NFA Closure Properties

Sipser pages pages 58-63

NFAs also have closure properties

- We have given constructions for showing that DFAs are closed under
    1. Complement
    2. Intersection
    3. Difference
    4. Union
- We will now establish that NFAs are also closed under
    1. Reversal
    2. Union
    3. Concatenation
    4. Kleene star

Proof Strategy

- As we did for DFAs, To prove these properties

1. We'll assume some language (or languages) are recognized byan NFA (or an ε-NFA)
2. The then that NFA must be a 5-tuple $\mathbf{A} = (\mathbf{Q}, \Sigma, \delta, \mathbf{q_0}, \mathbf{F})$
3. Then we'll use the pieces of the 5-tuple to create a new 5-tuple that is the NFA we want.
4. It is very similar to writing a program!

Reversal of ε-NFAs

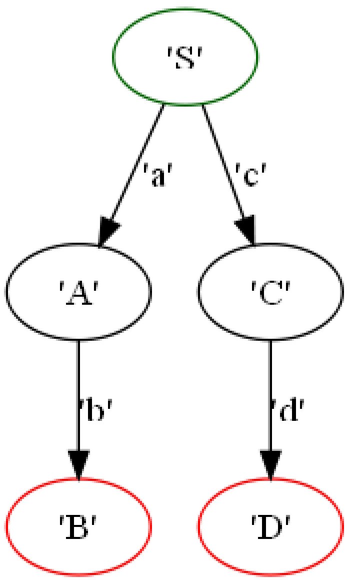
- Closure under reversal is easy using ε-NFAs. If you take such an automaton for L, you need to make the following changes to transform it into an automaton for L^{Rev}:

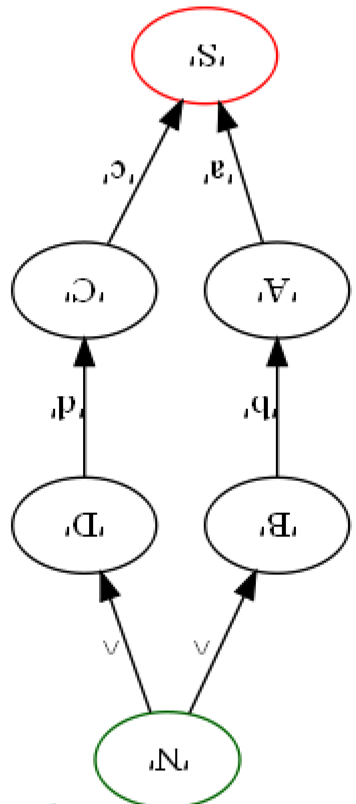
1. Reverse all arcs

2. The old start state becomes the only new final state.

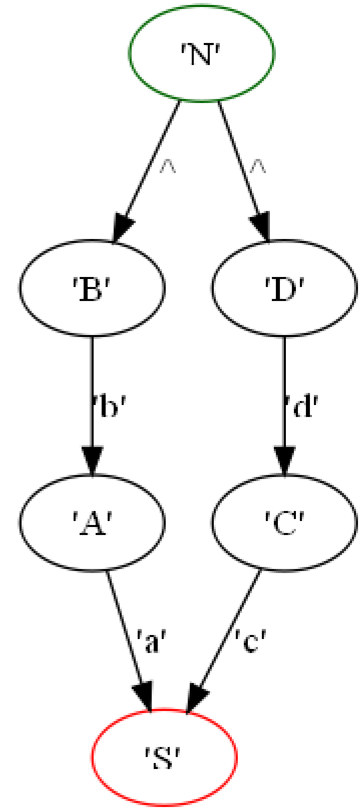
3. Add a new start state, and an ε-arc from it to all old final states.

Example





This is upside down on purpose



1. Reverse all arcs
2. The old start state becomes the only new final state.
3. Add a new start state, and an ε-arc from it to all old final states.

Union

- We showed that DFAs are closed under union by using the product construction. It is much easier to show NFAs closed under union because we have ε transitions.

- How?

Concatentation

- L • R = { x • y | x in L and y in R}

- To form a new ε-NFA that recognizes the concatenation of two other ε-NFAs with the same alphabet do the following
  - Union the states (you might have to rename them)
  - Add an ε-transition from each final state of the first to the start state of the second.

