

# Regular Grammars

# Definition

- A Regular Grammar is a quadruple  $G = (V, T, P, S)$ , where
  1.  $V$  is a finite set of *variables (nonterminals, syntactic categories)*
  2.  $T$  is a finite set of *terminals (alphabet)*
  3.  $P$  is a finite set of *productions* : rules of the forms
    1.  $V \rightarrow \Lambda$  ( $\lambda$ )
    2.  $V \rightarrow w$  ( $\beta$ )
    3.  $V \rightarrow V$  ( $\gamma$  rules)
    4.  $V \rightarrow wV$  ( $\delta$  rules) where  $w \in T^*$
  4.  $S$ , the *start symbol*, is an element of  $V$

# Example 1

Non-terminals = [S,B]

Terminals = [a,b]

Start = S

$S \rightarrow$

$S \rightarrow a S$

$S \rightarrow B$

$B \rightarrow b$

$B \rightarrow b B$

# Example 2

Non-terminals = [S,C]

Terminals = [a,b,c]

Start = S

$S \rightarrow a S$

$S \rightarrow b C$

$C \rightarrow$

$C \rightarrow c C$

# Example 3

Non-terminals = [A,B,C]

Terminals = [a,b]

Start = A

A  $\rightarrow$  a A

A  $\rightarrow$  a C

A  $\rightarrow$  b B

B  $\rightarrow$  a B

C  $\rightarrow$  b B

B  $\rightarrow$

# Derivation

- We say a grammar derives a string if
- Start with any rule whose LHS is the start symbol. Write down the RHS.
- Repeatedly, replace any Non-terminal,  $X$ , in the written down term, with rhs, where  $(X \rightarrow \text{rhs})$  is one of the productions.
- When there are no more Non-terminals, written down term is the derived string.

# Example

Non-terminals = [S,C]

Terminals = [a,b,c]

Start = S

S  $\rightarrow$  a S

S  $\rightarrow$  b C

C  $\rightarrow$

C  $\rightarrow$  c C

- Right-Hand-Side

- a S
- a a S
- a a b C
- a a b c C
- a a b c c C
- a a b c c

- Rule

- S  $\rightarrow$  a S
- S  $\rightarrow$  a S
- S  $\rightarrow$  b C
- C  $\rightarrow$  c C
- C  $\rightarrow$  c C
- C  $\rightarrow$

# Tree Derivation

Non-terminals = [S,C]

Terminals = [a,b,c]

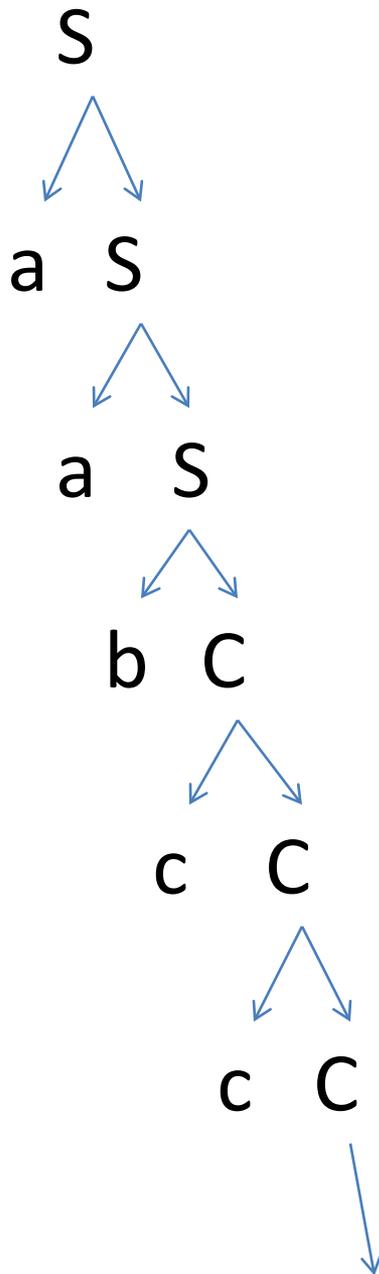
Start = S

S  $\rightarrow$  a S

S  $\rightarrow$  b C

C  $\rightarrow$

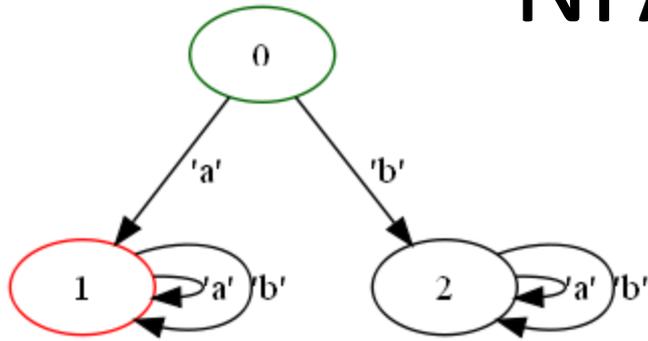
C  $\rightarrow$  c C



Derived string

**aabcc**

# NFA to RegGram



Non-terminals =  $[S_0, S_1, S_2]$

Terminals =  $[a, b]$

Start =  $S_0$

$S_0 \rightarrow a S_1$

$S_0 \rightarrow b S_2$

$S_1 \rightarrow a S_1$

$S_1 \rightarrow b S_1$

$S_2 \rightarrow a S_2$

$S_2 \rightarrow b S_2$

$S_1 \rightarrow$

For every transition

$I \xrightarrow{a} J$

Add a production

$S_I \rightarrow a S_J$

For every transition

$I \xrightarrow{\Lambda} J$

Add a production

$S_J \rightarrow$

For every final state  $K$

Add a production

$S_j \rightarrow$

# RegGram to GenNFA

Terminals = [a,b,c,d]

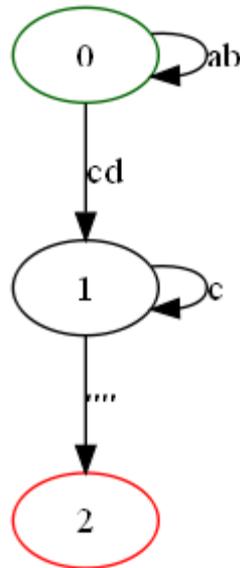
Start = S0

S0 -> a b S0

S0 -> c d S1

S1 ->

S1 -> c S1



The non-terminal become the states, but also invent a new final state F

For each kind of prod

1.  $V \rightarrow \Lambda$  ( $\lambda$ )
2.  $V \rightarrow w$  ( $\beta$ )
3.  $V \rightarrow V$  ( $\gamma$  rules)
4.  $V \rightarrow w V$  ( $\delta$  rules)

Add a transition

1.  $I \rightarrow \Lambda$  add  $I - \Lambda \rightarrow F$
2.  $I \rightarrow w$  add  $I - w \rightarrow F$
3.  $I \rightarrow w J$  add  $I - w \rightarrow J$
4.  $I \rightarrow J$  add  $I - \Lambda \rightarrow J$

# Simplify GenNFA

