CS311 – Computational Structures – HW5

Tuesday, October 23, 2012
due in class Tuesday, October 30, 2012

Answer each question below. Write your answers neatly on paper. Be sure your name is on the paper, and the paper is clearly identified as Homework 5.

1. **Proof by induction** (20 points).

   The natural numbers can be described by the following inductive definition.

   - Z is a natural number (zero)
   - if n is a natural number, then Sn is a natural number (the successor function).
   - Nothing else is a natural number

   Examples of using this description include

   (a) 0 is represented by Z

   (b) 3 is represented $S(S(SZ))$

   It is possible to define addition over the natural numbers using the equations.

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(1)  Z + n = n
(2)  (S m) + n = S(m + n)
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   - Carry out by hand the steps for $2+3=5$. Ie. use equations (1) or (2) to transform (S(S Z)) + (S(S(S Z))) into the representation for 5. (5 points)
   - Prove by structural induction that for all natural numbers i,j,k the following property holds: $(i+j)+k = i+(j+k)$
     That is that the definition for + is asscociative
     - What is the induction variable? It must be either i, j, or k. (2 points)
     - What is the formula as a function of the induction variable. (2 points)
     - Use the defintion of natural numbers to help you formulate the stucture of the proof (One case for each way you can make a natural number). Write down the parts of the proof based on the structure of a natural number. (4 points) Which cases have induction hypotheses?
     - Carry out the steps of the proof, label each step with the properties you use. (7 points)

2. **NFA to regular grammar** (20 points) Translate the following NFA over the alphabet $\{a, b, c\}$ into a regular grammar.

	0	1	2	3
	start	final	final	
a	$\{1\}$	$\emptyset$	$\emptyset$	$\{3\}$
b	$\emptyset$	$\{0\}$	$\emptyset$	$\emptyset$
c	$\{2\}$	$\{3\}$	$\emptyset$	$\{1, 2\}$

(a) Draw the NFA (5 points)

(b) Define the regular grammar in terms of a 4-tuple as described in the text and in the notes. (15 points)

3. **Pumping Lemma** (40 points). A pumping lemma argument that some language is not regular is always a proof by contradiction. Clearly state what you are assuming to be true. Clearly state how that leads to a contradiction. Restate what the contradiction shows to be true. Each proof should include re-stating the pumping lemma specialized to the proof you are constructing.

   - **There was a typo in this part of the question. It should be:**
     Show that the language $\{xyx | x \in \{0,1\}^+, y \in \{0,1\}^*\}$ is not regular (20 points). You should try this revised question as an exercise for preparing for the exam. We won't count this part of the question when we grade the homework.
   - The langauge of balanced parentheses can be defined inductively as follows:

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     (1) B -> ""
     (2) B -> B B
     (3) B -> ( B )
     ```

     (a) Give 4 examples of strings in the language described by B. Show which equations you used to derive each string (4 points).

     (b) Show that the language described by B is not regular (16 points).

4. Problem 4, Section 11.4, page 755. Left Biased RegGrammars. 10 points each. Write down the grammars. Explain in a sentence or two why the two formulations of Regular Grammars are equivalent.