

The Future of Parallel Computing

Special Purpose Mesh Architectures



P A R C

- **Why meshes ???**
- **Application specific parallel mesh architectures**

- Systolic Arrays
- Instruction Systolic Arrays
- PIPS
- Reconfigurable mesh
- Optical Highway

Fine grain
1983

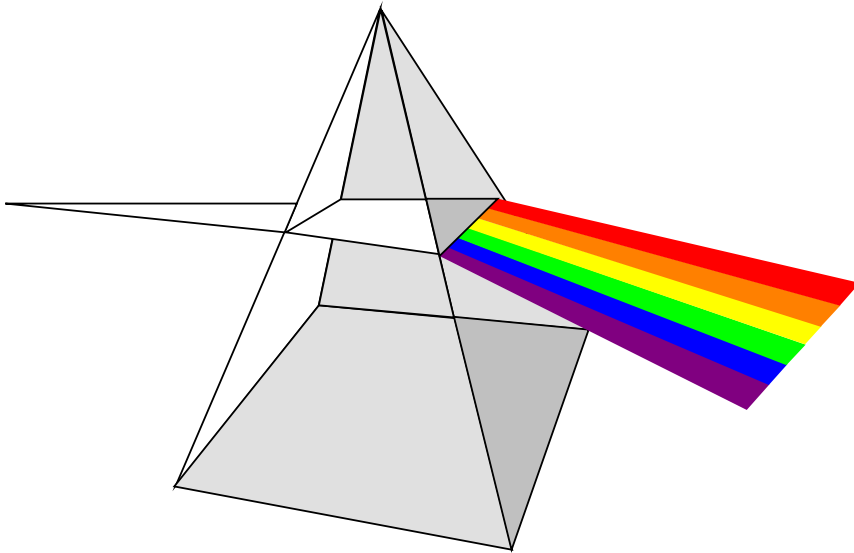


Coarse grain
1997

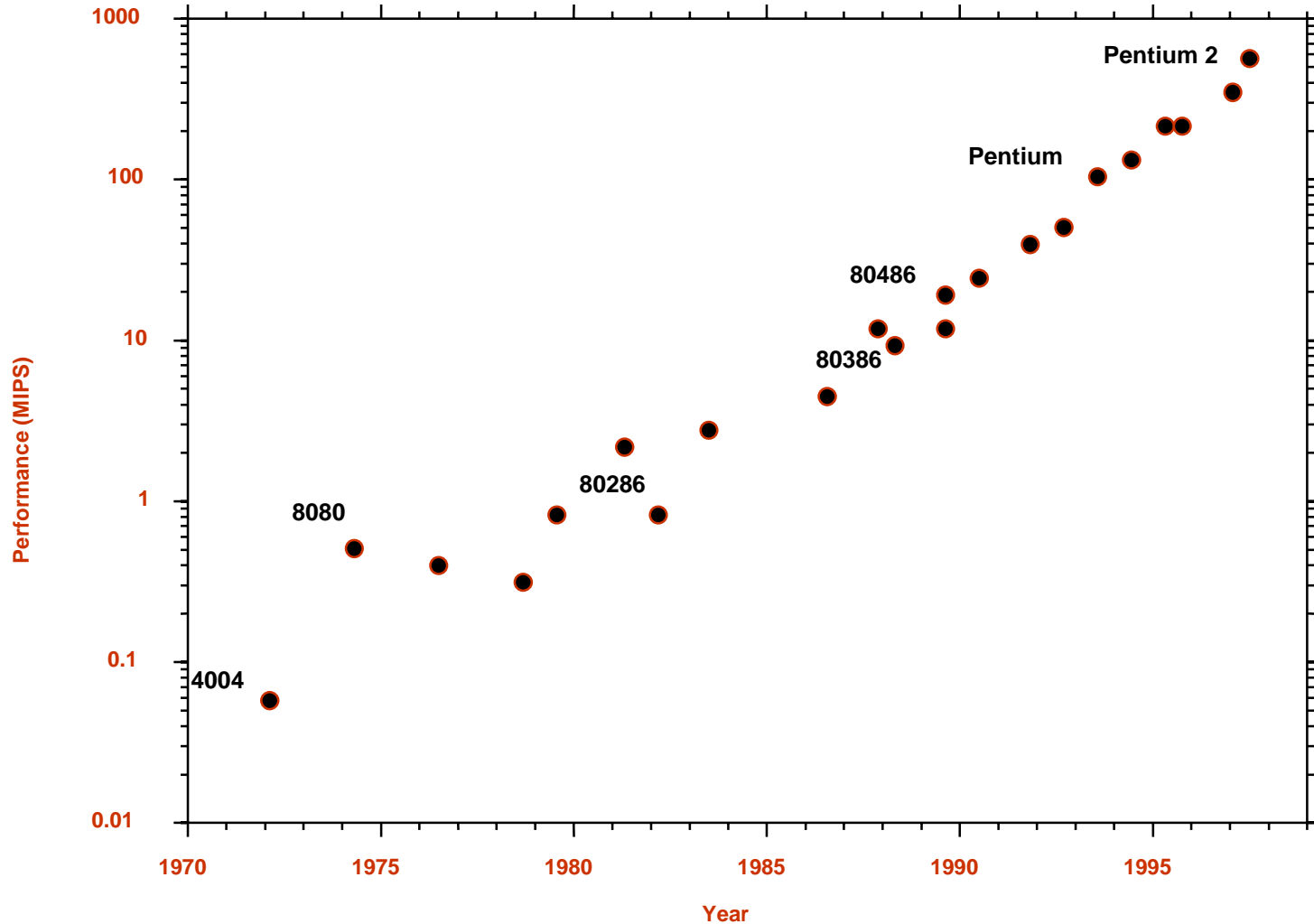
P A R C

Physical limits

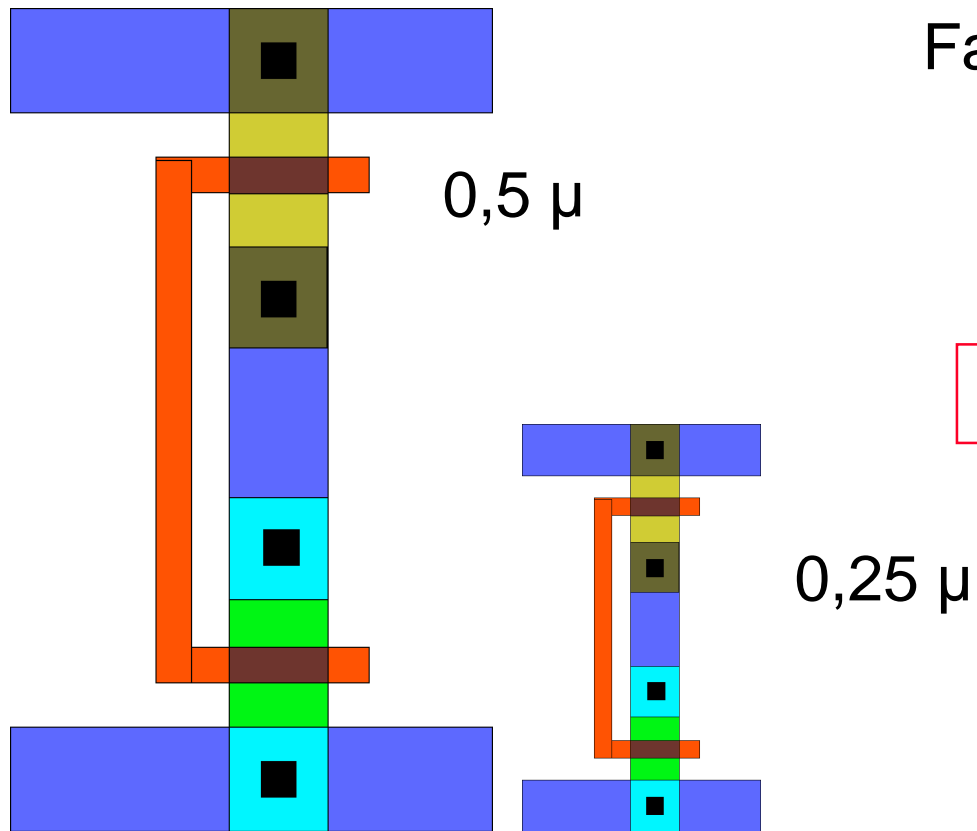
$c=300\,000\text{ km/sec}$



- 10^{12} OPS -- 0.3 mm/OP
- 1000 PEs with 10^9 OPS -- 30cm/OP
- massive parallelism
- distributed memory



PÄRC

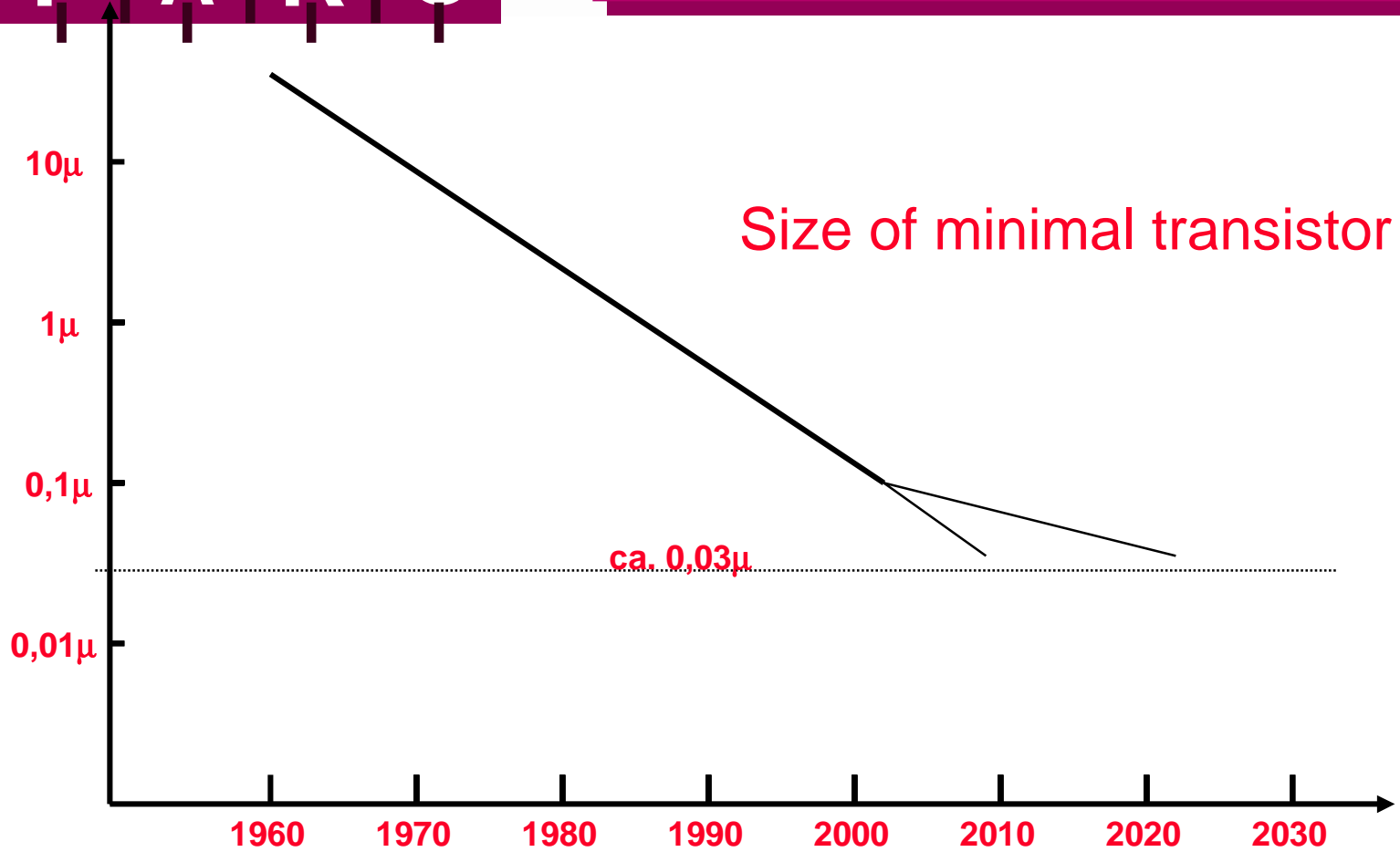


Scaling

Faktor 2:

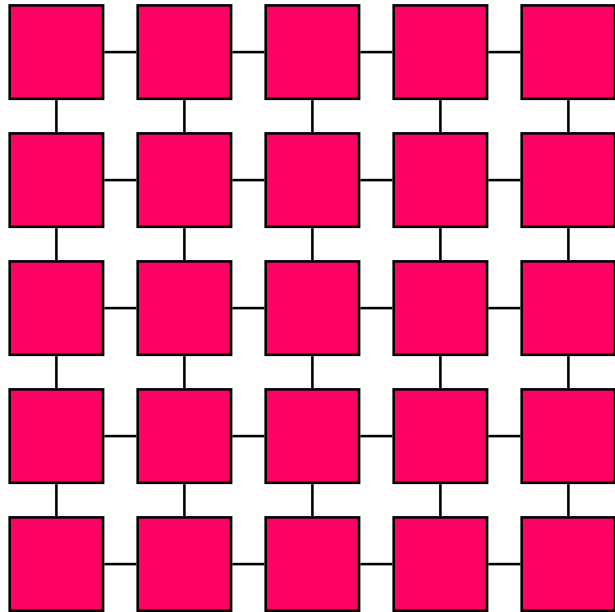
- 1/2 width
- 1/2 height
- 1/2 switching time

8 x performance!



PÄRC

Mesh/Torus

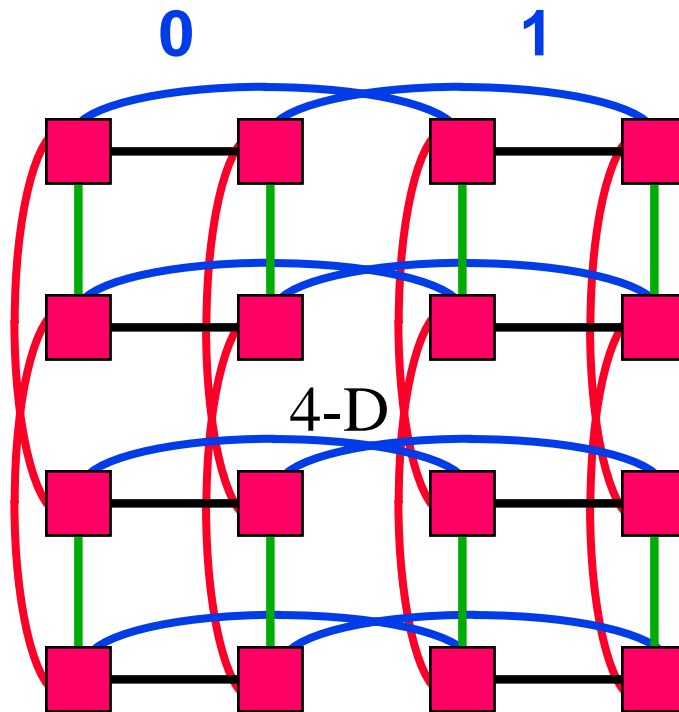
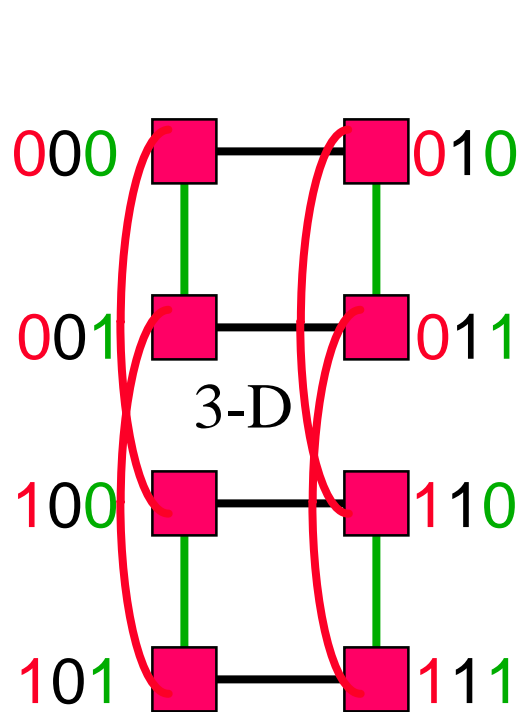
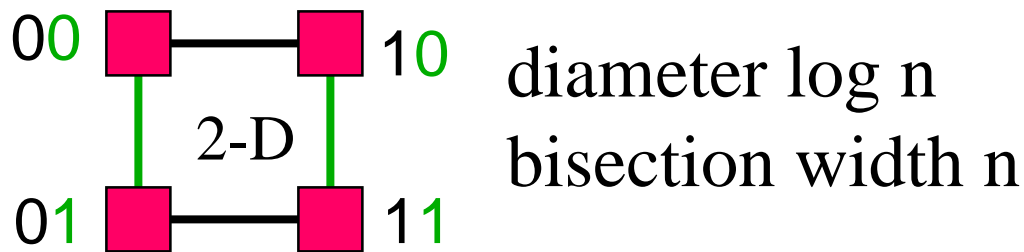


