CS311 – Computational Structures – HW1

Tuesday, October 1, 2013
due in class Tuesday, October 8, 2013

Answer each question below. You will turn this homework in using D2L as a pdf file. You may format your answers using some document processing software, or you may write it up with pencil and paper and scan it. In either case submit a pdf document. Be sure your submission is clearly identified as Homework 1, and contains your name and your email on the first line. The first line should look like:

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When doing a proof, set up the structure of the proof first, then carry out the steps. Use the formatting discussed in the class notes.

- Facts. List all the facts, including function definitions, and any facts about arithmetic you might use.
- State clearly the proposition you are proving.
- List each step. Justify each step with one of the facts.

Be sure and state what kind of proof: Proof by induction (state all the cases, what the induction variable is, and what the inductive hypotheses are), Proof by Contradiction (state what is to be proved, state what contradiction you reach), etc. and then format the proof (using numbering, indentation, boxes, or other lexigraphic conventions) so that the structure is evident in the proof steps.

1. The sum of the first n integers $(1 + 2 + 3 + \ldots N)$ can be computed by the formula $(1 + N) * (N/2)$. Prove by induction over the natural numbers that this formula is correct. Format your proof as discussed in the class notes and in the instructions above.
2. For each part give a relation that satisfies the condition. (Exercise 0.7 page 26 Sipser)
   (a) Reflexive and symetric but not transitive.
   (b) Reflexive and transitive but not symmetric.
   (c) Symetric and transitive, but not reflexive.

Warning, part C is harder than part A and B.

3. Construct DFAs for the following languages over $\{a, b\}^\star$. Your answer should be in the form of a state-diagram (a picture or graph representing the DFA).

   (a) Strings with an even number of a's

   (b) Strings with an even number of b's

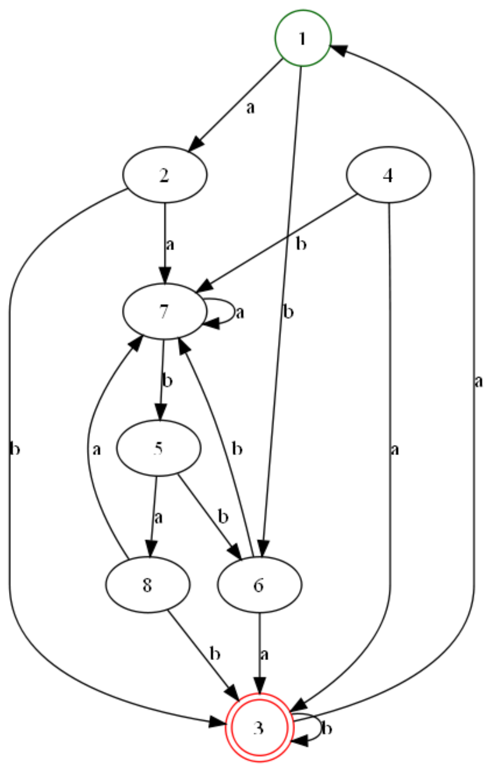
   (c) Strings that contain the substring aa

   (d) Strings that contain the substring abb

   (e) The empty language

   (f) The language consisting of the empty string

4. Consider the state-diagram below,



Note: the start state is state 1. This machine generated graph indicates start states with green circles.

Then recall the formal definition of computation by DFA (page 40 of the text). Prove that the DFA from question 4 above either accepts or rejects the strings:

- aaaaabba

- baaa
- babbbaab

If it accepts the string, exhibit the path, and argue that it meets all 3 conditions. If it rejects, find a path and show what condition it fails to meet.

5. For any string $w = w_1 w_2 \ldots w_n$, the reverse of w, writen w^R is the string in reverse order. $w_n w_{n-1} \ldots w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R. (Problem 1.31, page 88 Sipser).

6. Read the description of finite state transducer (FST) in problem 1.24 (page 87 Sispser). Then do question 1.25 (also on page 87 of Sipser). The problem asks you to formalize a finite state transducer (much like the formalization of finite automaton on page 35), and to formalize the notion of computation of a finite state transducer (much like the formal definition of computation for finite automaton on page 40). In essence you must formalize the notion of when an FST relates an input string x with an output string y.