

# CS311 – Computational Structures – HW8

Thursday, May 26, 2011  
due in class Thursday, June 2, 2011

Answer each question below. Write your answers neatly on paper. Be sure your name is on the paper, and the paper is clearly identified as Homework 8.

1. Problem 8, Section 13.2, page 843. Post system for the odd palindromes over  $\{a,b,c\}$  (10 points)
2. In the lambda calculus, computation is performed by substitution. There are two important properties that must be preserved by substitution. When substituting the term *new* for the variable  $x$  in the term *old*
  - Free variables in *new* must remain free in the resulting term.
  - Only Free occurrences of  $x$  must be replaced in *old*

Perform the following substitutions, by writing down the result (5 points each).

- substitute (`fst x`) for  $z$  in  $(\lambda y . z y)$
  - substitute (`snd x`) for  $y$  in  $(\lambda w . (\lambda y . y) y)$
  - substitute (`third x`) for  $y$  in  $(\lambda w . (\lambda x . y) y)$
  - substitute (`snd x`) for  $y$  in  $(\lambda w . (\lambda y . y) y)$
3. In the lambda calculus there are 3 kinds of terms
    - variables.  $x, y, z$  etc.
    - applications.  $f x$  (formed by juxtaposition of two other terms  $f$  and  $x$ ).
    - lambda terms.  $\lambda v . t$  where  $v$  is a variable and  $t$  is a term.

An application term  $(f x)$  where  $f$  is a lambda term  $(\lambda v . e)$  *reduces* to substitute  $x$  for  $v$  in  $e$ .

Given the following definitions

- $\text{add} = \lambda x . \lambda y . \lambda z . \lambda s . x (y z s) s$
- $2 = \lambda z . \lambda s . s (s z)$
- $1 = \lambda z . \lambda s . s z$

reduce the following term  $((\text{add } 2) 1)$ . Write your answer as a series of steps. Each step does one of two things. Either it replaces a definition with its value, or performs 1 reduction step. You should continue until no more steps are possible. Hint: it took me 10 steps. (20 points)

4. Problem 3, part b, Section 14.1, page 857 (15 points)
5. Problem 4, Section 14.1, page 857 (15 points)
6. Problem 6, parts b and d, Section 14.1, page 857 (20 points)