2D mesh

diameter \sqrt{n}
bisection width \sqrt{n}

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Hypercube



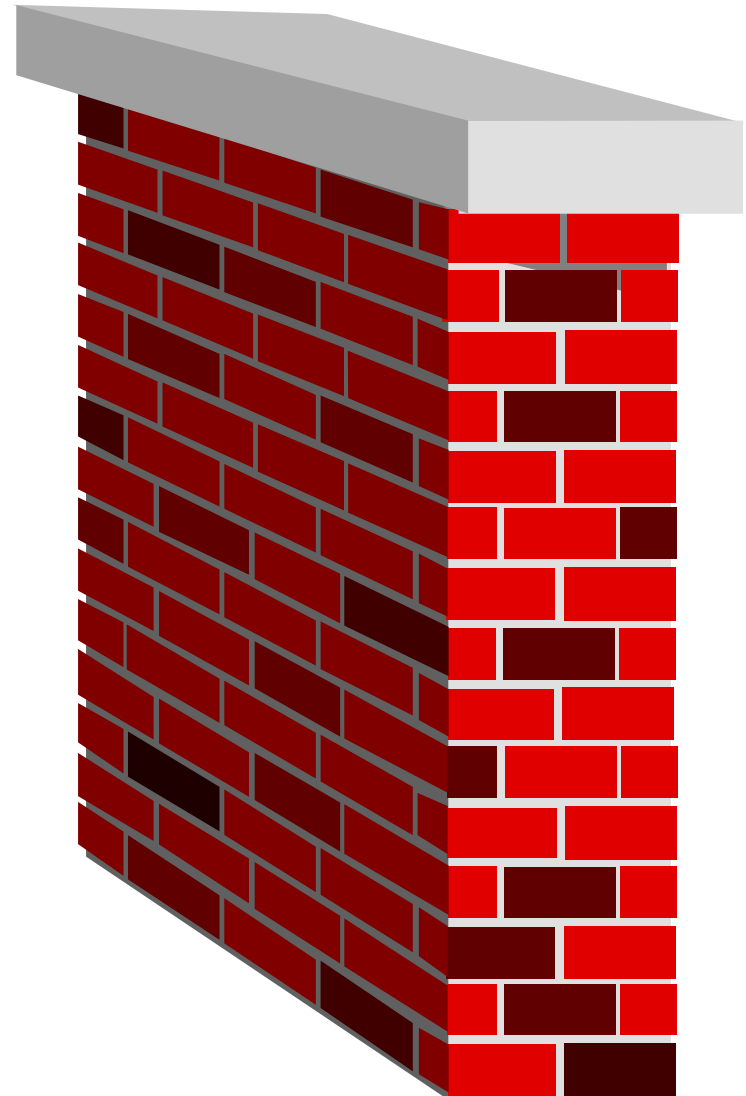
P A R C

VLSI

**Very
Large
Scale
Integration**

- simple cells
- few types
- regular architecture
- short connections

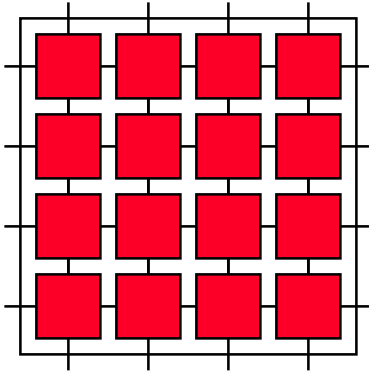
mesh -- torus



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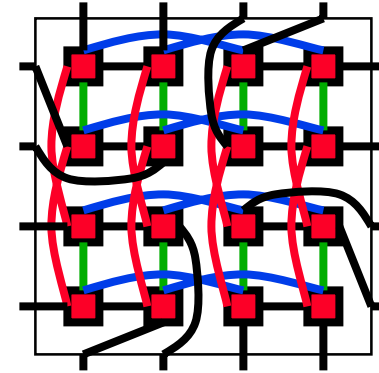
Pin limitations

diameter 256

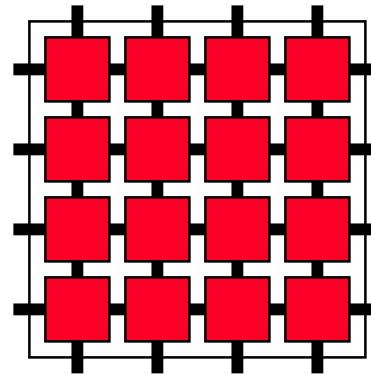


16 pins

diameter 16



16x12 pins

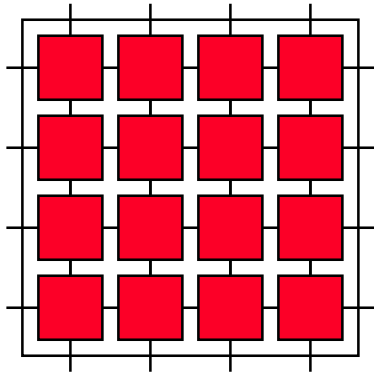


16x16 pins

PÄRC

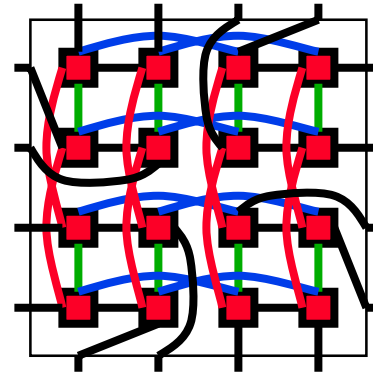
Bisection width

Bisection width 256



25 cm

Bisection width 32K



32 m

PARC

Programming

- **SA** --- **S**ystolic **A**rray
- **SIMD** --- **S**ingle **I**nstruction **M**ultiple **D**ata
- **ISA** --- **I**nstruction **S**ystolic **A**rray
- **MIMD** --- **M**ultiple **I**nstruction **M**ultiple **D**ata



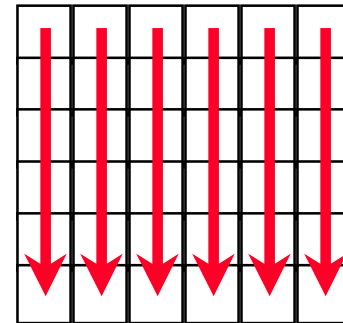
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parallel merge

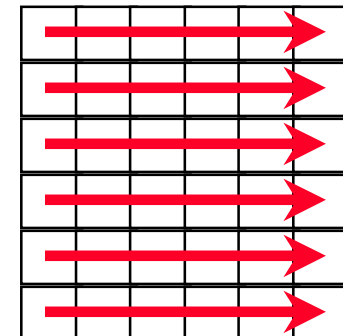
initial situation:

x1	x2	x3	x4	x5	x6
x7	...	↓		→	
		↓	...	x17	x18
y1	y2	y3	y4	y5	y6
y7	...	↑		←	
		↑	...	y17	y18

1.) sort columns
(odd-even-transposition sort)



2.) sort rows
(odd-even-transposition sort)

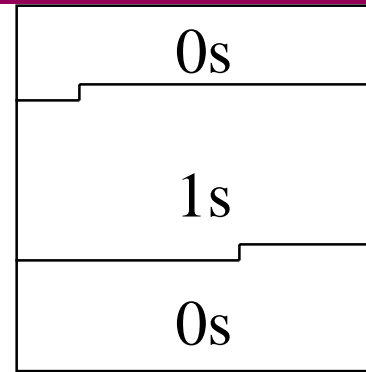


sorted !!!!

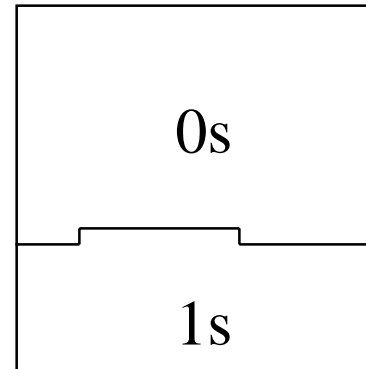
P A R C

0-1 principle

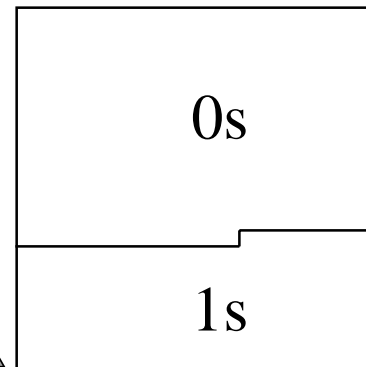
- The 0-1 principle states that if all sequences of 0 and 1 are sorted properly than this is a correct sorter.
- The sorter must be based on moving data.



initially



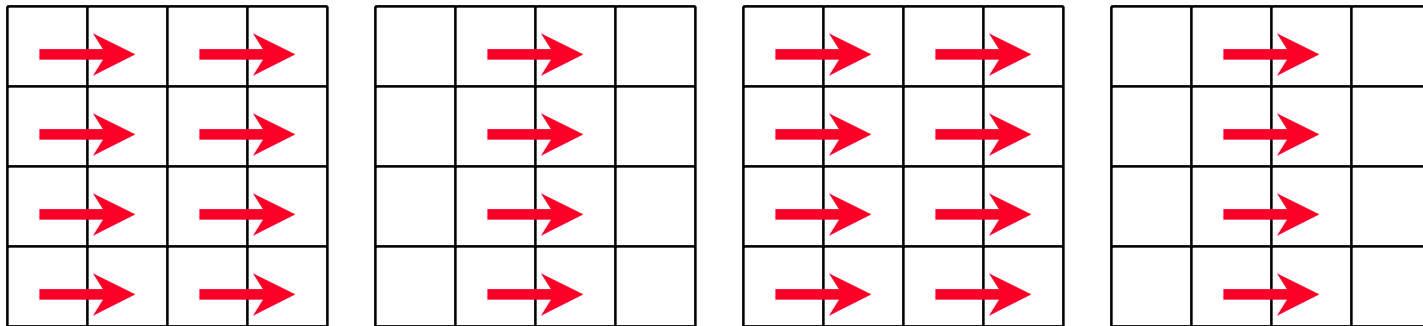
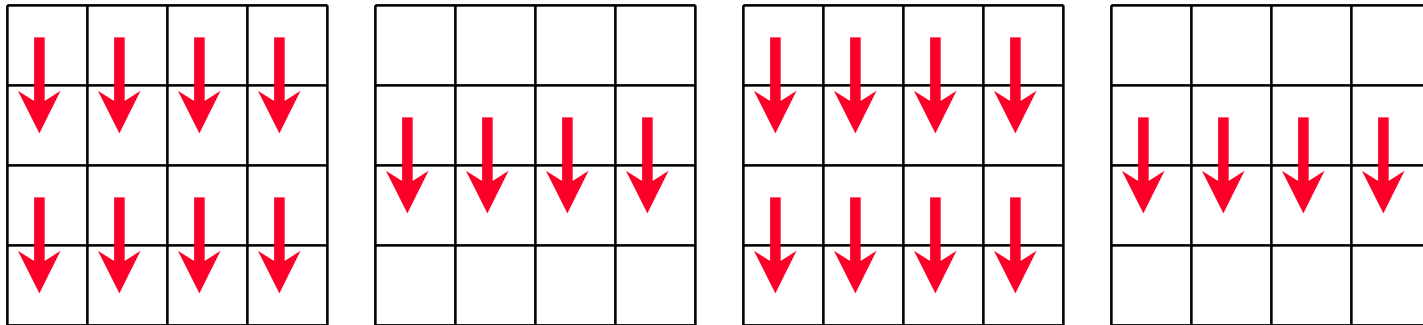
after vertical
sort



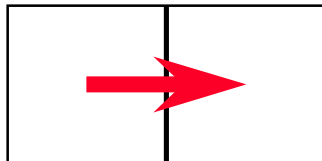
after horizontal
sort

P A R C

MIMD-mesh (clocked)



