CS311 – Computational Structures – HW2

Tuesday, October 2, 2012
due in class Tuesday, October 9, 2012

Answer each question below. Write your answers neatly on paper. Be sure your name is on the paper, and the paper is clearly identified as Homework 2. When doing a proof, set up the structure of the proof first, then carry out the steps. The structure tells what style of proof: Proof by induction (state all the cases, what the induction variable is, and what the inductive hypotheses are), Proof by Contradiction (state what is to be proved, state what contradiction you reach), etc. and then formats the proof (using numbering, indentation, boxes, or other lexigraphic conventions) so that the structure is evident in the proof steps.

1. If an alphabet A contains k elements, how many elements does A^n contain? Use induction to prove your answer.

2. Prove that set intersection distributes over set union: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Remember that to prove sets S_1 and S_2 are equal, you must show that $S_1 \subseteq S_2$ and $S_2 \subseteq S_1$ (or alternatively, show $x \in S_1 \Rightarrow x \in S2$ and $x \in S_2 \Rightarrow x \in S1$).

3. Let the alphabet $A = \{a, b\}$, and let f be a function from A^* to integers that counts the number of as in the given string. A formal definition of f is given by:

$$f(\varepsilon) = 0$$
$$f(\alpha a) = f(\alpha) + 1$$
$$f(\alpha b) = f(\alpha)$$

   Prove that $f(\alpha\beta) = f(\alpha) + f(\beta)$ for any two strings α and β in A^*. (Hint: Use structural induction on the second string.)

4. Construct DFAs for the following languages over $\{a, b\}^*$:
   (a) Strings with an even number of a's
   (b) Strings with an even number of b's
   (c) Strings that contain the substring aa
   (d) Strings that contain the substring abb
   (e) The empty language
   (f) The language consisting of the empty string

5. Illustrate the following constructions by drawing a DFA (or providing a transition table) for each of the following languages:
   (a) Union of languages (a) and (b) in problem 4
   (b) Intersection of languages (a) and (b)
   (c) Complement of language (d)