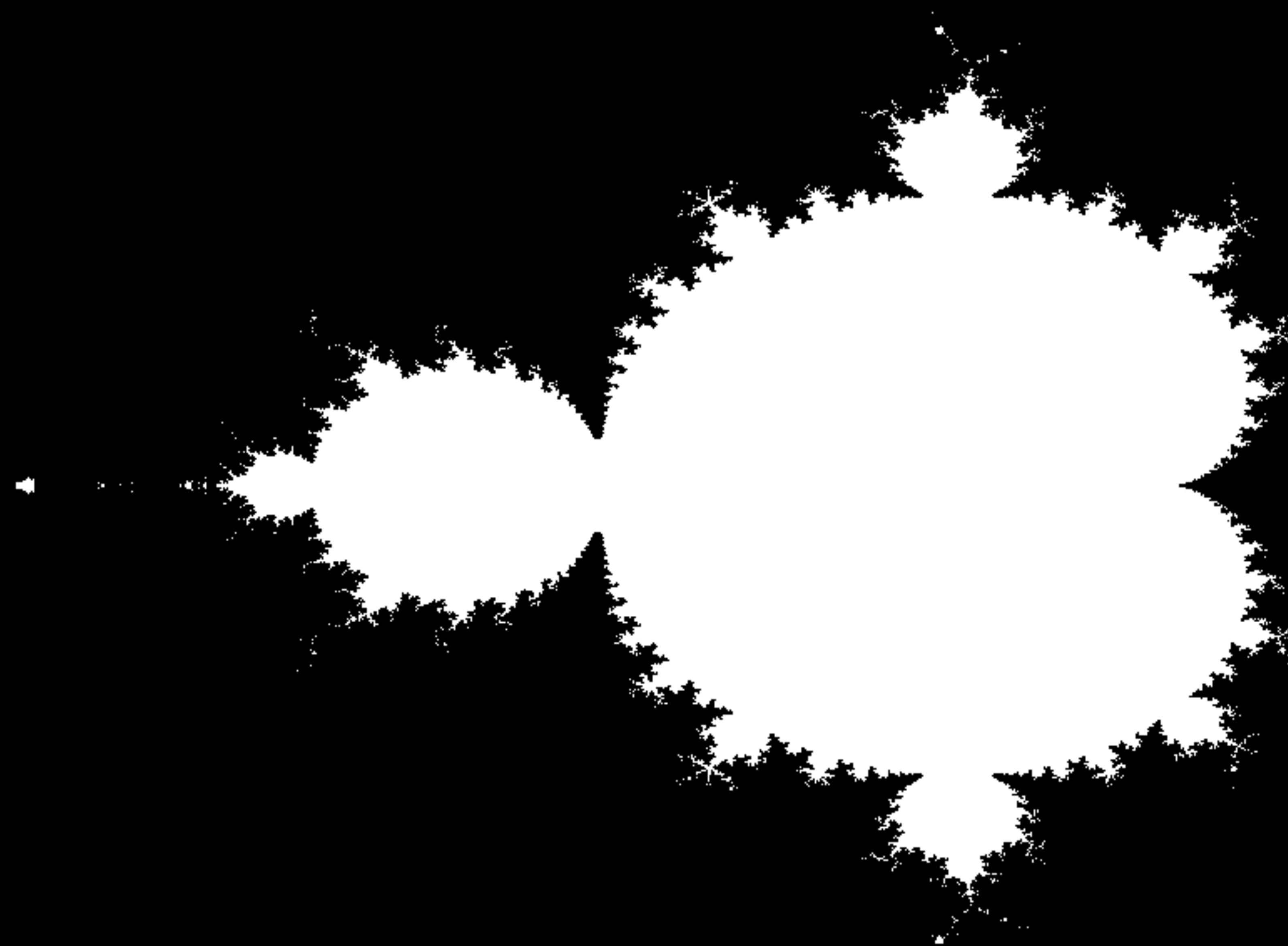
Testing for Membership:

```
fairlyClose        :: Point -> Bool
fairlyClose (u,v) = (u*u + v*v) < 100
```