min

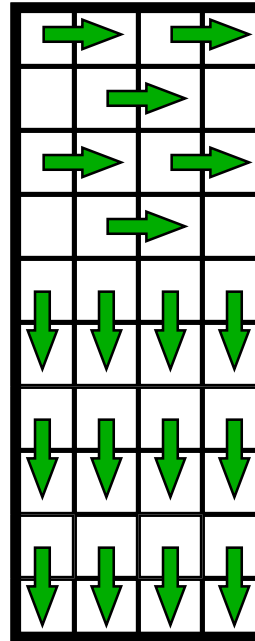


max

Time: $2n$

P Å R C

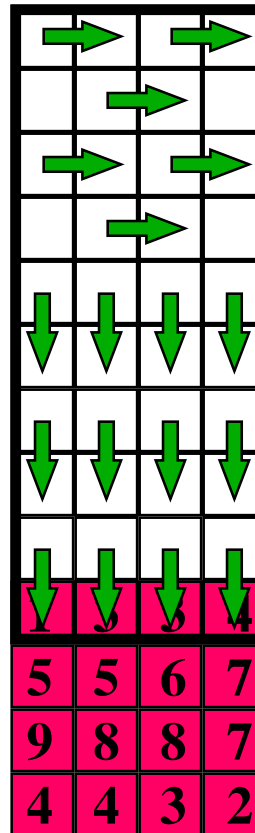
systolic merge



1	3	3	4
5	5	6	7
9	8	8	7
4	4	3	2

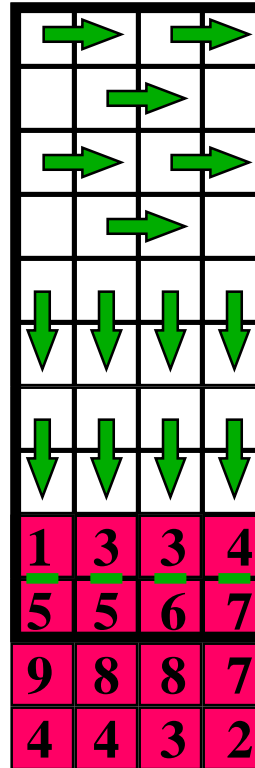
P Å R C

systolic merge



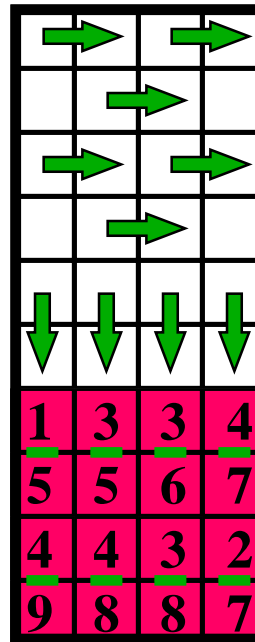
P Å R C

systolic merge



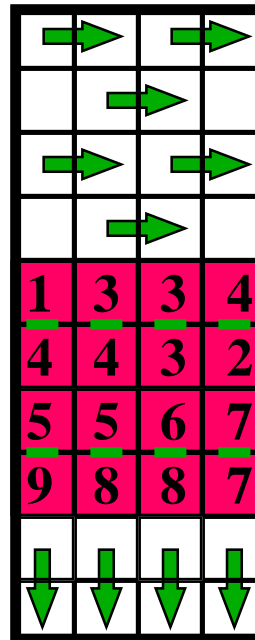
P Å R C

systolic merge



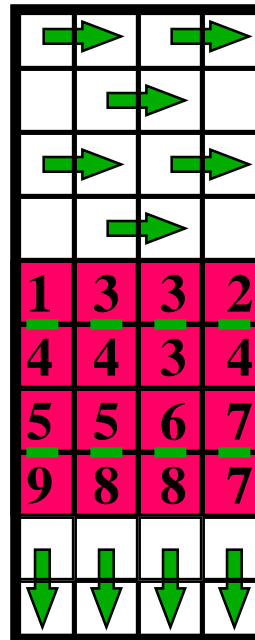
P Å R C

systolic merge



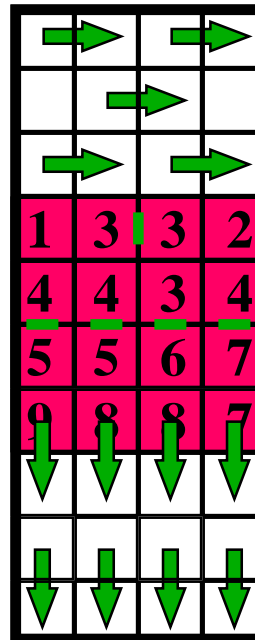
P Å R C

systolic merge



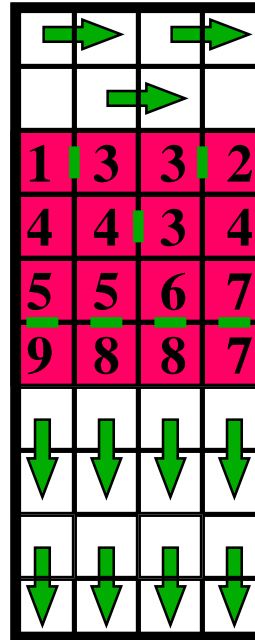
P Å R C

systolic merge



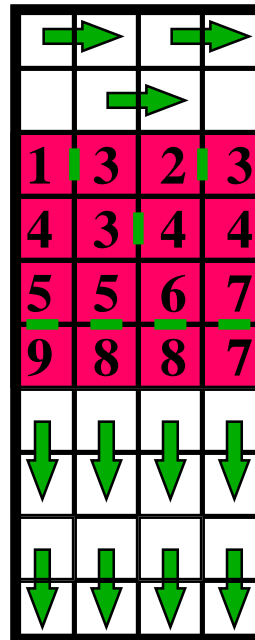
P Å R C

systolic merge



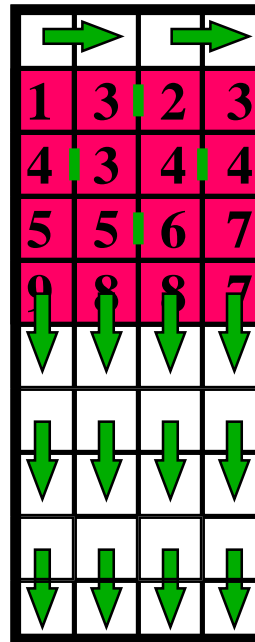
P Å R C

systolic merge



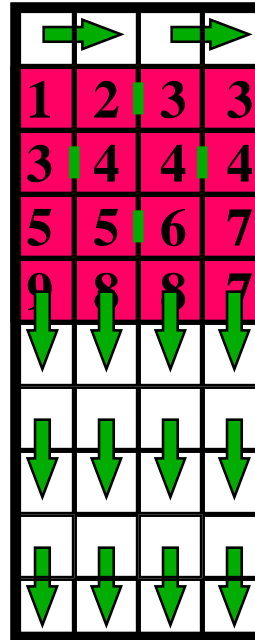
P Å R C

systolic merge



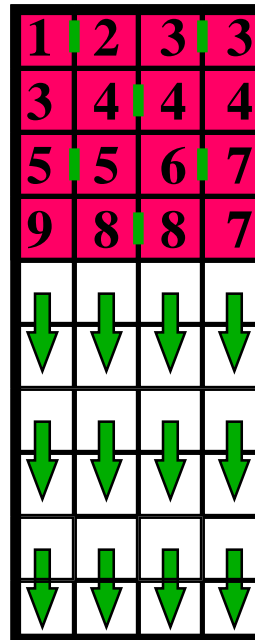
P Å R C

systolic merge



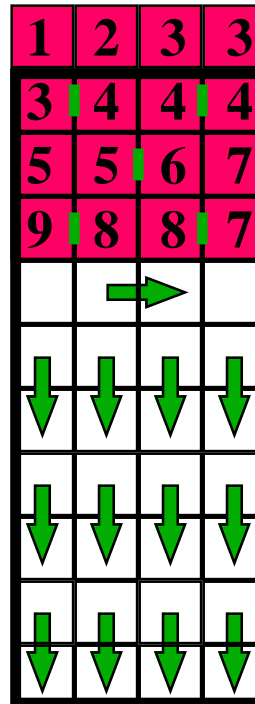
P A R C

systolic merge



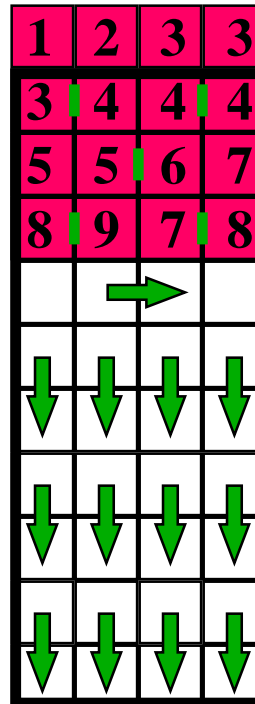
P Å R C

systolic merge



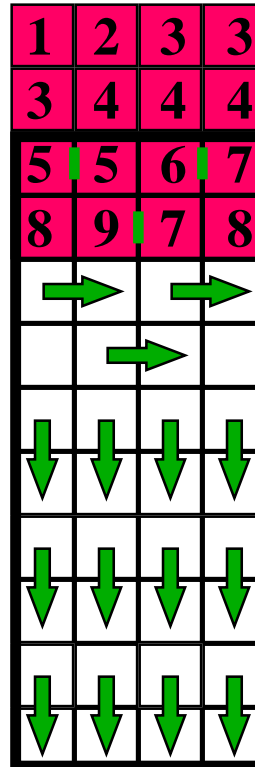
P A R C

systolic merge



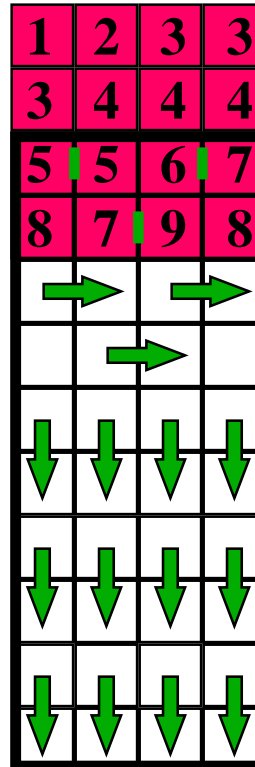
P Å R C

systolic merge



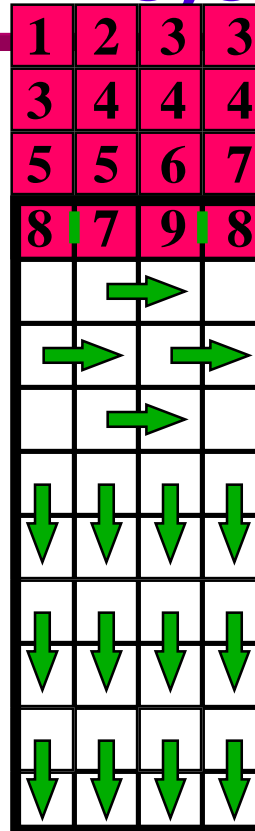
P Å R C

systolic merge



P Å R C

systolic merge

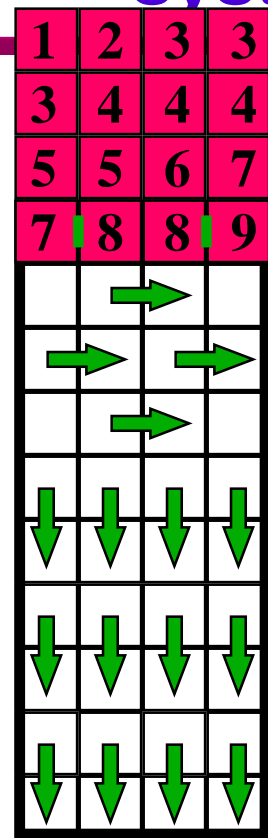


P Å R C

- sorted !!!



systolic merge





Characteristics of SAs

Extremely high cost-performance
no flexibility -- long development time

Suitable for special signal
processing tasks ???

1. Lang, H.W., Schimmler, M., Schmeck, H., Schröder, H., “A Fast Sorting Algorithm for VLSI”, Proc. 10th ICALP, Barcelona, July 1983, *Lecture Notes in Computer Science*, 154, pp408--419, 1983
2. Lang, H.W., Schimmler, H., Schröder, H., “Pattern Matching in Binary Trees on a Mesh-Connected Processor Array”, *VLSI: Algorithms and Architectures*, Bertolazzi and Luccio (eds.), North Holland, pp113--124, 1984
3. Schmeck, H., Schröder, H., “Dictionary Machines for Different Models of VLSI”, *IEEE Transactions on Computers*, C-34, pp472--475, 1985
4. Lang, H.W., Schimmler, M., Schmeck, H., Schröder, H., “Realistic Comparisons of Sorting Algorithms for VLSI”, *Foundations of Data Organisation*, Ghosh, Kambayashi and Tanaka (eds.), Plenum Press, pp309--318, 1987
5. Schröder, H., “VLSI-Sorting Evaluated under the Linear Model”, *Journal of Complexity* 4, pp 330-355, December 1988
6. Schmeck, H., Schröder, H., Starke, C., “Systolic s^2 -Way Merge Sort is Nearly Optimal”, in *IEEE Transactions on Computers*, Vol 38, Nr 7, pp 1052-1056, 1989
7. Murthy, V.K., Schröder, H., “Systolic Arrays for Parallel G-Inversion and Finding Petri Net Invariants”, in *Parallel Computing*, Vol. 11, Nr. 3, pp 349-359, 1989
8. E.V. Krishnamurthy, M. Kunde, M. Schimmler, H. Schröder, “Systolic algorithm for tensor products of matrices --- implementation and applications”, *Parallel Computing* 13, 301-308, 1989

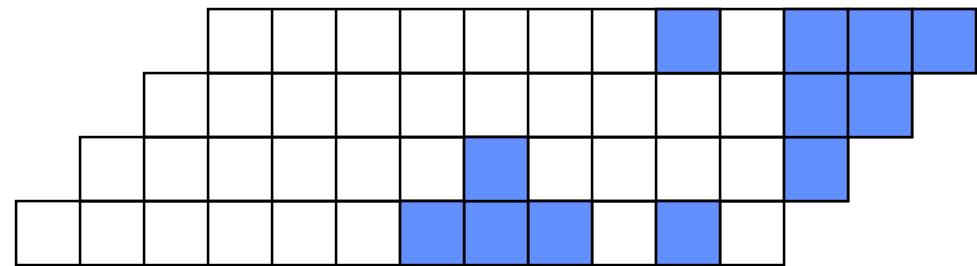
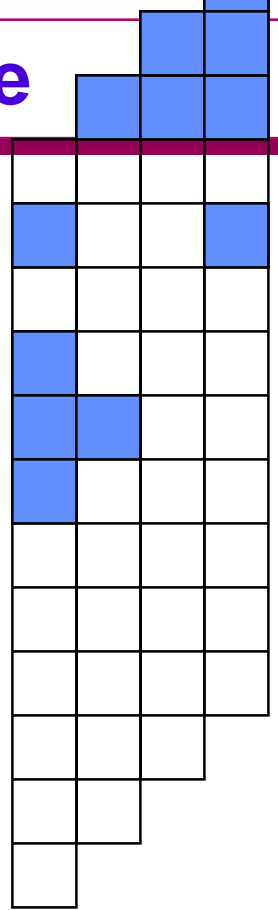
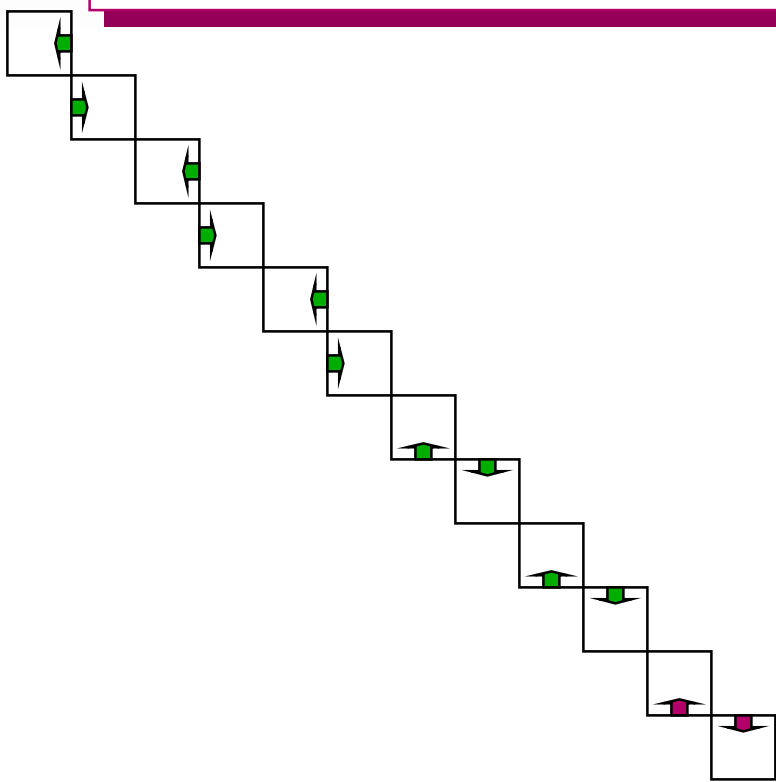
9. Schimmler, M., Schröder, H., “A Simple Systolic Method to Find all Bridges on an undirected Graph”, *Parallel Computing* 12, 107-111, 1989
10. P. Lenders, H. Schröder, “A programmable systolic device for Image Processing -- based on Mathematical Morphology”, *Parallel Computing*, 13, pp 337-344, 1990
11. Schröder, H., Krishnamurthy, E.V., “Systolic Algorithms for Polynomial Interpolation and Related Problems”, *Parallel Computing*, 17, pp 493-504, 1991
12. Schröder, H., Krishnamurthy, E.V., “Systolic Algorithms for Multivariate Approximation Using Tensor Products of Basis Functions”, *Parallel Computing*, 17, pp 483-492, 1991
13. B. Beresford-Smith, J. Breckling, H. Schröder, “Systolic Codebook Generation”, *Transactions on Acoustics, Speech, & Signal Processing*, pp 144-149, Vol.1, Nr. 2, April 1993
14. Schröder, H., “Partition Sorts for VLSI”, Proceedings GI-13, Jahrestagung, Hamburg, October 1983, *Informatik-Fachberichte* 73, pp101--116, 1983
15. Schröder, H., “VLSI-gerechte Sortierverfahren”, PARS-Workshop, Erlangen, April 1984, Mitteilungen GI-PARS Nr. 2, pp134--143, 1984
16. Schröder, H. “VLSI-Sorting under the Linear Model”, Proceedings of the Tenth Australian Computer Science Conference, pp330--340, Deakin University, February 1987
17. B. Beresford-Smith, J. Breckling, H. Schröder, “Systolic Devices for Speech Processing”, CompEuro 89, Hamburg, Germany, May 1989

