NFA Closure Properties

NFAs also have closure properties

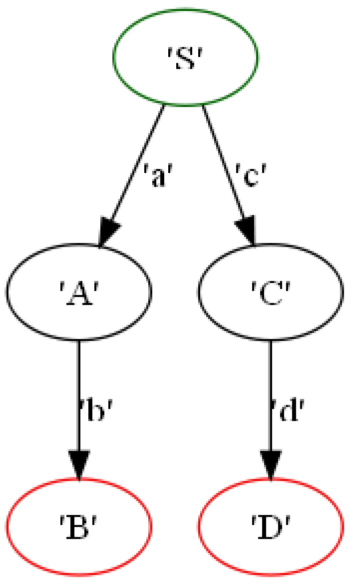
- We have given constructions for showing that DFAs are closed under
  1. Complement
  2. Intersection
  3. Difference
  4. Union
- We will now establish that NFAs are closed under
  1. Reversal
  2. Kleene star
  3. Concatenation

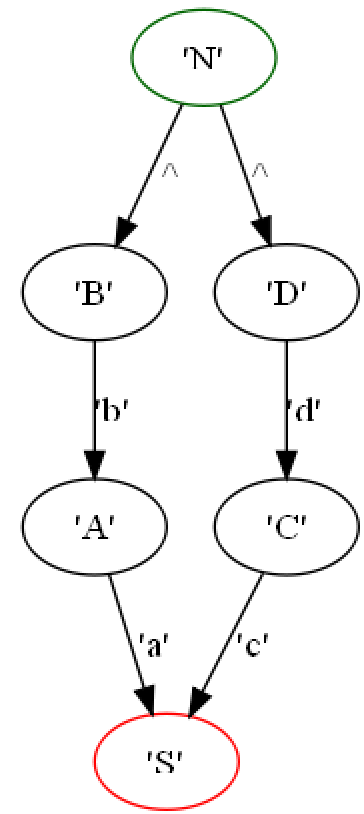
Reversal of Λ-NFAs

- Closure under reversal is easy using Λ-NFAs. If you take such an automaton for L, you need to make the following changes to transform it into an automaton for L^{Rev}:

1. Reverse all arcs
2. The old start state becomes the only new final state.
3. Add a new start state, and an Λ-arc from it to all old final states.

Example





1. Reverse all arcs
2. The old start state becomes the only new final state.
3. Add a new start state, and an Λ-arc from it to all old final states.

Concatentation

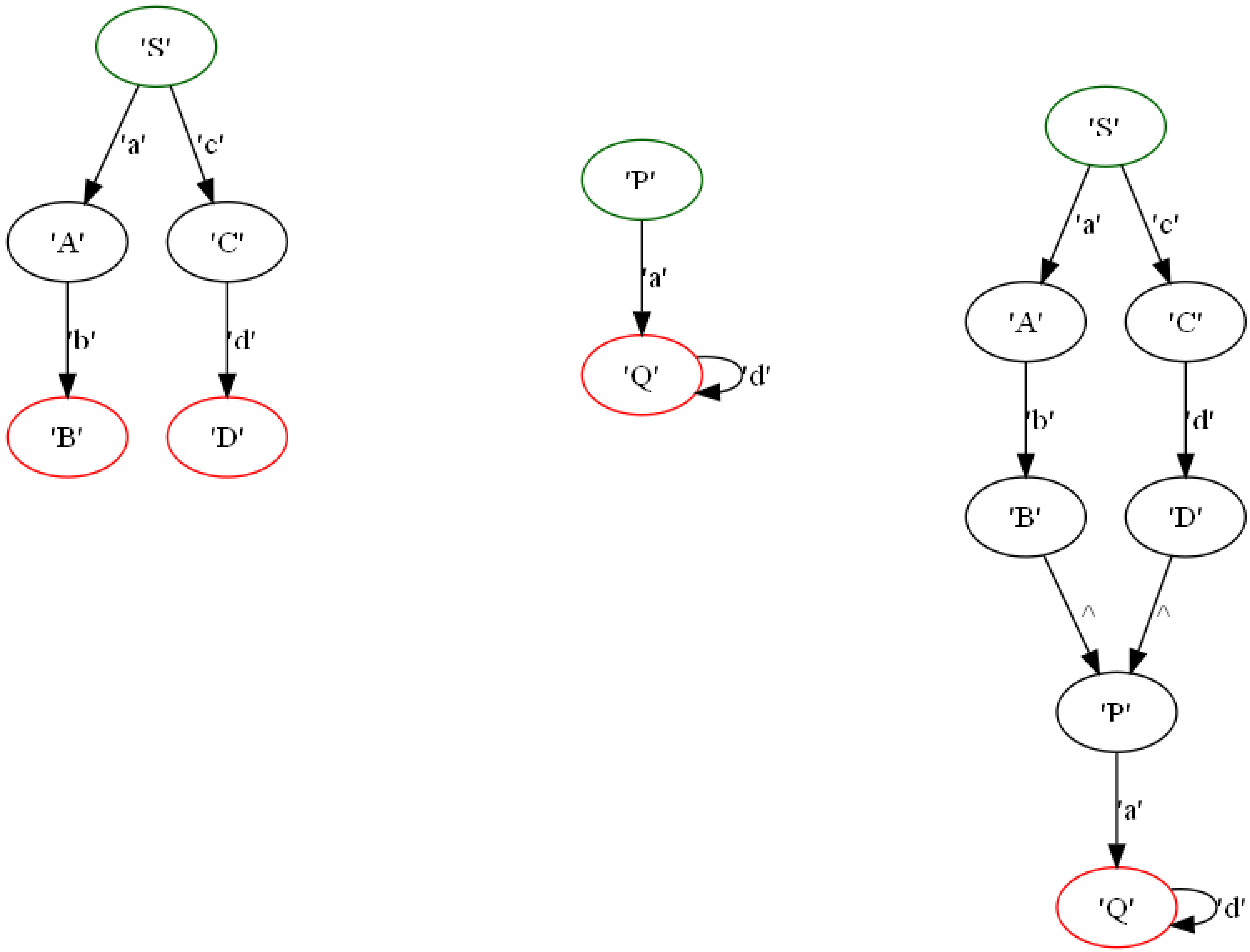
- $L \bullet R = \{ x \bullet y \mid x \text{ in } L \quad \text{and} \quad y \text{ in } R\}$

- To form a new Λ-NFA that recognizes the concatenation of two other Λ-NFAs with the same alphabet do the following
  – Union the states (you might have to rename them)
  – Add an Λ transition from each final state of the first to the start state of the second.

Formally

- Let
  - $L = (Q_L, A, s_L, F_L, T_L)$
  - $R = (Q_R, A, s_R, F_R, T_R)$
- $L \bullet R = = (Q_L \cup Q_R, A, s_L, F_R, T)$

$$
\begin{array}{llll}
\text{Where} & T\ s\ \Lambda & \mid\ s \in F_L & = S_R \\
 & T\ s\ c & \mid\ s \in Q_L & = T_L\ s\ c \\
 & T\ s\ c & \mid\ s \in Q_R & = T_R\ s\ c
\end{array}
$$


'S'
'a'
'c'
'A'
'C'
'b'
'd'
'B'
'D'
'P'
'a'
'Q'
'd'
'S'
'a'
'c'
'A'
'C'
'b'
'd'
'B'
'D'
^
^
'P'
'a'
'Q'
'd'