An almost arbitrary constant

```
inMandelbrotSet  :: Point -> Bool
inMandelbrotSet p = all fairlyClose (mandelbrot p)
```

This could take a long time …



Pragmatics:

- For points very close to the edge, it may take many steps to determine whether the sequence will converge or not.
- It is impossible to determine membership with complete accuracy because of rounding errors
- And besides, the resulting diagram is really dull!
- If life gives you lemons … make lemonade!

Approximating Membership:

```
fracImage              :: [color] -> Point -> color
fracImage palette = (palette!!)
                    . length
                    . take n
                    . takeWhile fairlyClose
                    . mandelbrot
                    where n = length palette - 1
```

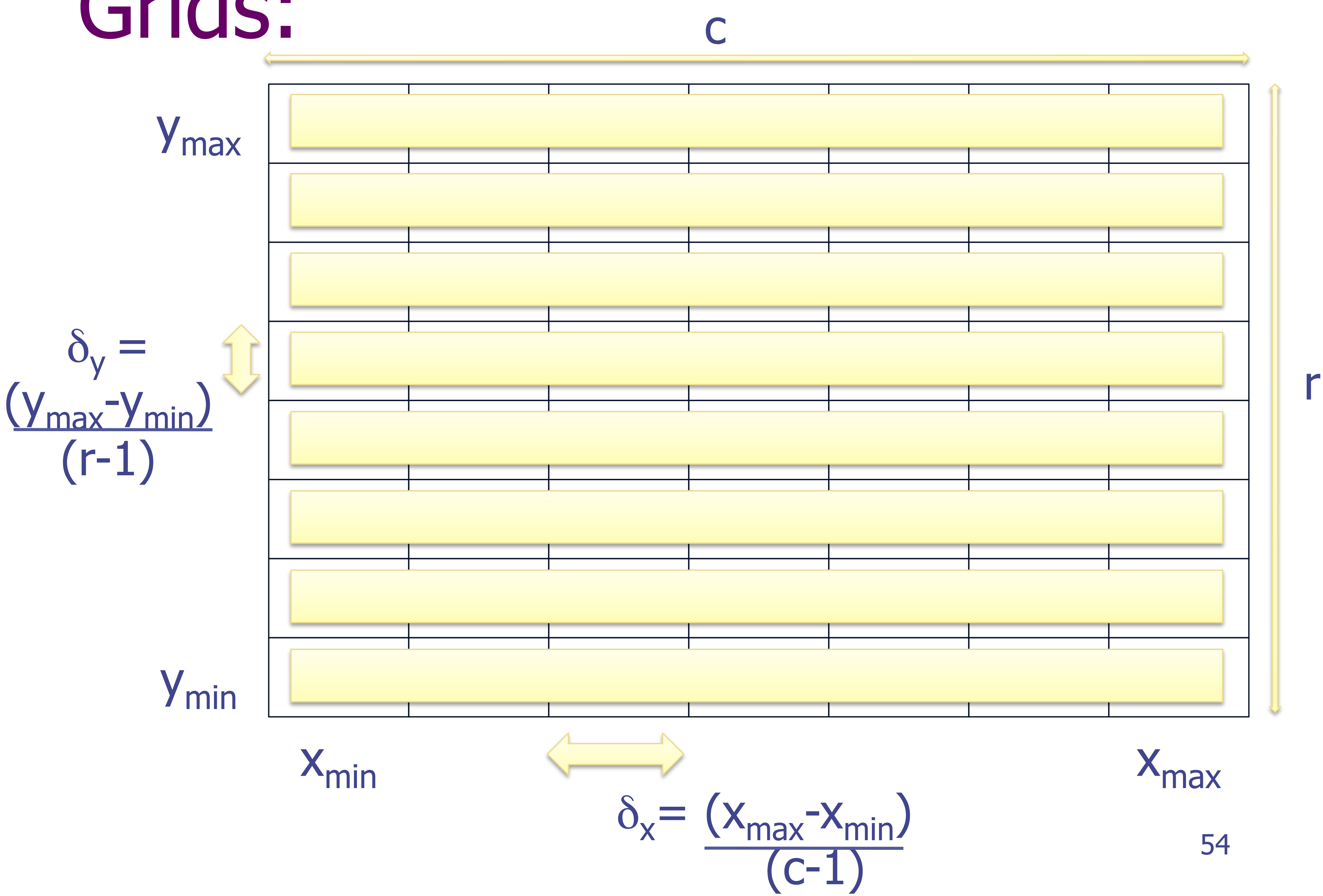
Only looks at initial prefix

A pipeline of functions ...

Now we're using a palette of multiple colors instead of a monochrome membership!

But how are we going to render this?

Grids:


c
r
y_{max}
y_{min}
x_{min}
x_{max}
$\delta_y = \frac{(y_{max}-y_{min})}{(r-1)}$
$\delta_x = \frac{(x_{max}-x_{min})}{(c-1)}$

Grids:

```
type Grid a = [[a]]
```

Give meaningful names to types

```
grid :: Int -> Int -> Point -> Point -> Grid Point
grid c r (xmin,ymin) (xmax,ymax)
      = [[ (x,y) | x <- for c xmin xmax ]
                 | y <- for r ymin ymax ]
```

List comprehensions

```
for          :: Int -> Float -> Float -> [Float]
for n min max = take n [min, min+delta ..]
    where delta = (max-min) / fromIntegral (n-1)
```

Capture recurring pattern

Some Sample Grids:

```
mandGrid  = grid 79 37 (-2.25, -1.5) (0.75, 1.5)

juliaGrid = grid 79 37 (-1.5, -1.5) (1.5, 1.5)
```

Names make it easier to refer to previously defined values!

Images:

Allow for different types of "color"

```
type Image color = Point -> color

sample :: Grid Point -> Image color -> Grid color
sample points image
        = map (map image) points
```

Functions are just regular values ...

Putting it all together:

```
draw :: [color] ->
          Grid Point ->
            (Grid color -> pic) -> pic
draw palette grid render
      = render (sample grid (fracImage palette))
```