P A R C

ISA merge

 $C := \min\{C, CE\}$

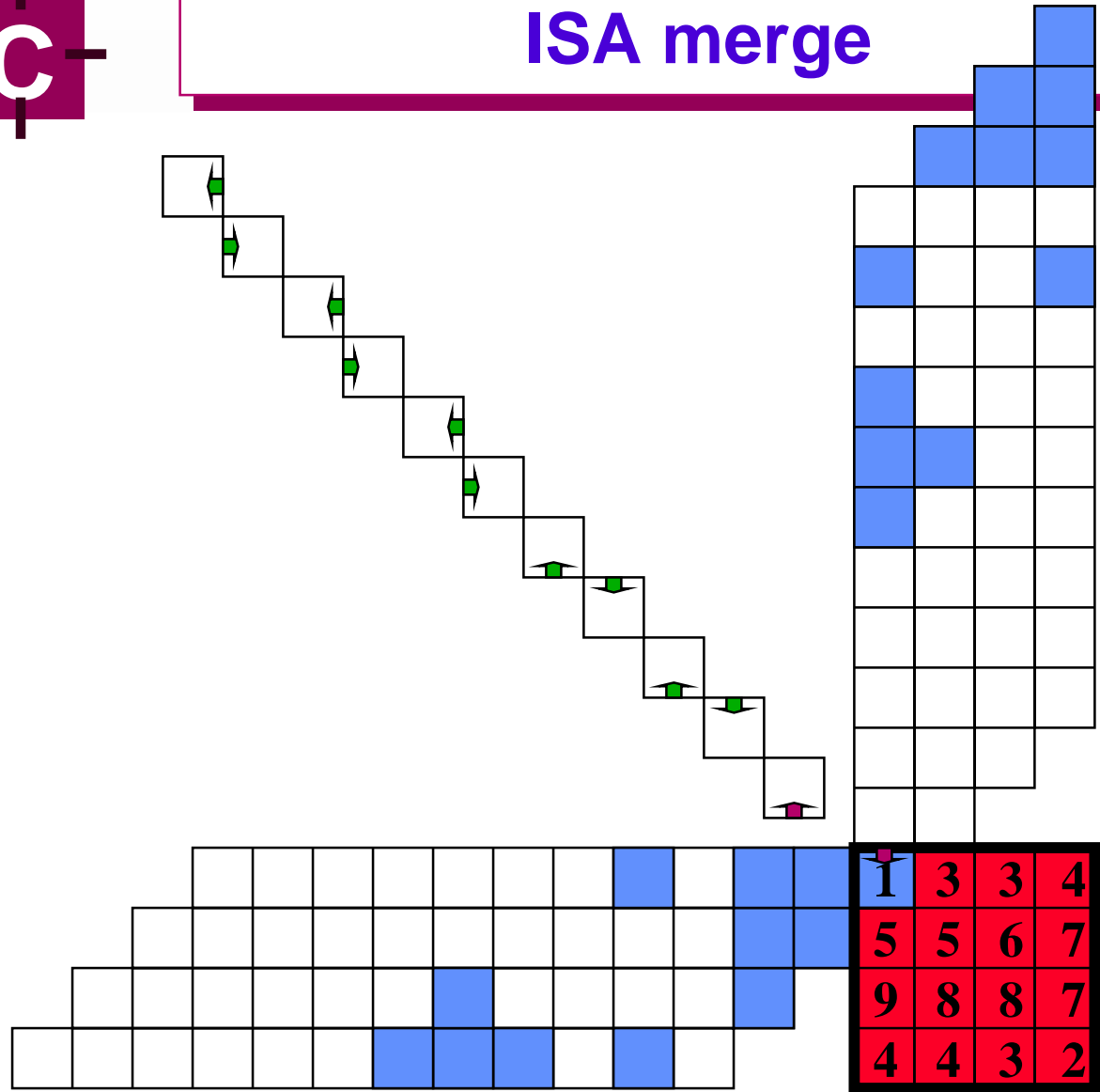
 $C := \max\{C, CW\}$



1	3	3	4
5	5	6	7
9	8	8	7
4	4	3	2

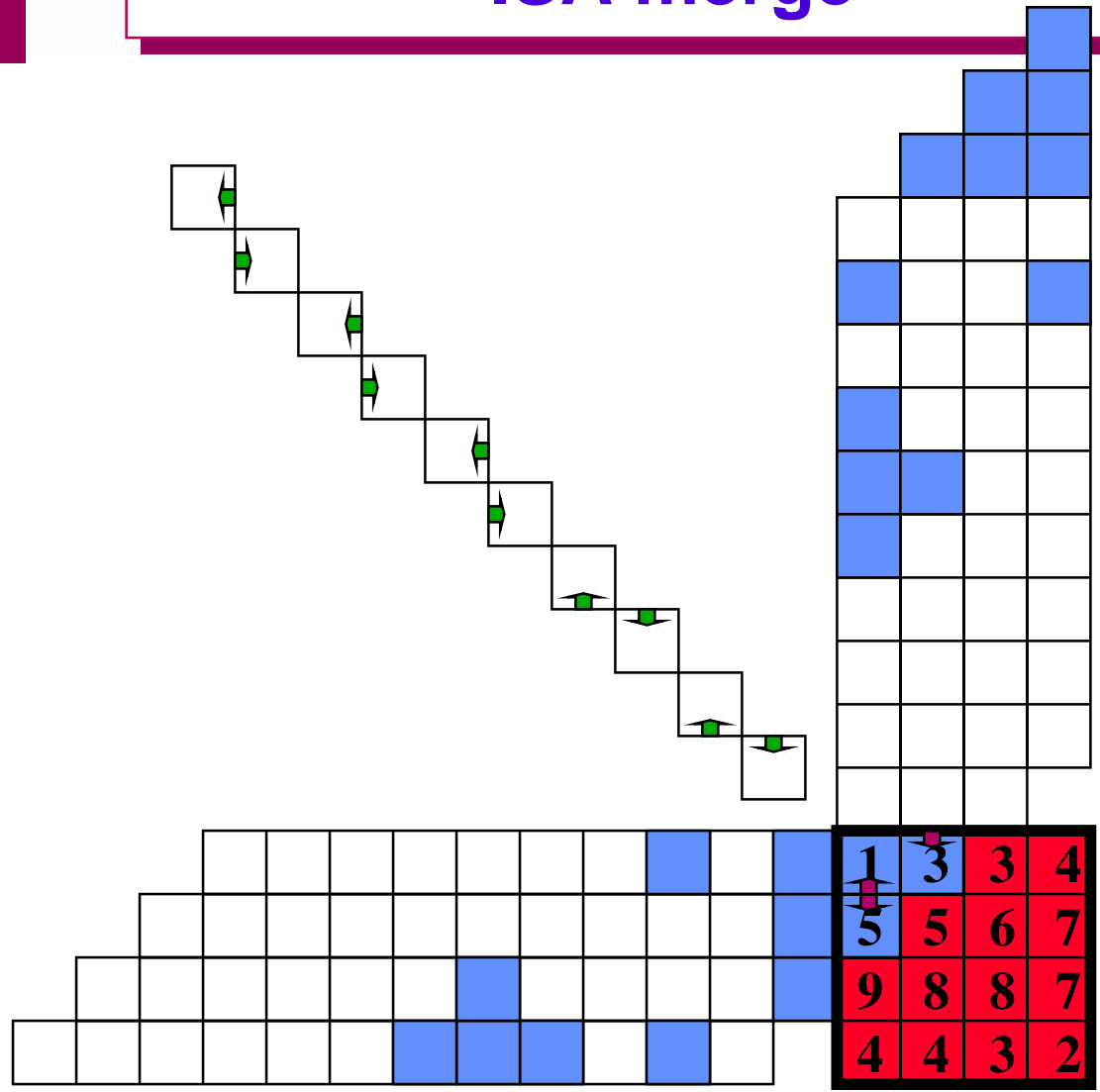
P Å R C

ISA merge



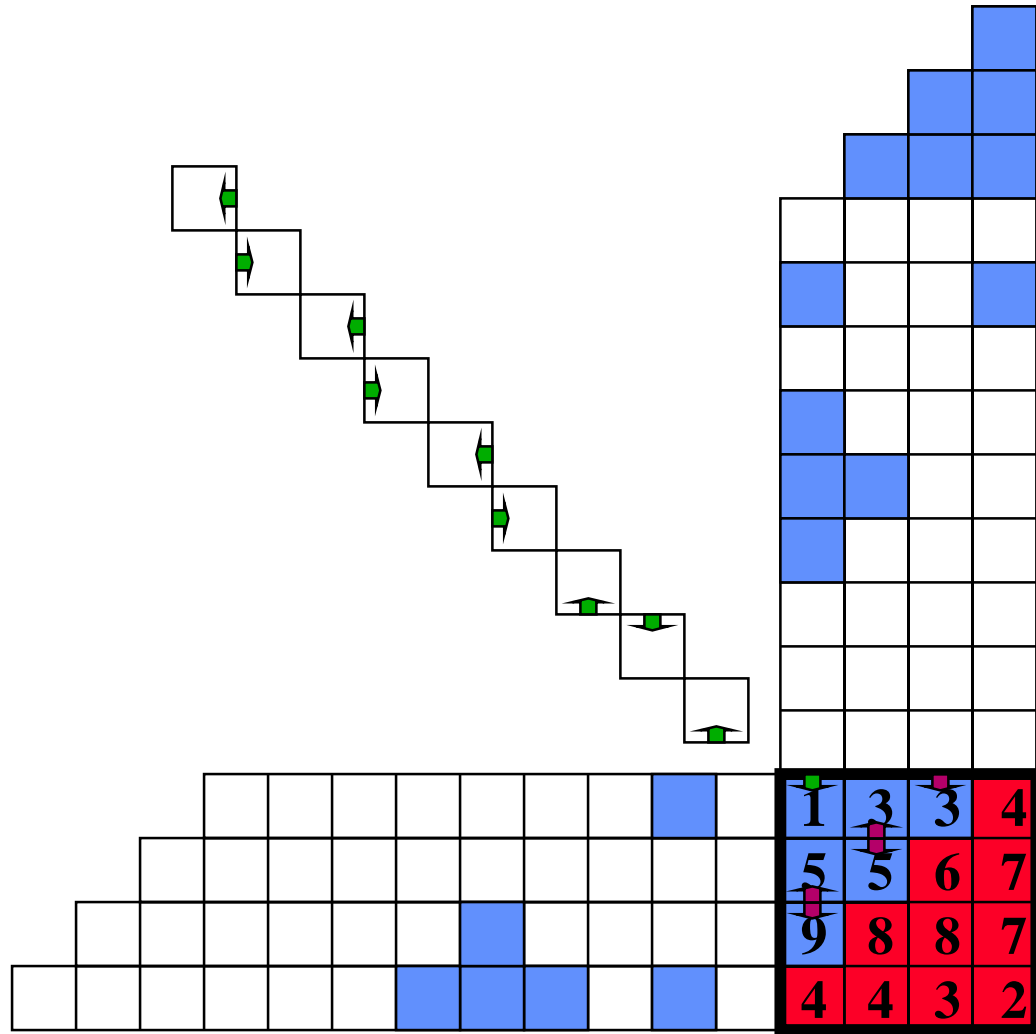
P Å R C

ISA merge



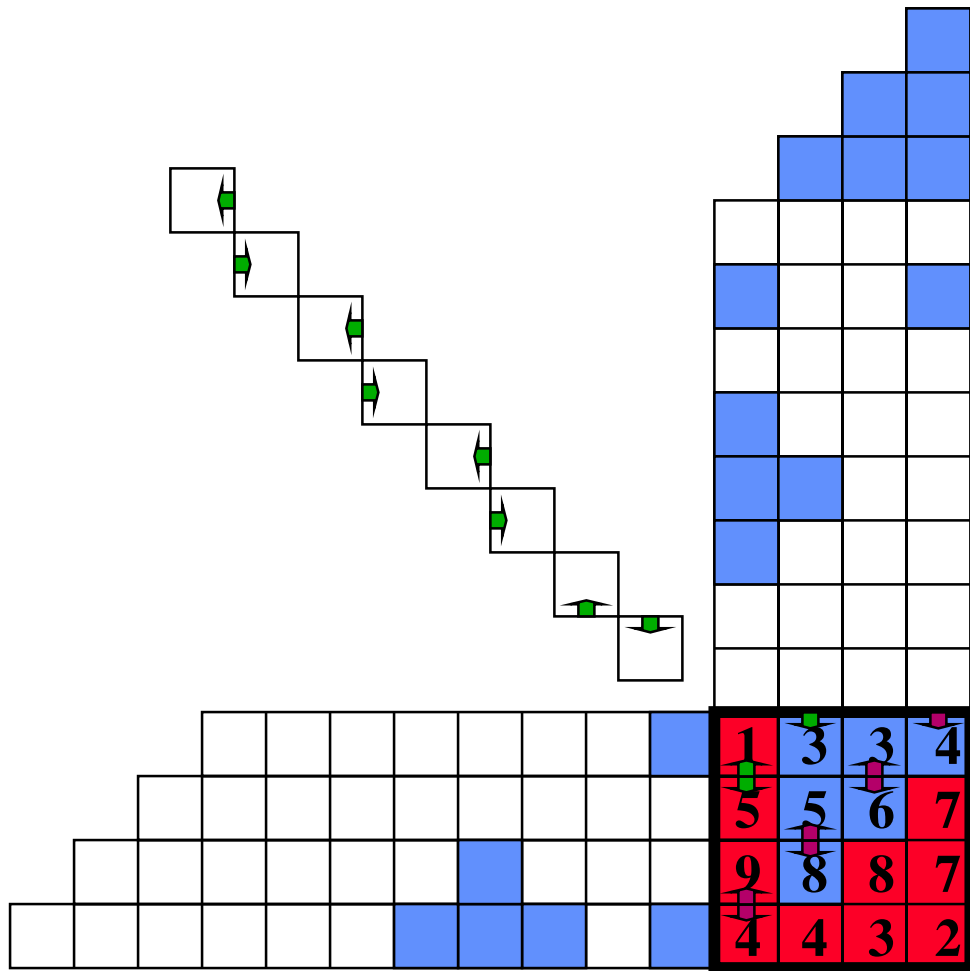
P Å R C

ISA merge



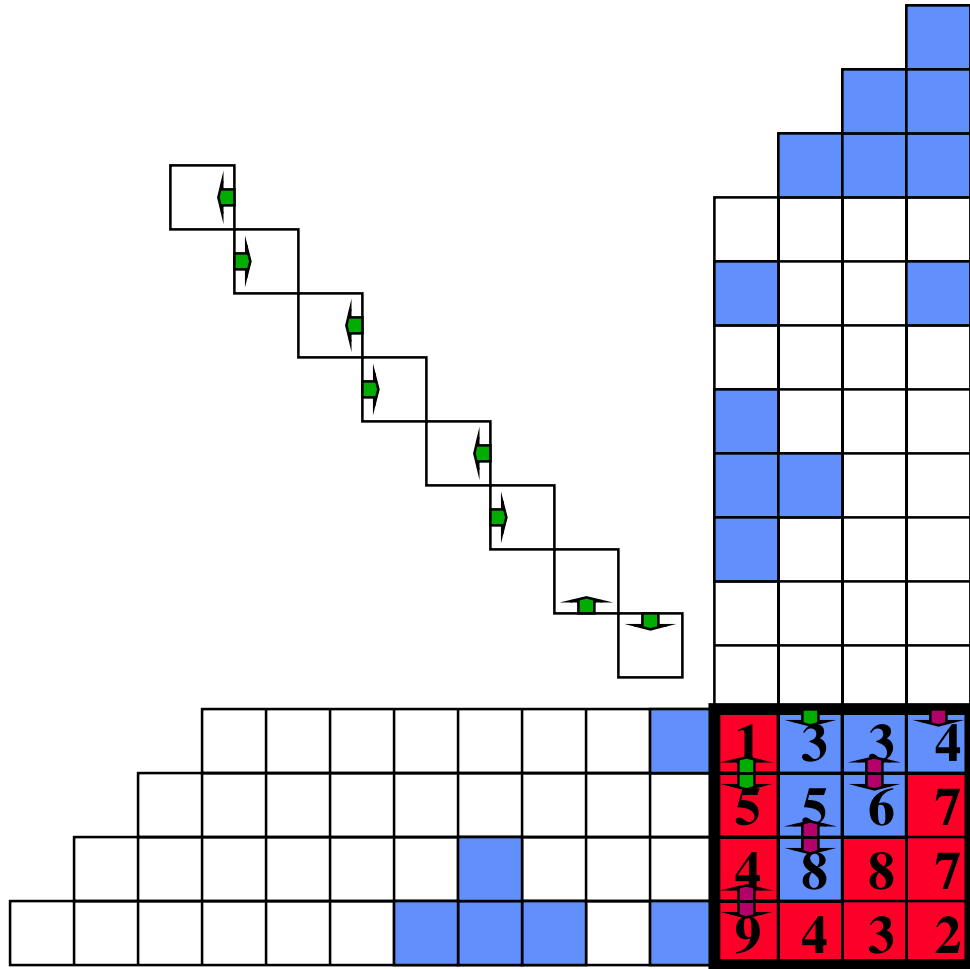
P Å R C

ISA merge