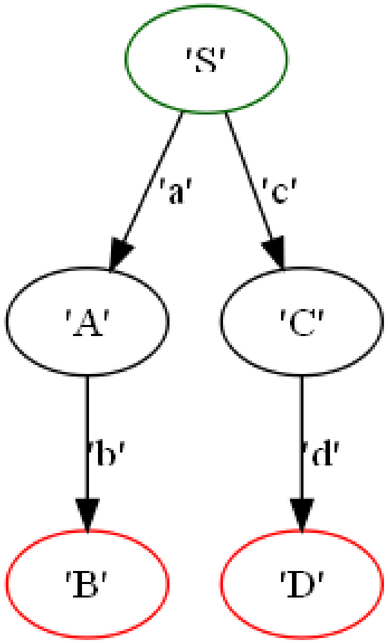
Formally

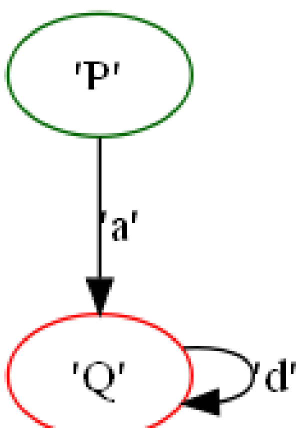
- Let
  - $L = (Q_L, A, T_L, s_L, F_L)$
  - $R = (Q_R, A, T_R, s_R, F_R)$
- $L \bullet R = = (Q_L \cup Q_R, A, T, s_L, F_R)$

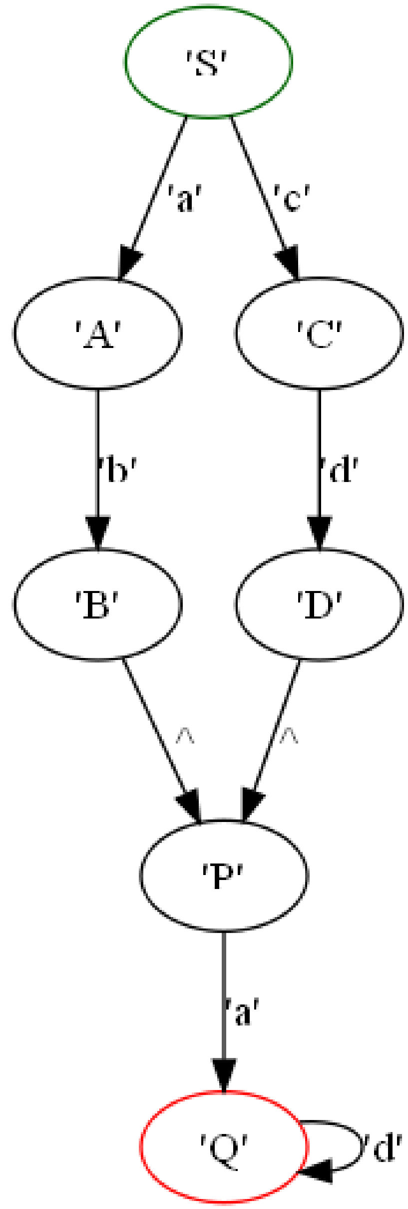
Where $T\ s\ \varepsilon\ |\ s \in F_L = S_R \cup T_L\ s\ \varepsilon$

$T\ s\ c\ |\ s \in Q_L = T_L\ s\ c$

$T\ s\ c\ |\ s \in Q_R = T_R\ s\ c$


'S'
'a'
'c'
'A'
'C'
'b'
'd'
'B'
'D'


'P'
'a'
'Q'
'd'

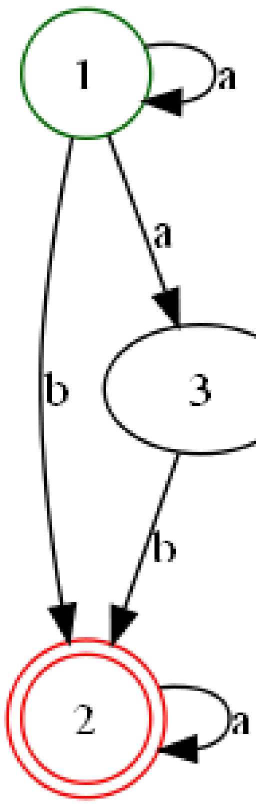

'S'
'a'
'c'
'A'
'C'
'b'
'd'
'B'
'D'
^
^
'P'
'a'
'Q'
'd'

Kleene - Star

- If a language L is recognized by an NFA then so is the language L*

- Add a new state.
- Make it the start state in the new NFA.
- Add an ε-arc from this state to the old start state.
- Add ε-arcs from every final state to this new state.

Example



- Add a new state.
- Make it the start state in the new NFA, and an accepting state.
- Add an ε-arc from this state to the old start state.
- Add ε-arcs from every final state to this new state