Example 1:

```
charPalette :: [Char]
charPalette  = "    ,.`\"~:;o-!|?/<>X+={^O#%&@8*$"

charRender  :: Grid Char -> IO ()
charRender   = putStr . unlines

example1 = draw charPalette mandGrid charRender
```

```
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```
draw charPalette mandGrid charRender
```



```
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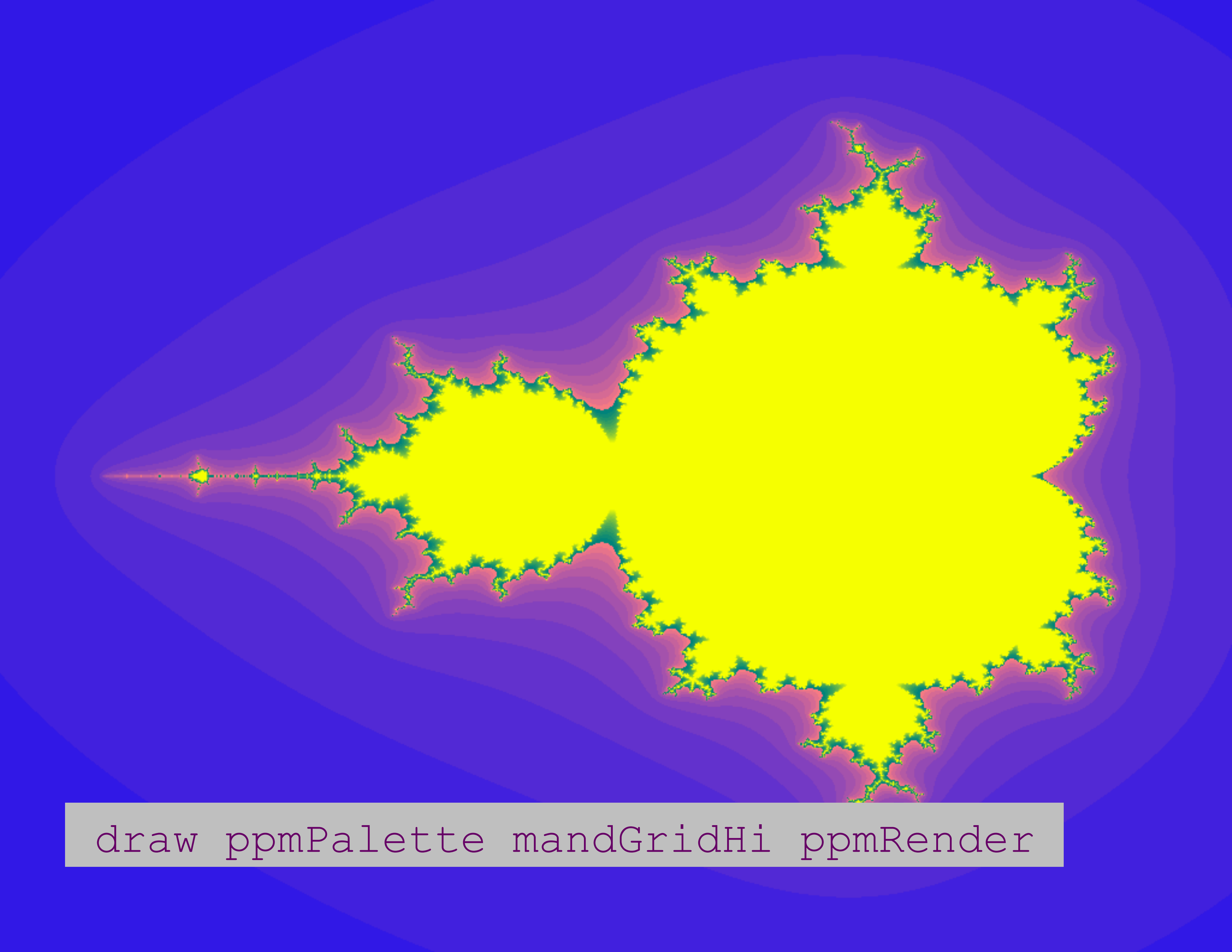
```

Example 2:

```
type PPMcolor = (Int, Int, Int)

ppmPalette :: [PPMcolor]
ppmPalette  = [ (((2*i) `mod` (ppmMax+1)), i, ppmMax-i)
              | i <- [0..ppmMax] ]
ppmMax      = 31 :: Int

ppmRender  :: Grid PPMcolor -> [String]
ppmRender g = ["P3", show w ++ " " ++ show h, show ppmMax]
              ++ [ show r ++ " " ++ show g ++ " " ++ show b
                 | row <- g, (r,g,b) <- row ]
              where w = length (head g)
                    h = length g
```



draw ppmPalette mandGridHi ppmRender

Down with Tangling!

- Changes to a program may require modifications of the source code in multiple places
- The implementation of a program feature may be "tangled" through the code
- Programs are easier to understand and maintain when important changes can be isolated to a single point in the code (and, perhaps, turned into a parameter)
- A simpler example:
  - Calculate the sum of the squares of the numbers from 1 to 10
  - sum (map square [1..10])

Summary:

- An appealing, high-level approach to program construction in which independent aspects of program behavior are neatly separated
- It is possible to program in a similar compositional / calculational manner in other languages …
- … but it seems particularly natural in a functional language like Haskell …