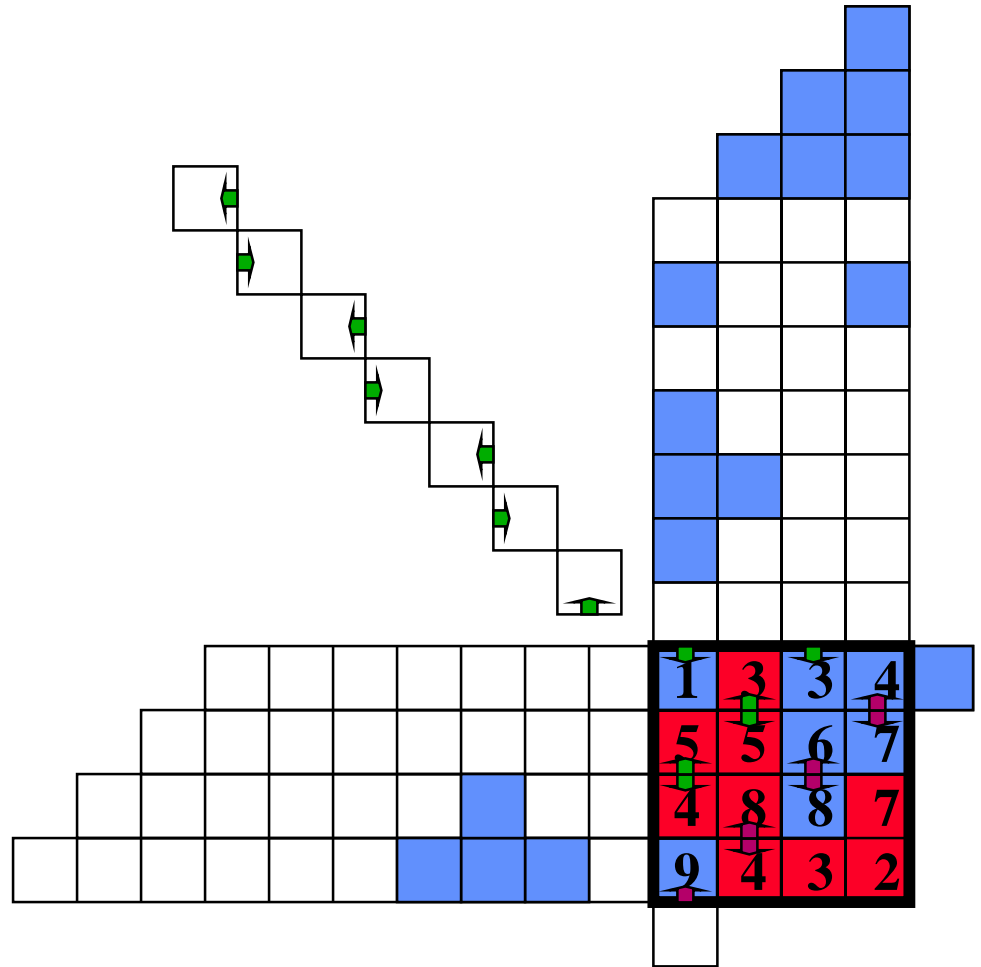
P Å R C

ISA merge



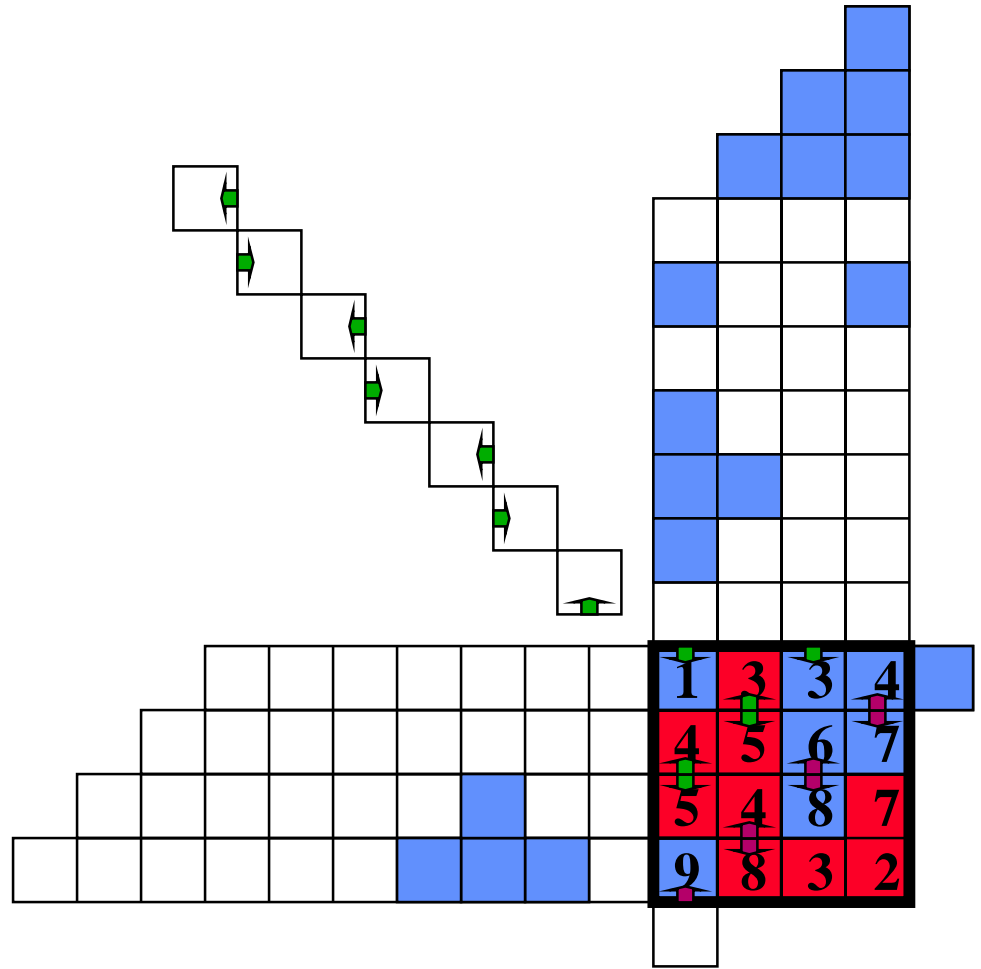
P Å R C

ISA merge



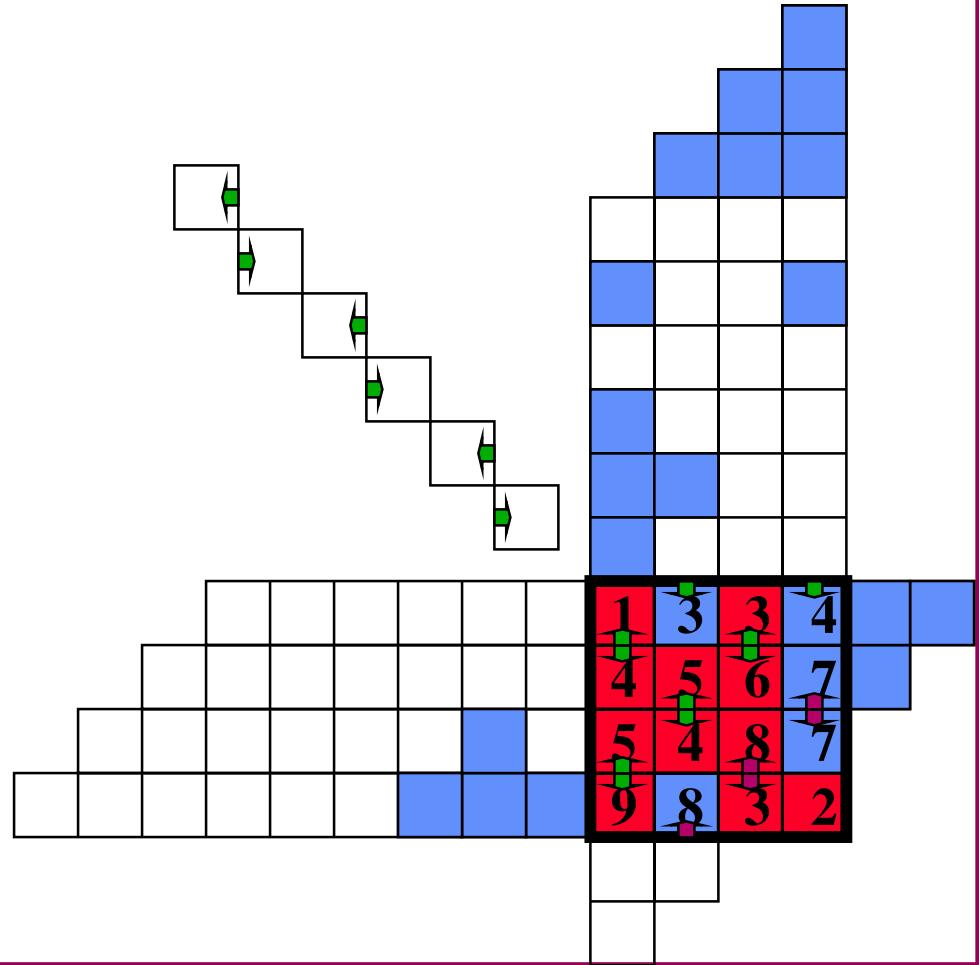
P Å R C

ISA merge



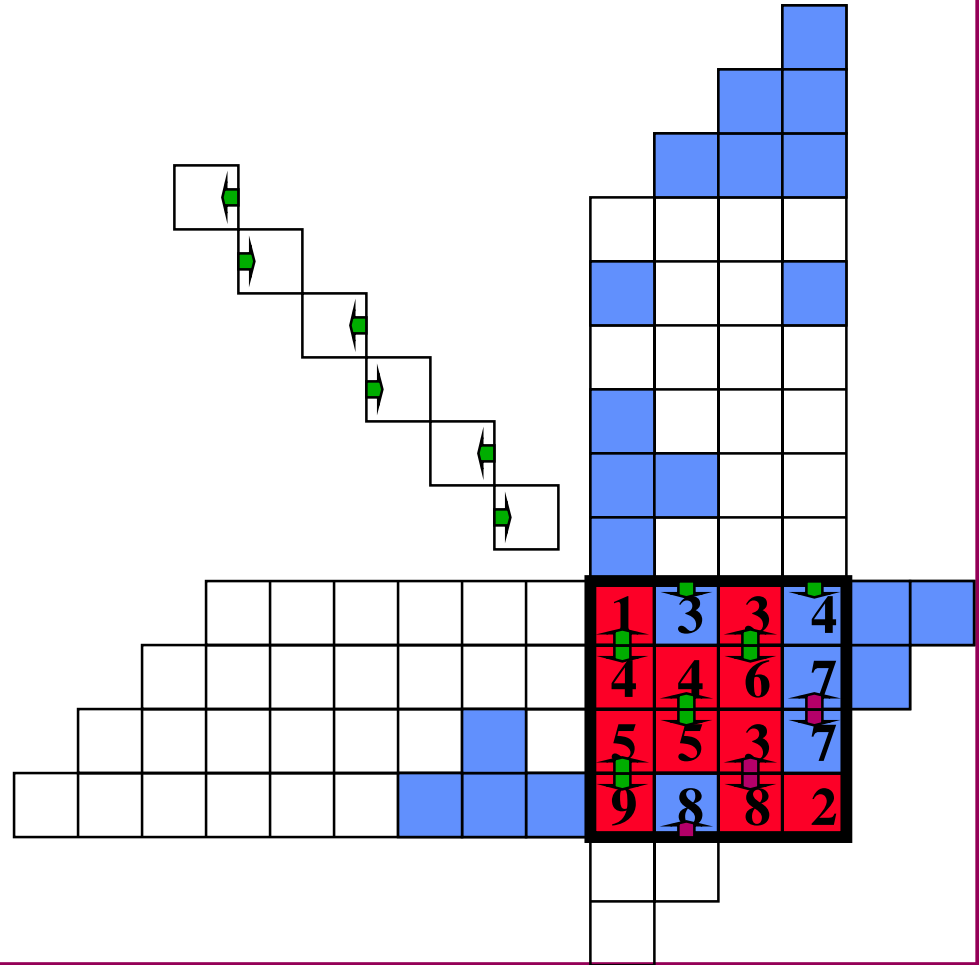
P Å R C

ISA merge



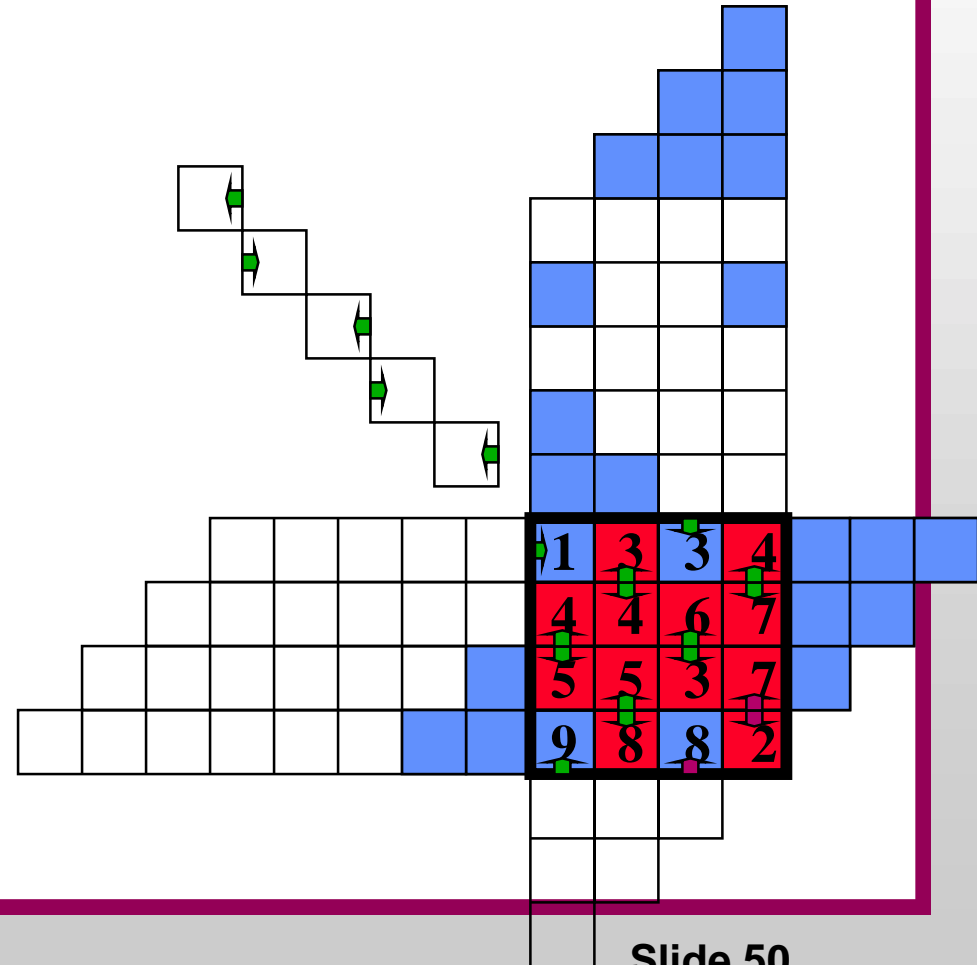
P Å R C

ISA merge



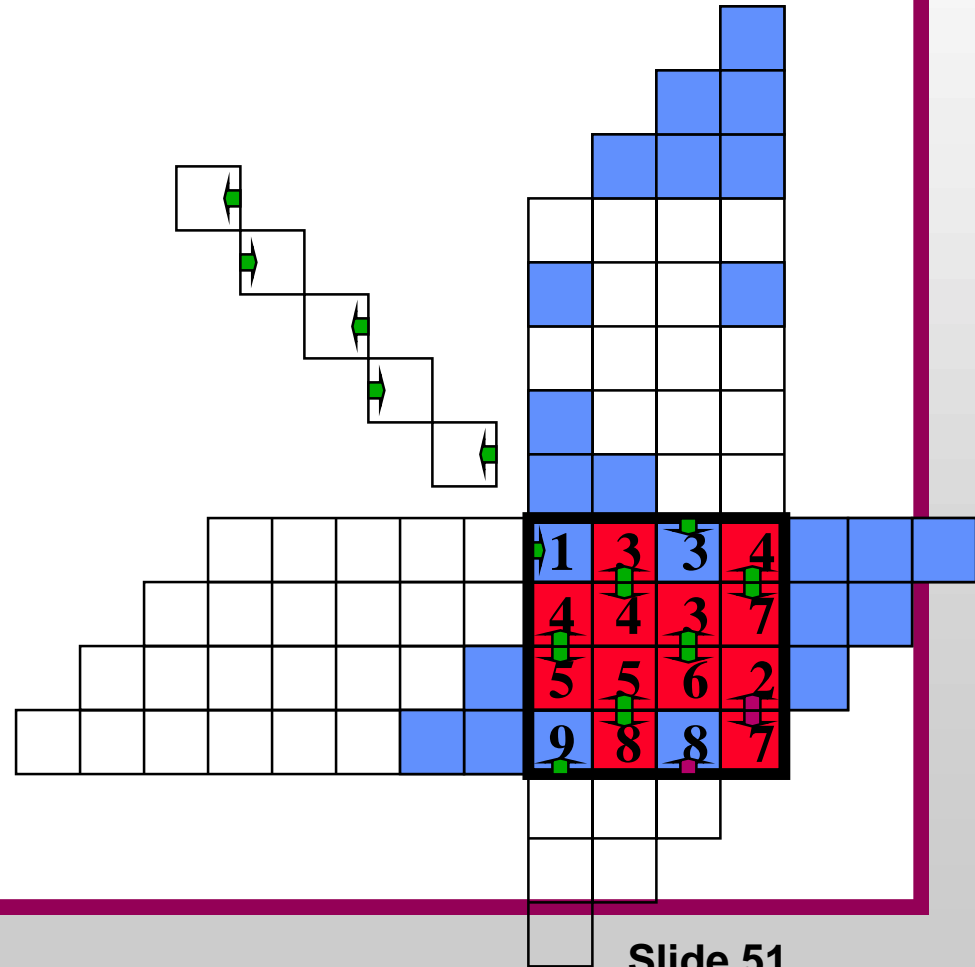
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ISA merge



