NFA with epsilon transitions

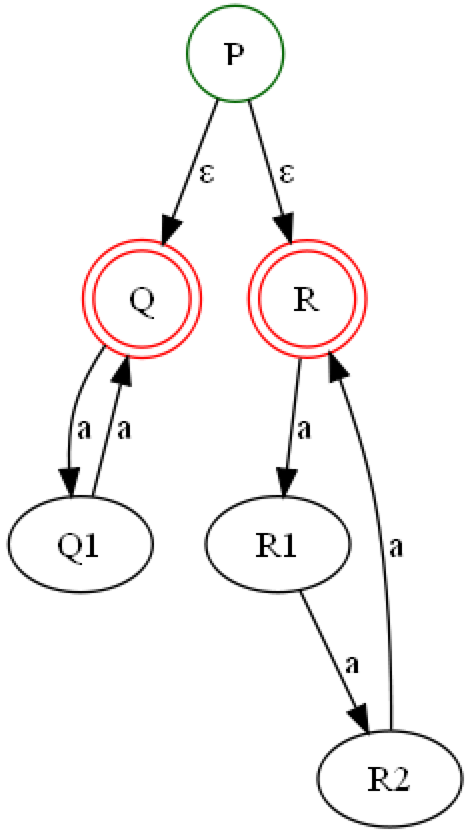
Sipser pages 47-54

NFA's with ε –Transitions

- We extend the class of NFAs by allowing instantaneous (ε) transitions:
  1. The automaton may be allowed to change its state without reading the input symbol.
  2. In diagrams, such transitions are depicted by labeling the appropriate arcs with ε.
  3. Note that this does not mean that ε has become an input symbol. On the contrary, we assume that *the symbol* ε *does not belong to any alphabet*.

example

- { a^n | n is even or divisible by 3 }



This is the version of the NFA on page 53 of Sipser.

Definition

- **A** ε-NFA is a quintuple $\mathbf{A=(Q,\Sigma,\delta,q_0,F)}$ where

  - $\mathbf{Q}$ is a set of *states*
  - Σ is the alphabet of *input symbols*
  - $\mathbf{q_0} \in \mathbf{Q}$ is the *initial state*
  - $\mathbf{F} \subseteq \mathbf{Q}$ is the set of *final states*
  - δ: $\mathbf{Q} \times \Sigma_\varepsilon \longrightarrow P(\mathbf{Q})$ is the *transition function*

- Note ε is never a member of Σ
- Σ_ε is defined to be $(\Sigma \cup \varepsilon)$

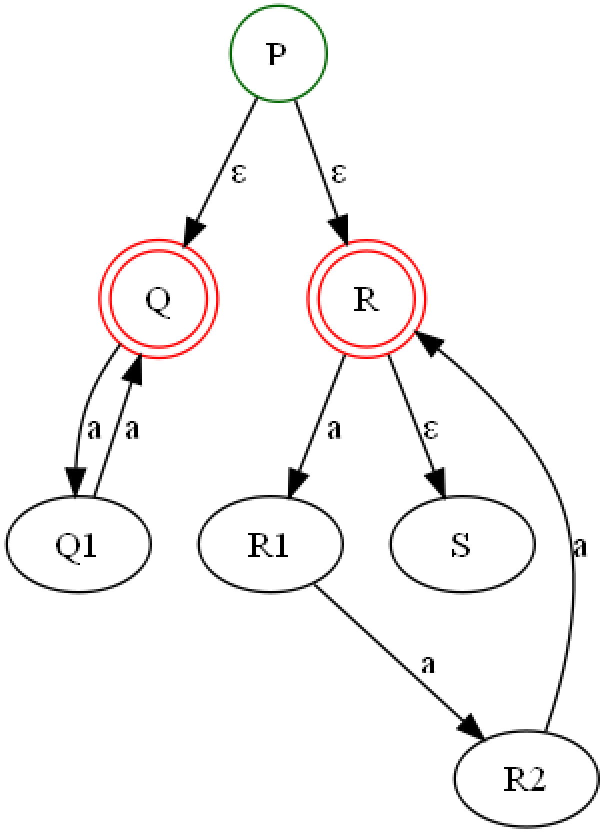
ε-NFA

- ε -NFAs add a convenient feature but (in a sense) they bring us nothing new: they do not extend the class of languages that can be represented. Both NFAs and ε-NFAs recognize exactly the same languages.

- ε-transitions are a convenient feature: try to design an NFA for the even or divisible by 3 language that does not use them!
  - Hint, you need to use something like the product construction from union-closure of DFAs

ε-Closure

- ε-closure of a state
- The *ε-closure* of the state q, denoted ECLOSE(q), is the set that contains q, together with all states that can be reached starting at q *by following only ε-transitions*.

