CS311 – Computational Structures – HW8

Assined Thursday, November 15, 2012
due in class Tuesday, November 27, 2012

Answer each question below. Write your answers neatly on paper. Be sure your name is on the paper, and the paper is clearly identified as Homework 8.

This assignment gives you experience programming with primitive recursive functions. In the lecture notes we introduced a grammar for an algebra of primitive recursive functions.

```
Term ::= Z
       | S
       | P N
       | C  Term  [ Term1, ... ,TermN ]
       | PR  Term  Term

N ::=  1 | 2 | 3 | 4 | ...
```

In lecture notes and in the text below, primitive recursive functions over natural numbers are defined. I have also written a Haskell interpreter for that formalization. It is in the file NaturalPR.hs. for those who might want to study it. This file is available on the class index page.

For this assignment you have two options, either of which is acceptable. You may do either of the following:

1. do the original assignment below with pencil and paper,
2. do the original assignment below as a programming exercise (possibly starting from NaturalPR.hs)

1 Original Assignment

1. In lecture I presented five schemas for defining primitive recursive functions. They are as follows:

   (a) [Zero] There is a constant function zero of every arity.

$$Z(x_1, \ldots, x_k) = 0$$

(b) [Successor] There is a successor function of arity 1.

$$S(x) = x + 1$$

(c) [Projection] There are projection functions for every argument position of every arity.

$$P\ i\ (x_1, \ldots, x_k) = x_i \qquad \text{where } k > 0,\ i \leq k$$

(d) [Composition (also called substitution)] The composition of the function f of arity k with functions $g_1, \ldots g_k$, each of arity l, defines a $C\ f\ [g_1 \ldots g_k]$ of arity l satisfying:

$$C\ f\ [g_1 \ldots g_k](x_1, \ldots, x_l) = f\ (g_1(x_1, \ldots, x_l), \ldots, g_k(x_1, \ldots, x_l))$$

(e) [Primitive Recursion] The arity k function defined by primitive recursion from a function h of arity $k-1$ and a function g of arity $k+1$ is indicated PR h g. It satisfies:

$$\begin{array}{lcl} \text{PR}\ h\ g(0, x_2, \ldots, x_k) & = & h(x_2, \ldots, x_k) \\ \text{PR}\ h\ g(x+1, x_2, \ldots, x_k) & = & g(x, \text{PR}\ h\ g(x, x_2, \ldots, x_k), x_2, \ldots, x_k) \end{array}$$

In lecture we showed how to define addition by primitive recursion:

$$\text{add} = \text{PR}(P1)(C\ S\ [P\ 2])$$

Using primitive recursion define:

(a) Multiplication
(b) Constant true function (e.g. $true(x_1, \ldots, x_n) = 1$)
(c) Constant false function (e.g. $false(x_1, \ldots, x_n) = 0$)
(d) If-then-else (e.g. $\text{ITE}(1, x, y) = x, \text{ITE}(0, x, y) = y$)
(e) Or
(f) And
(g) Not
(h) Minus (e.g. $x - y$ when $x > y$ and 0 otherwise)
(i) Integer equality
(j) The factorial function

Note that each of these solutions is a term in the grammar given above.