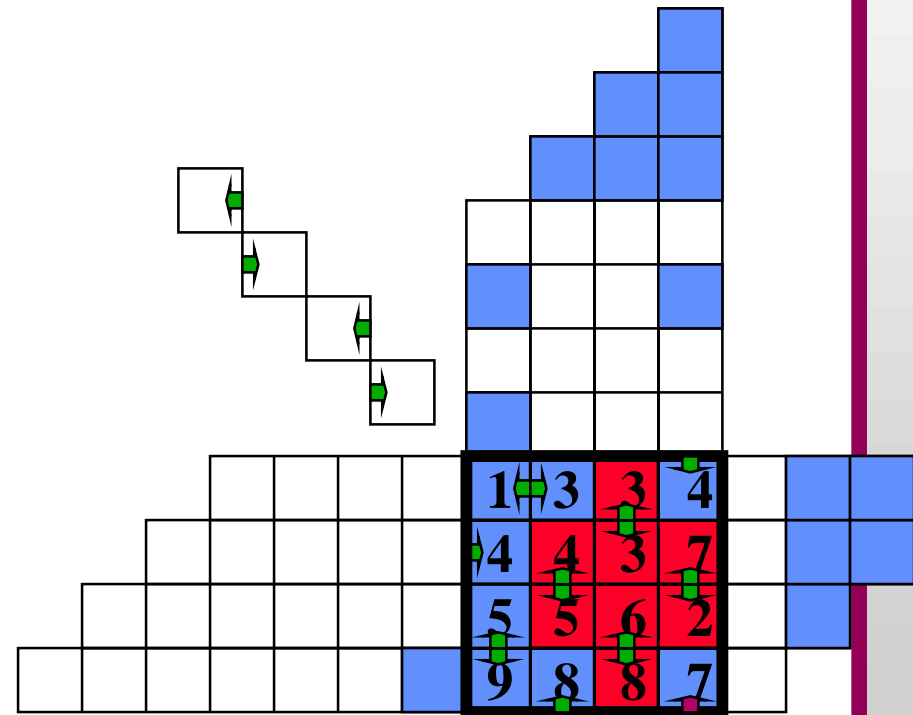
P Å R C

ISA merge



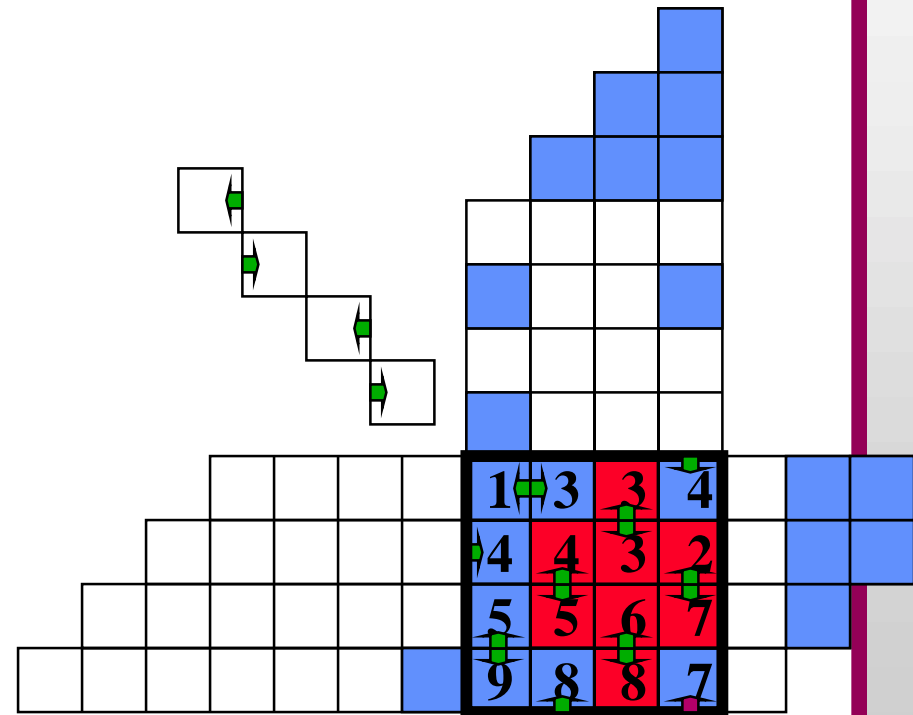
P Å R C

ISA merge



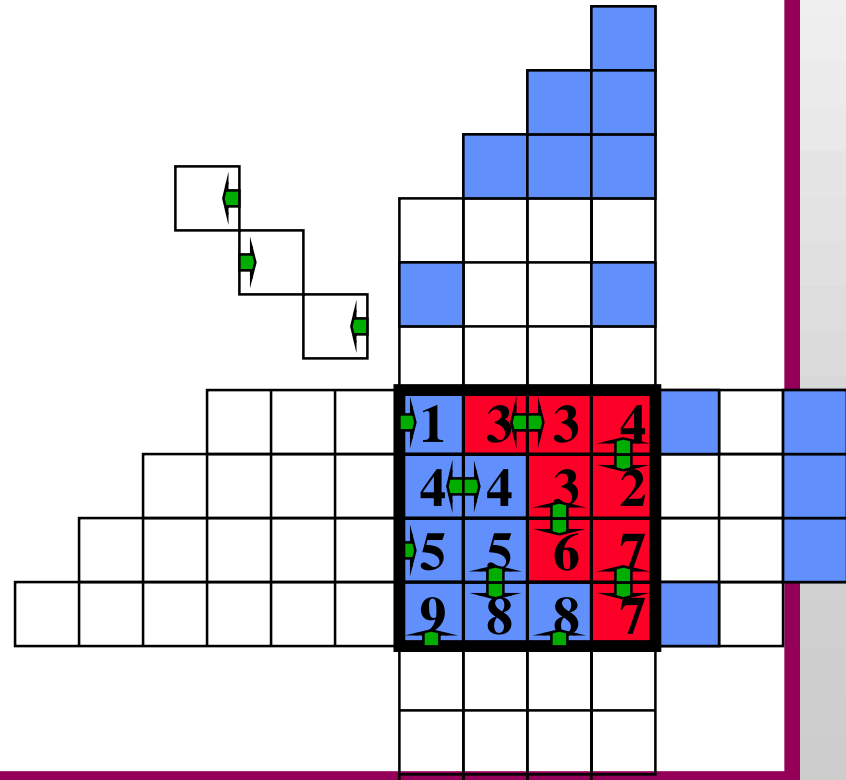
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ISA merge



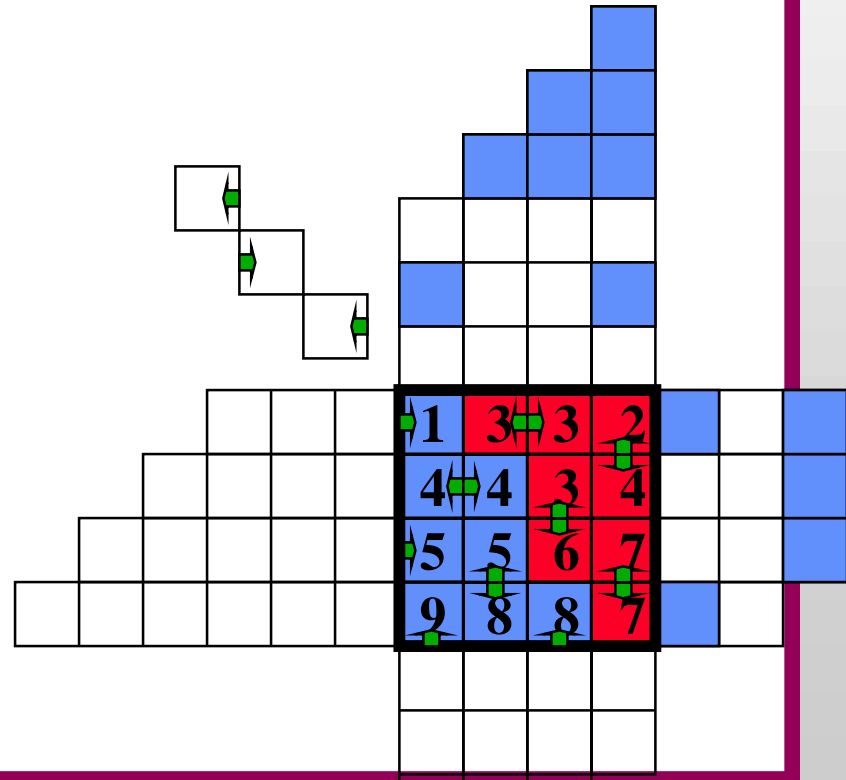
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ISA merge



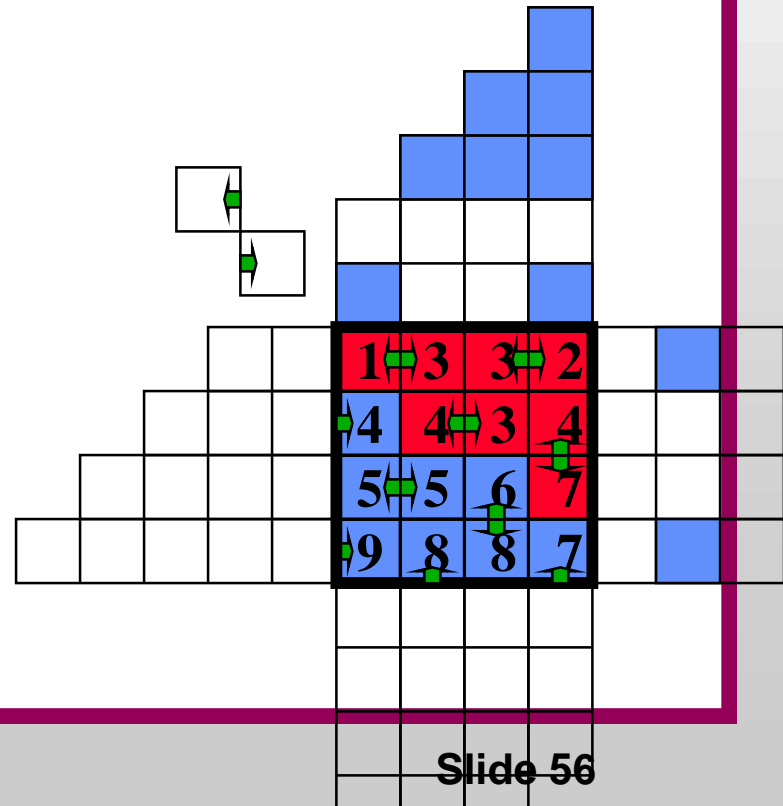
P Å R C

ISA merge



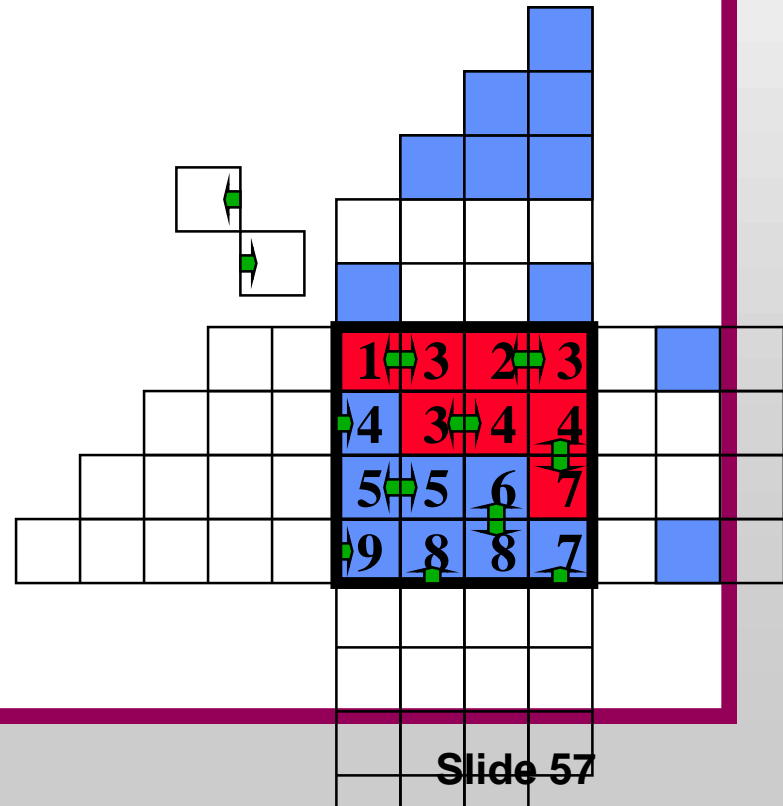
P Å R C

ISA merge



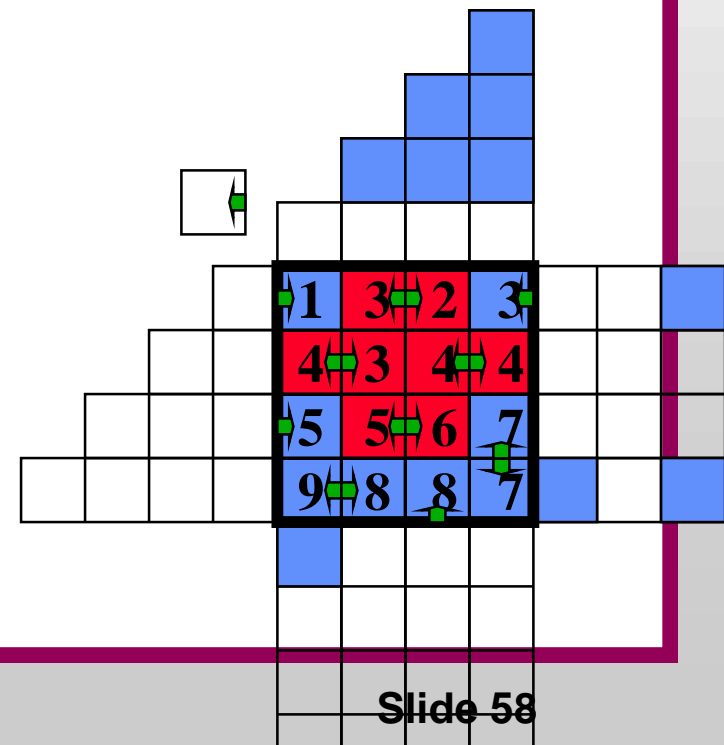
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ISA merge



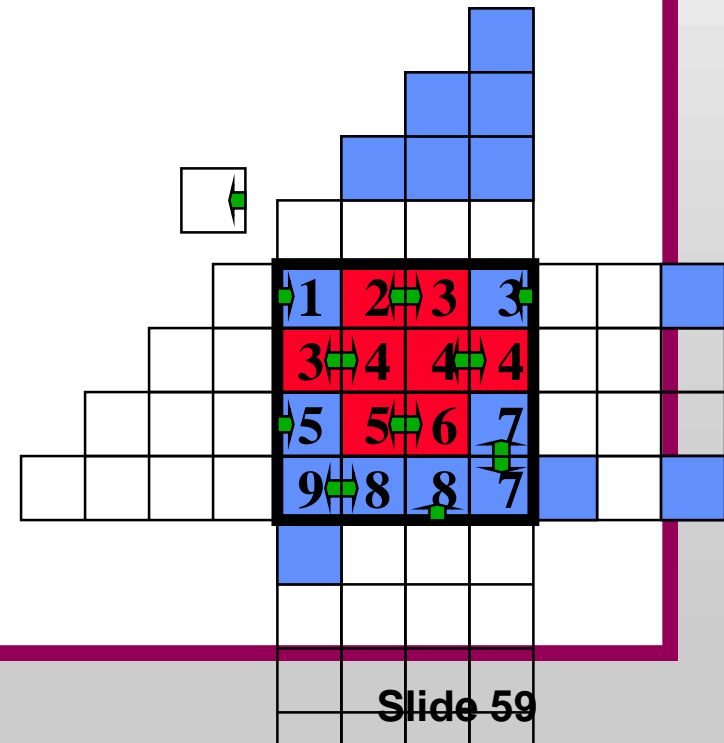
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ISA merge



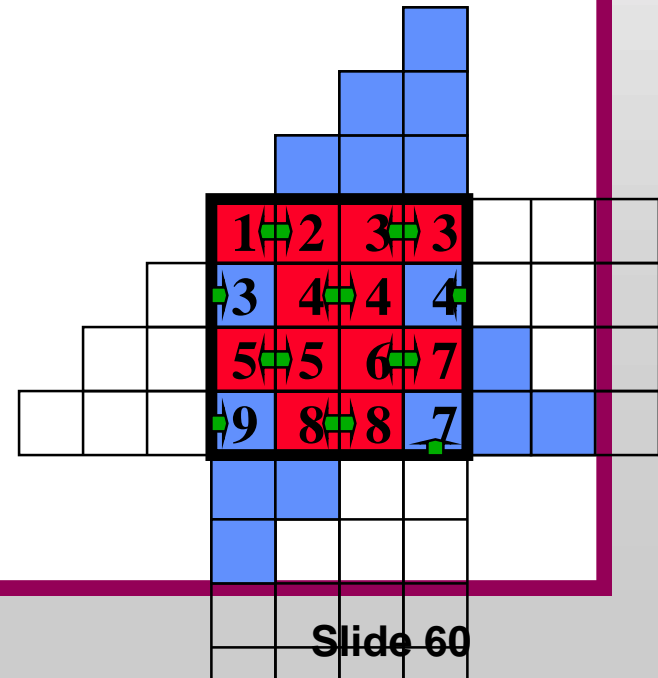
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ISA merge



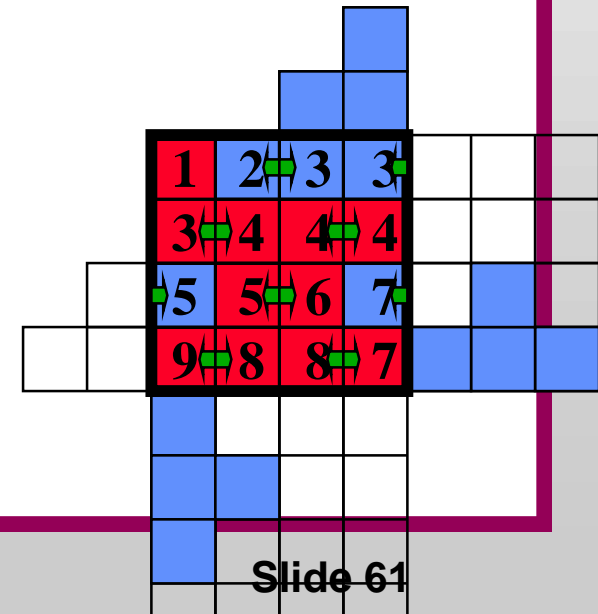
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ISA merge



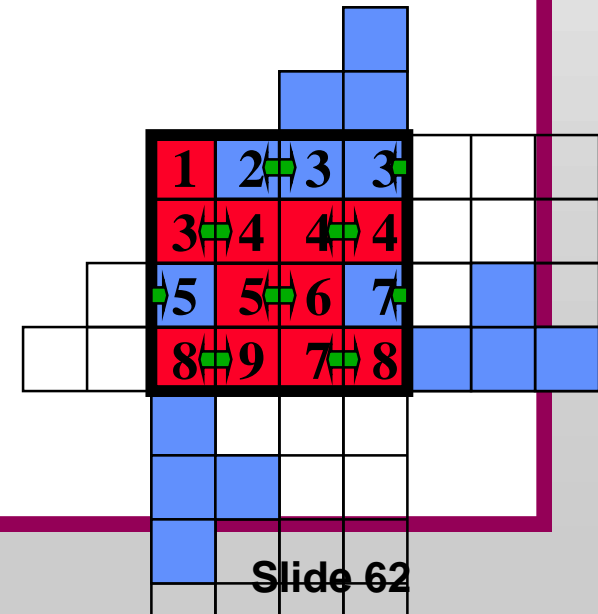
P Å R C

ISA merge



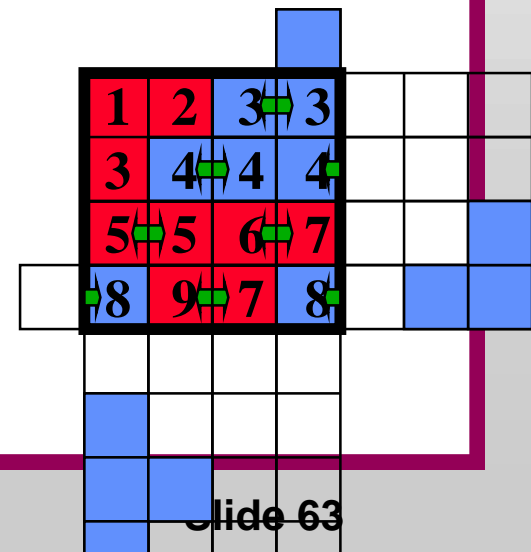
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ISA merge