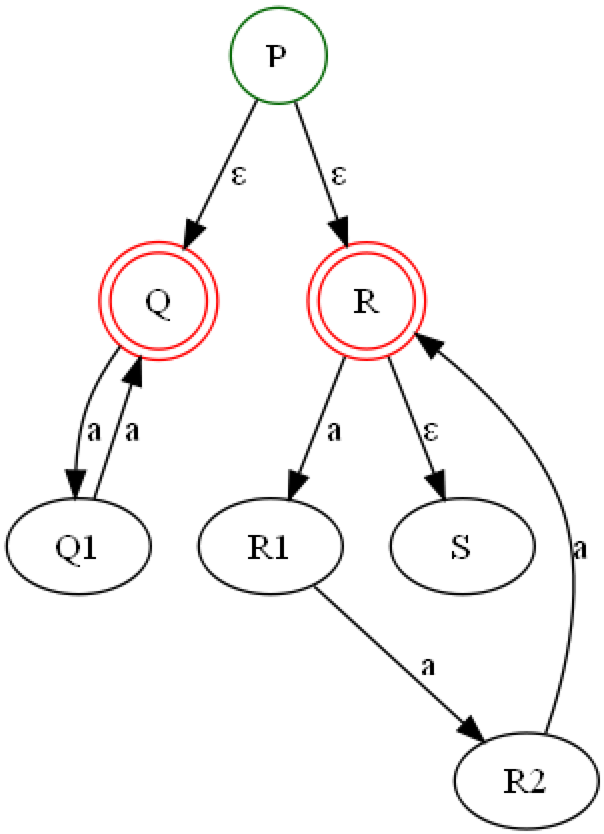


- In the above example:
- ECLOSE(P) ={P,Q,R,S}
- ECLOSE(R)={R,S}
- ECLOSE(x)={x} for the remaining 5 states
  {Q,Q1,R1,R2,R2}

Computing eclose

- Compute eclose by adding new states until no new states can be added

- Start with [P]
- Add Q and R to get [P,Q,R]
- Add S to get [P,Q,R S]
- No new states can be added

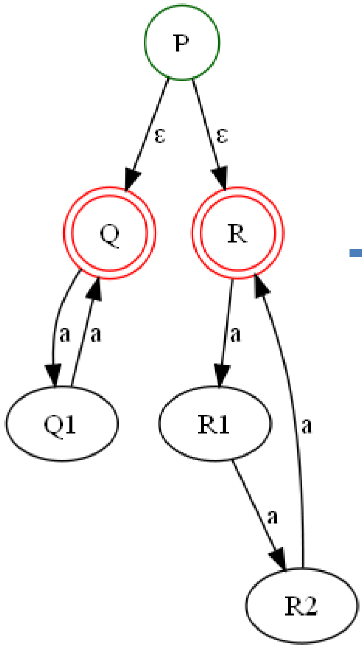


Elimination of ε-Transitions

- Given an ε-NFA N, this construction produces an NFA N' such that L(N')=L(N).

- The construction of N' begins with N as input, and takes 3 steps:

  1. Make p an accepting state of N' iff ECLOSE(p) contains an accepting state of N.
  2. Add an arc from p to q labeled a iff there is an arc labeled a in N from some state in ECLOSE(p) to q.
  3. Delete all arcs labeled ε.



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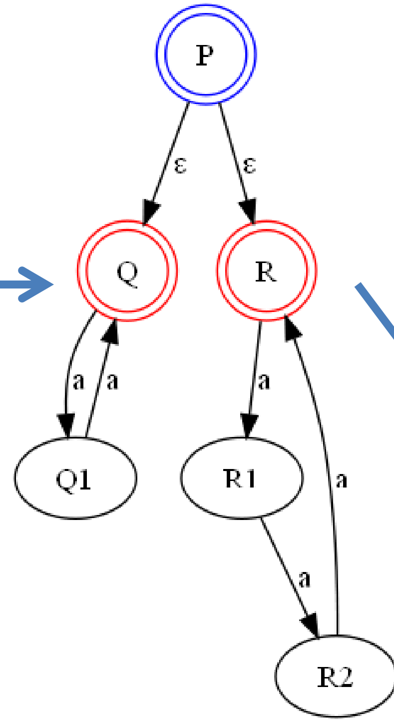

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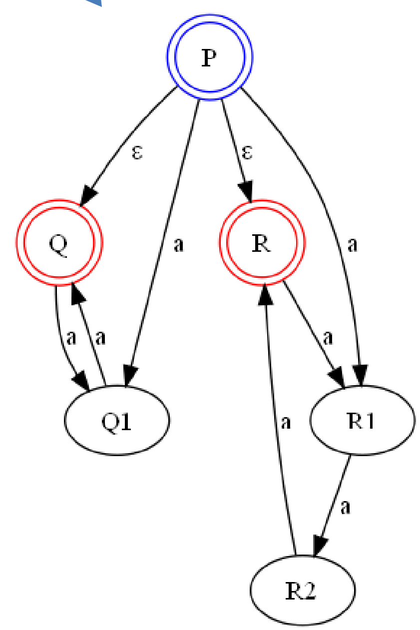

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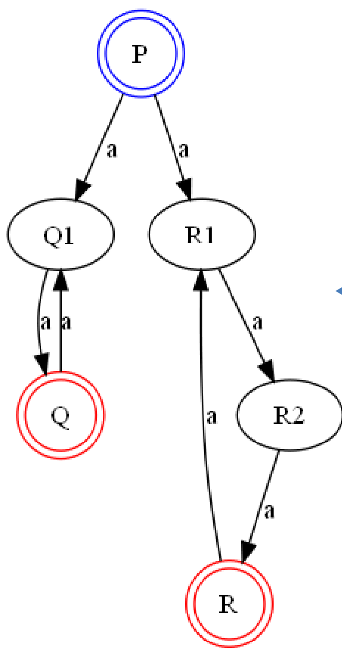

Why does it work?

- The language accepted by the automaton is being preserved during the three steps of the construction: $L(N)=L(N_1)=L(N_2)=L(N_3)$

- Each step here requires a proof. A Good exercise for you to do!

Theorem

- Any NFAe can be turned into an NFA
- How?