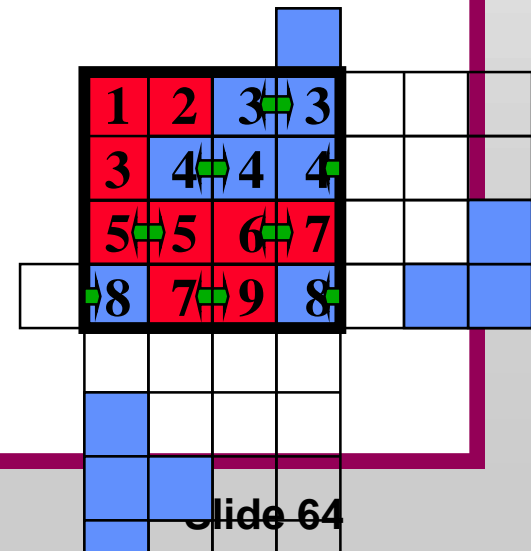
P Å R C

ISA merge



P Å R C

ISA merge



P Å R C

ISA merge

1	2	3	3		
3	4	4	4		
5	5	6	7		
8	7	9	8		

P Å R C

ISA merge

1	2	3	3			
3	4	4	4			
5	5	6	7			
7	8	8	9			

P Å R C

ISA merge

1	2	3	3		
3	4	4	4		
5	5	6	7		
7	8	8	9		

P Å R C

ISA merge

1	2	3	3		
3	4	4	4		
5	5	6	7		
7	8	8	9		

P Å R C

ISA merge

1	2	3	3
3	4	4	4
5	5	6	7
7	8	8	9

P Å R C

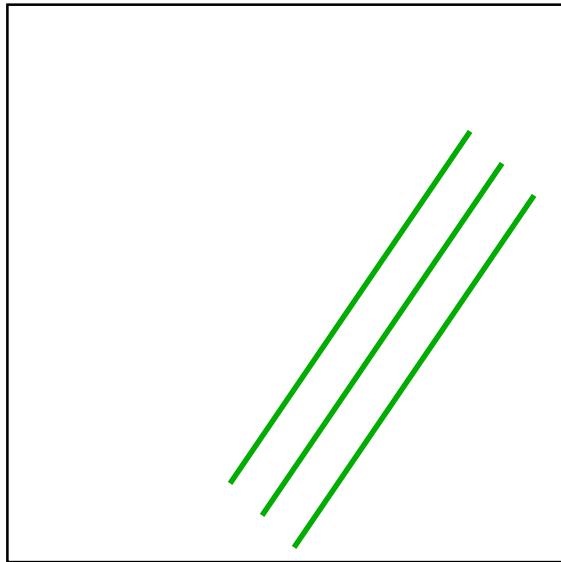
ISA merge

1	2	3	3
3	4	4	4
5	5	6	7
7	8	8	9

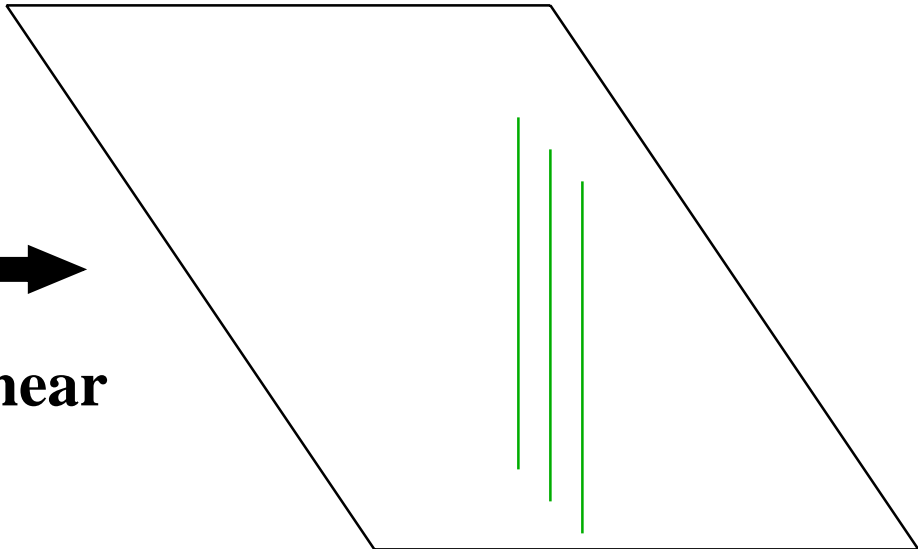
P A R C

Hough transform on the ISA

- good line detection method



→
shear



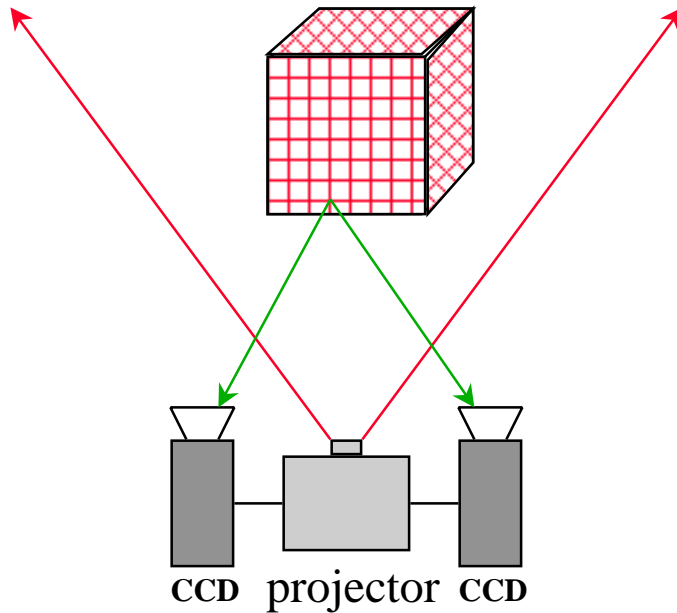
↓
 Σ

Fast tomography

P A R C

robot vision

- stereo vision



Special features:

fast aggregate functions (sum, carry)

fast local communication

no local memory

typical improvement over PC: Factor 20-30

Areas of application for ISA:

automatic optical quality control

real time signal processing

computer graphics /visualization

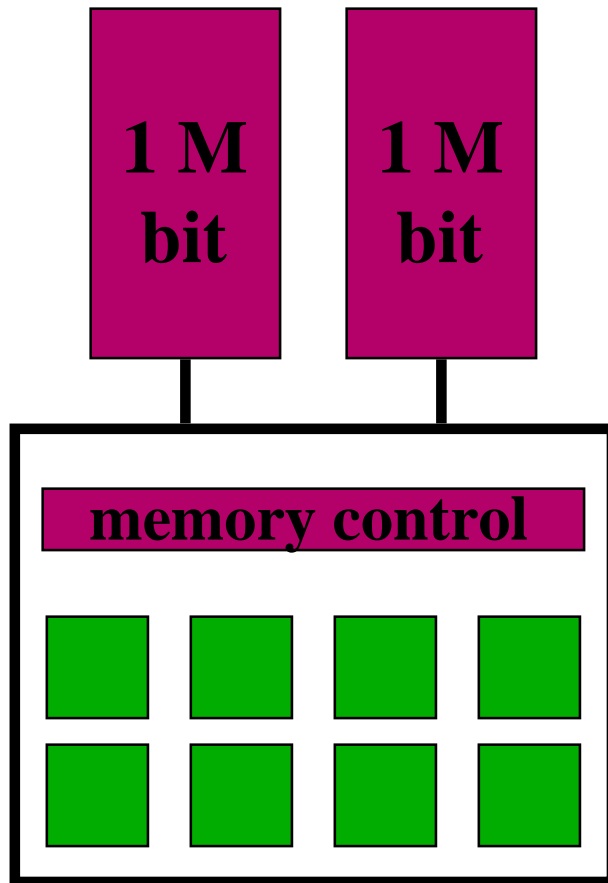
linear equations

Cryptography --> Tele-medicine ?

1. Schröder, H., “The Instruction Systolic Array --- A Tradeoff between Flexibility and Speed”, *Computer Systems Science and Engineering*, Vol 3 No 2, April 1988
2. Schröder, H., “Top-Down Designs of Instruction Systolic Arrays for Polynomial Interpolation and Evaluation”, in *Journal of Parallel and Distributed Computing*, Vol. 6, pp 692-703, 1989
3. Kunde, M., Lang, H.W., Schimmler, M., Schmeck, H., Schröder, H., “The Instruction Systolic Array and its Relation to other Models of Parallel Computers”, *Parallel Computing*, Vol 7, pp 25-39, 1988
4. Schröder, H., Krishnamurthy, E.V., “Instruction Systolic Array Computation of the Characteristic Polynomial of a Hessenberg Matrix”, *Parallel Computing*, 17, pp 273-278, 1991
5. H. Schröder, P. Strazdins, “Programm compression on the ISA”, *Parallel Computing* 17, 207-219, 1991
6. B. Pham, H. Schröder, “An Instruction Systolic Device for Quadratic Surface Generation”, CompEuro 89, Hamburg, West Germany, May 1989
7. P. Lenders, H. Schröder, P. Strazdins, “Microprogramming Instruction Systolic Arrays”, *MICRO 22*, Dublin, August 1989
8. B. Schmidt, M. Schimmler, H. Schröder, “Morphological Hough Transform on the Instruction Systolic Array”, Euro-Par '97 – Parallel Computing, LNCS 1300, Springer Verlag, pp. 798-806, 1997.

P Å R C

PIPS (1990-94)



32x32 torus
16 bit parallel
communication
16 bit add
prefetch

BHP -- CSIRO -- NU -- ADFA 1.4 M



Special features:

local memory

SIMD-torus

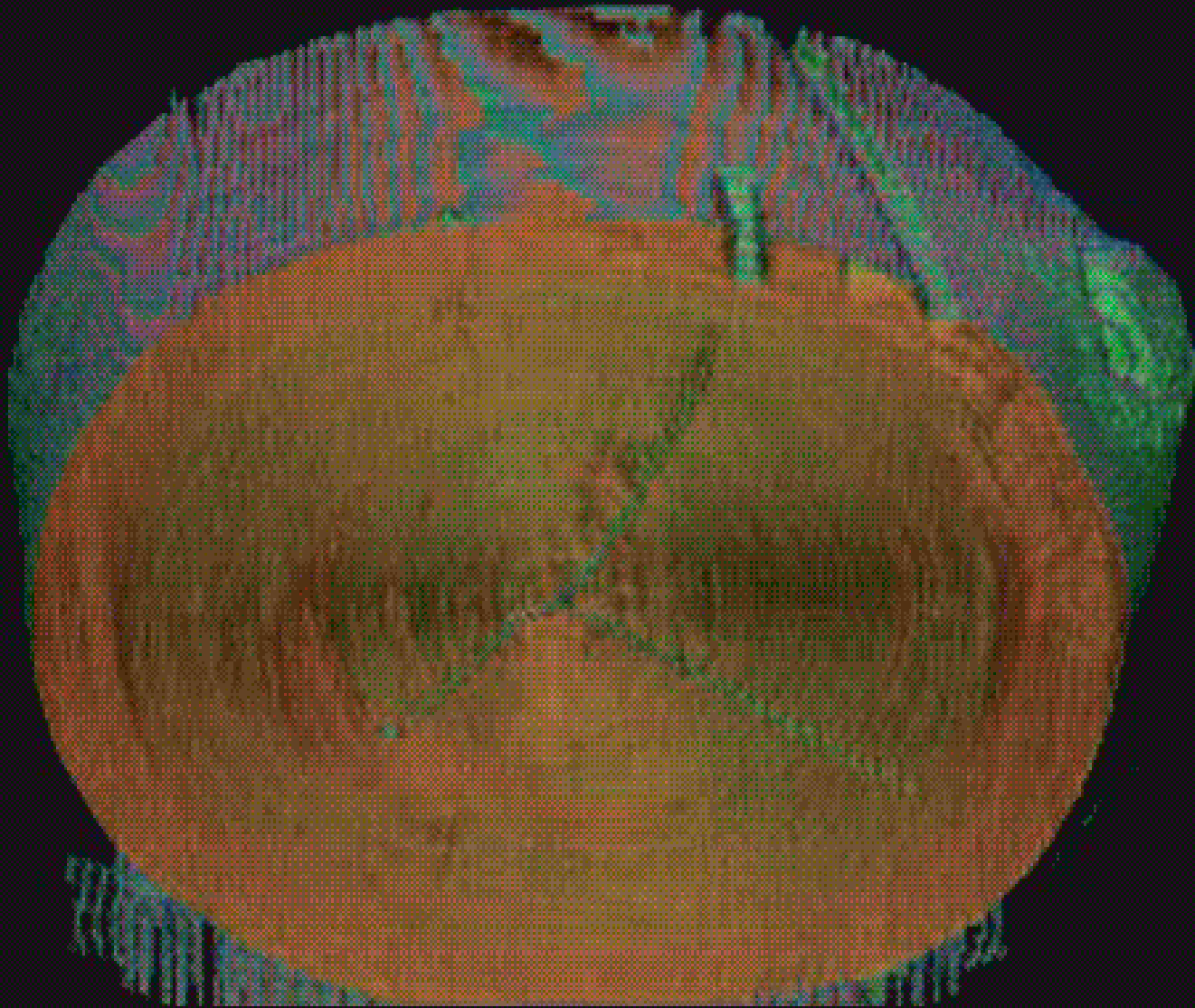
memory pre-fetch

Applications:

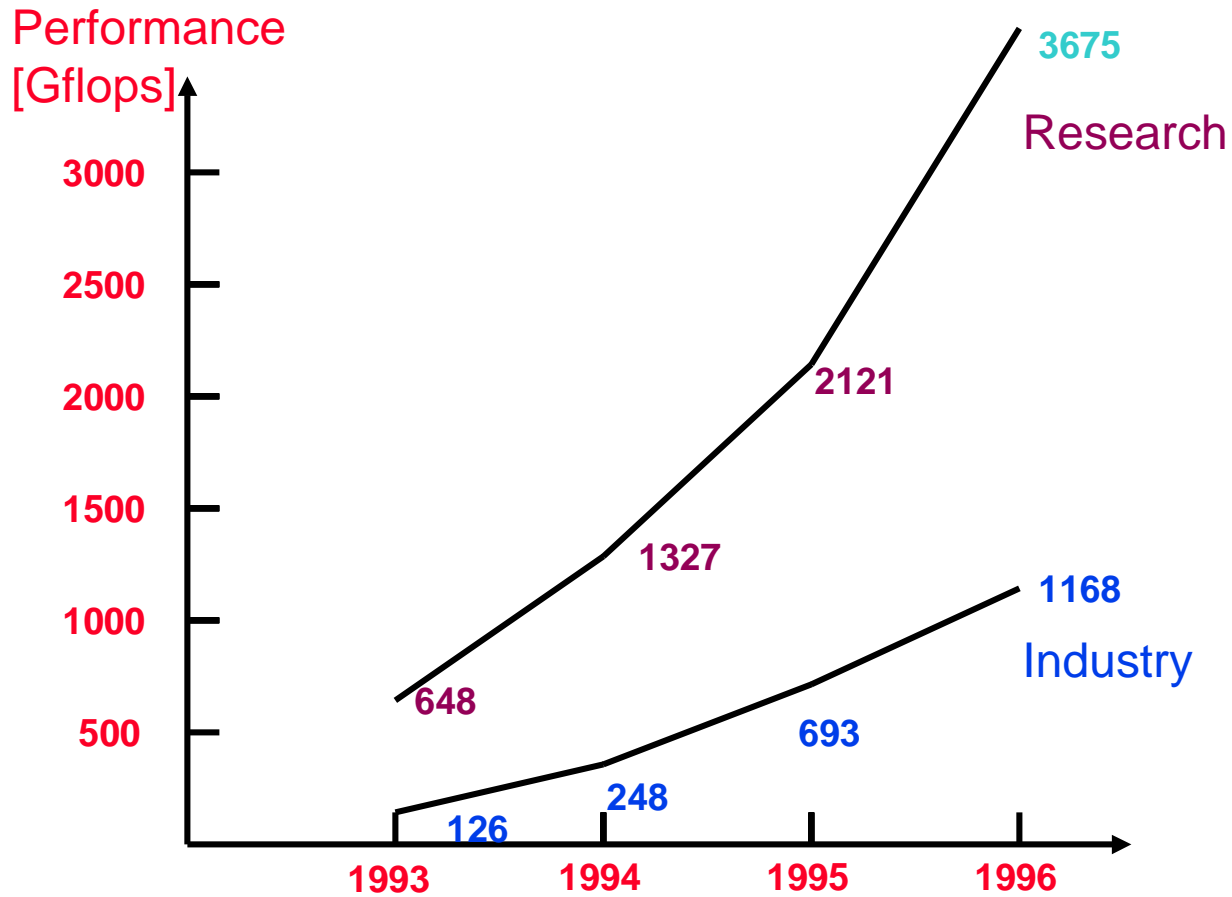
visualization

3D-simulation (CFD, FEM)

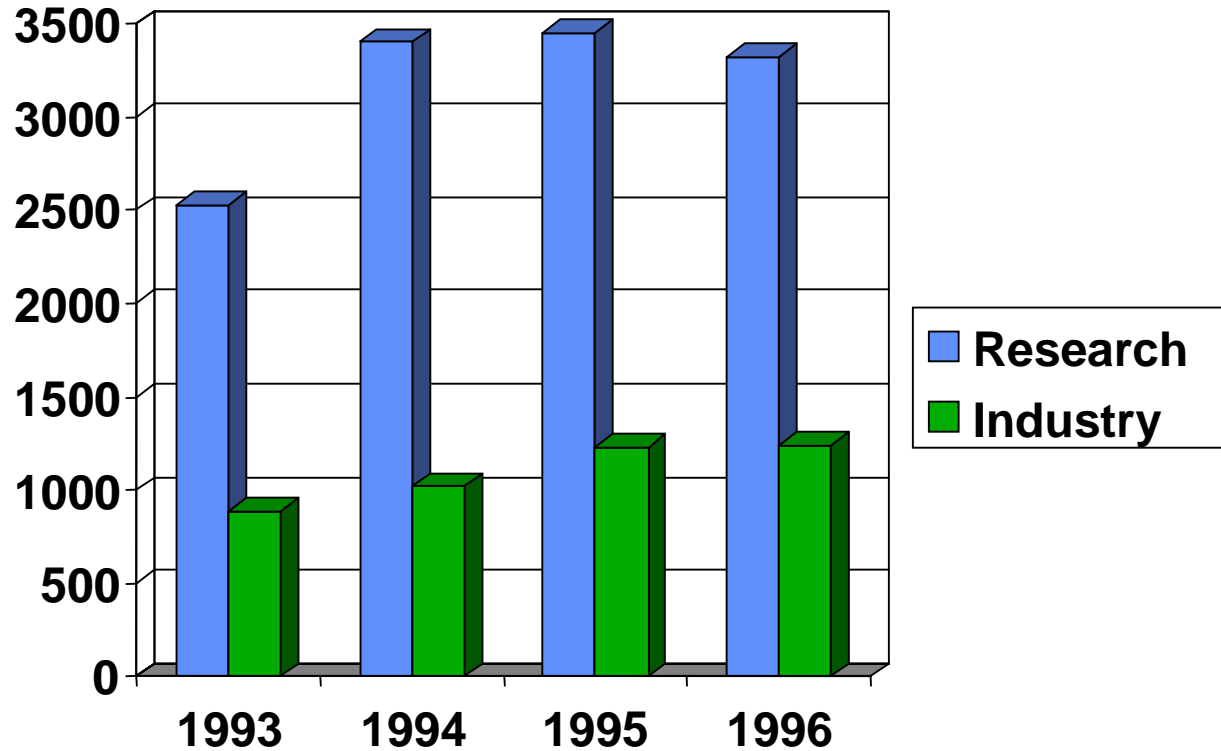
Eucalyptus Regnans



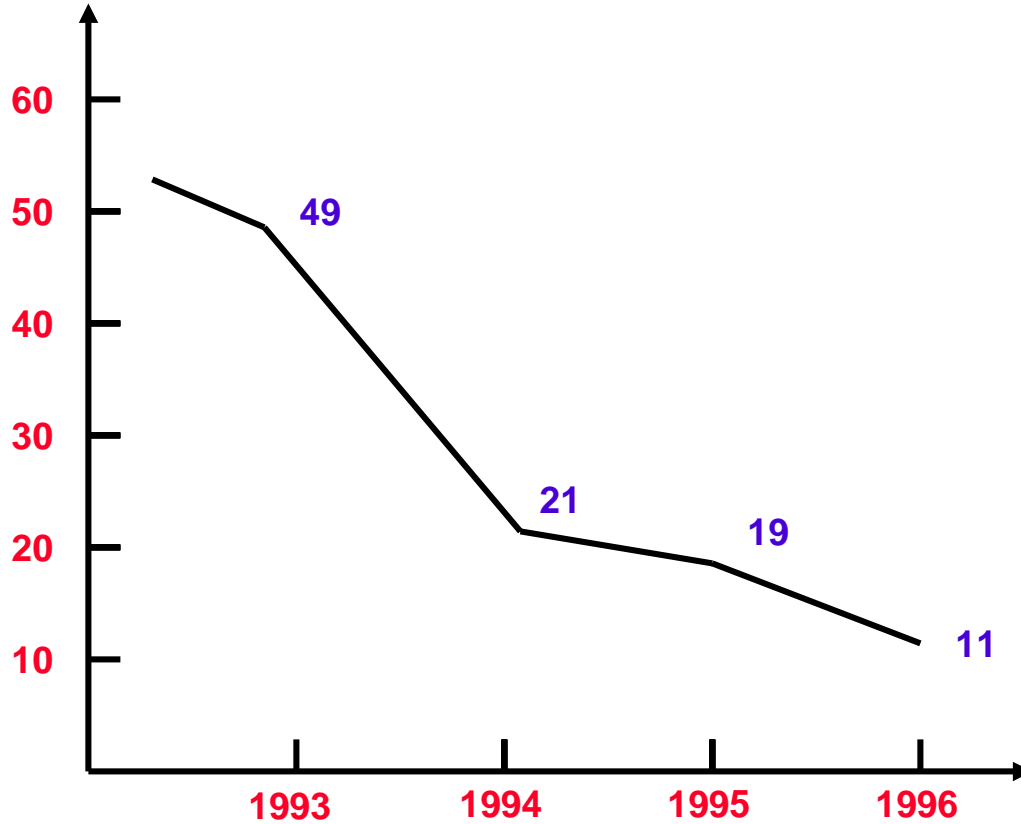
1. A. Spray, K.T. Lie, H. Schröder, “Test Strategies Employed in a Massively Parallel Visualization Engine”, PRFTS, Melbourne, December 1993
2. A. Spray, H. Schröder, K.T. Lie, “A Low-Cost Machine for Real-Time Visualization”, 5th International Symposium on IC Technology, Singapore, 1993
3. H. Schröder et al, “PIPADS: A Vertically Integrated Parallel Image Processing and Display System”, 5th Australian Supercomputing Conference, Melbourne, 1992.
4. R.Lang, E.Plesner, H.Schröder, A.Spray, “An efficient systolic architecture for the one-dimensional wavelet transform”, in Wavelet Applications, Harold H. Szu, Editor, Proc. SPIE 2242, p925-935, 1994.
5. R. Lang, A. Spray, H. Schröder, “2D wavelet transform on a SIMD torus of scanline processors”, *Australian Computer Science Communications*, Vol 17, Nr 17, 271-277, 1995.
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7. A. Spray, H. Schröder, K.T. Lie, E. Plesner, P. Bray, “PIPADS --- A Low-Cost Real-Time Visualization Tool”, Supercomputing, Melbourne, 1993.
8. H. Schröder, A. Spray, “PIPS a massively parallel image processing system”, Workshop PARAGRAPH'94, Linz, March 1994.
9. H. Schröder, “3D-Visualisation based on Scan-line Image Processing”, invited speaker, Workshop MWTAI'97 in Missen-Wilhams, March 1997



Investments into parallel computers
[M\$]



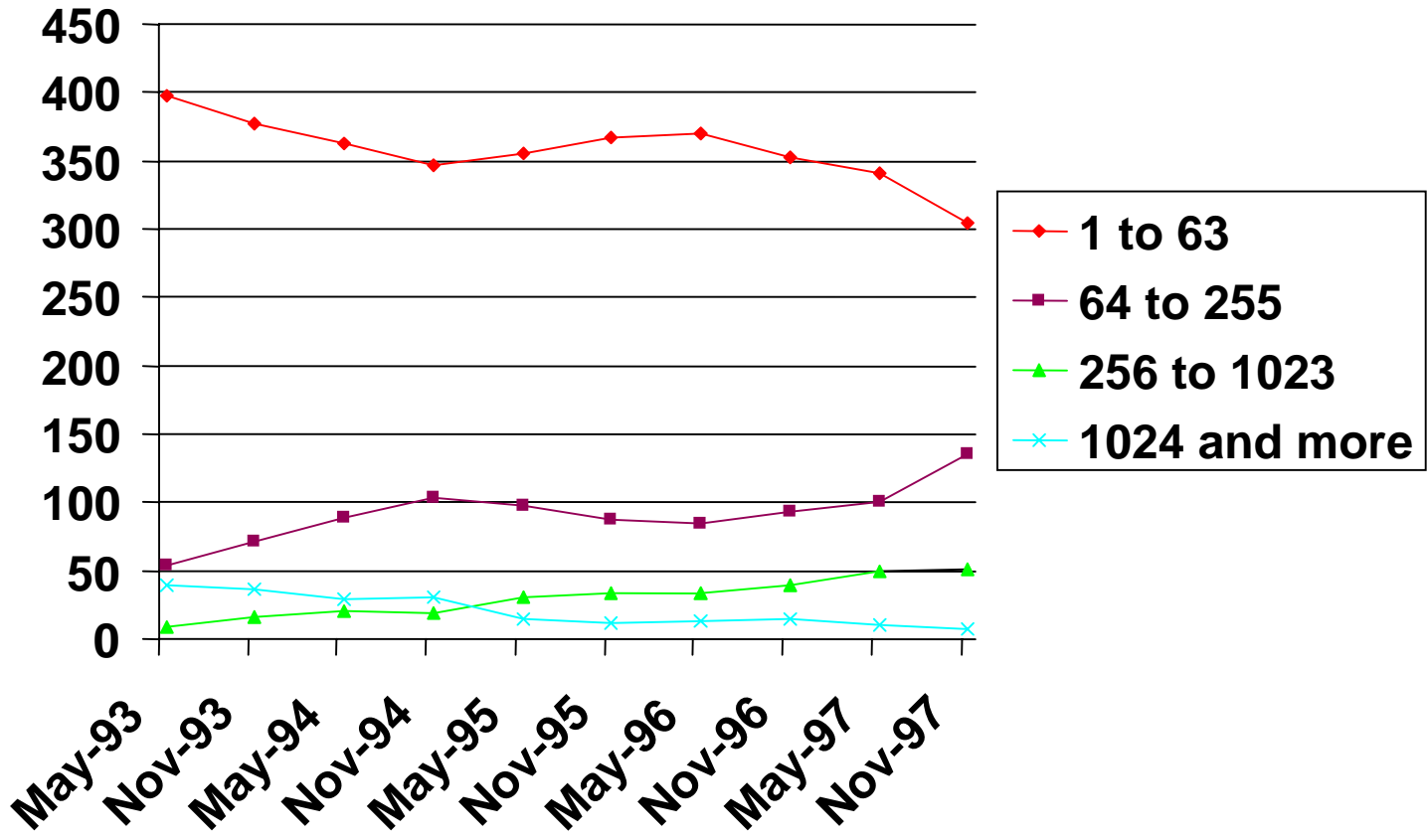
Number of
manufacturers



P A R C

Degree of Parallelism

Number of new Systems



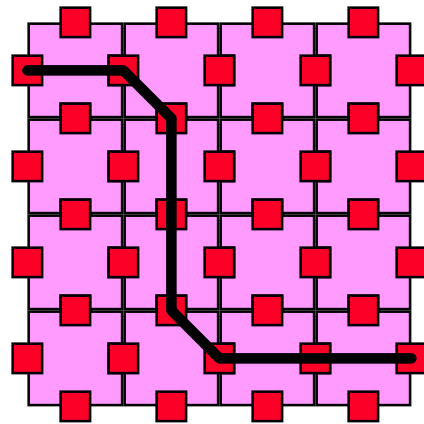
Cost * computation time

- Parallel computers with standard components
- Imbedded parallel systems

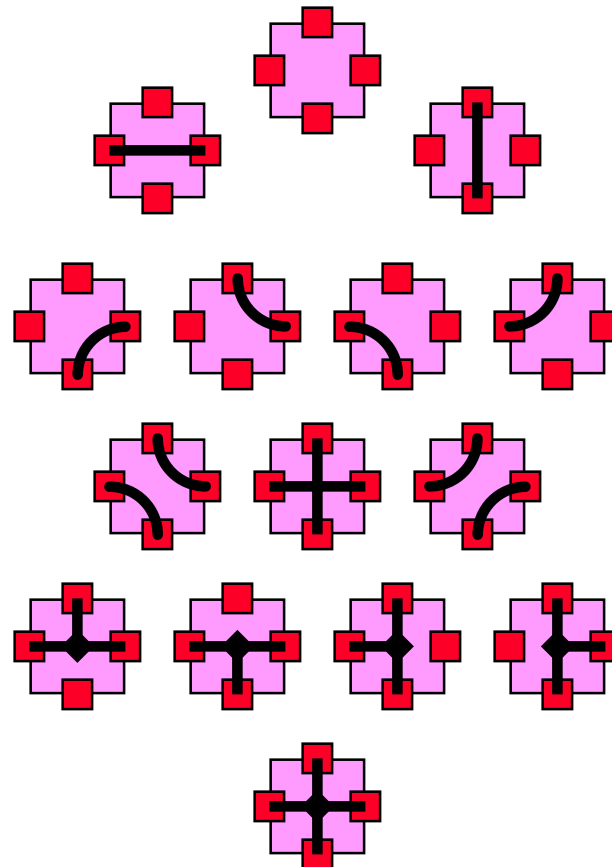
P A R C

reconfigurable mesh

reconfigurable mesh =
mesh + interior connections



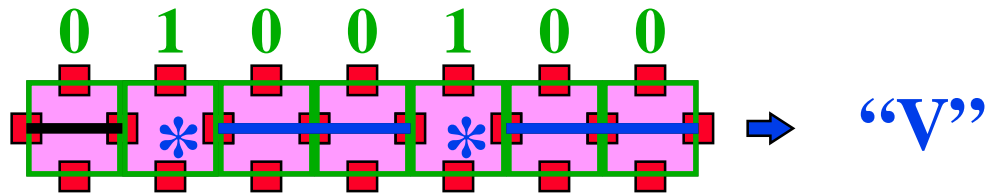
low cost



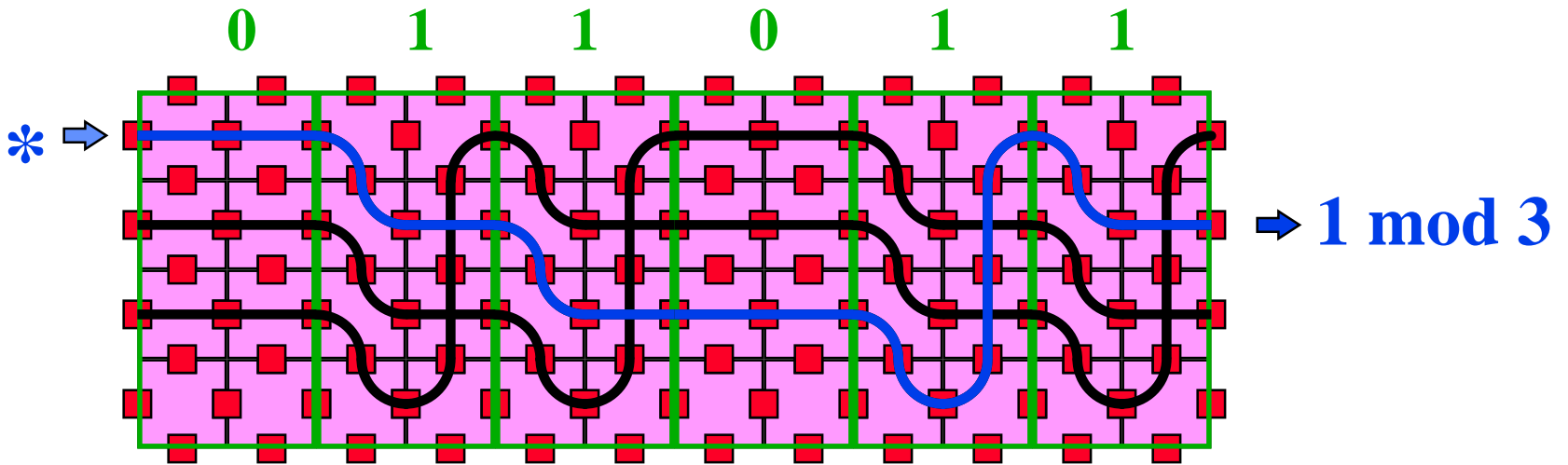
15 positions

P A R C

global OR and modulo 3



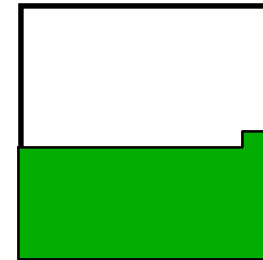
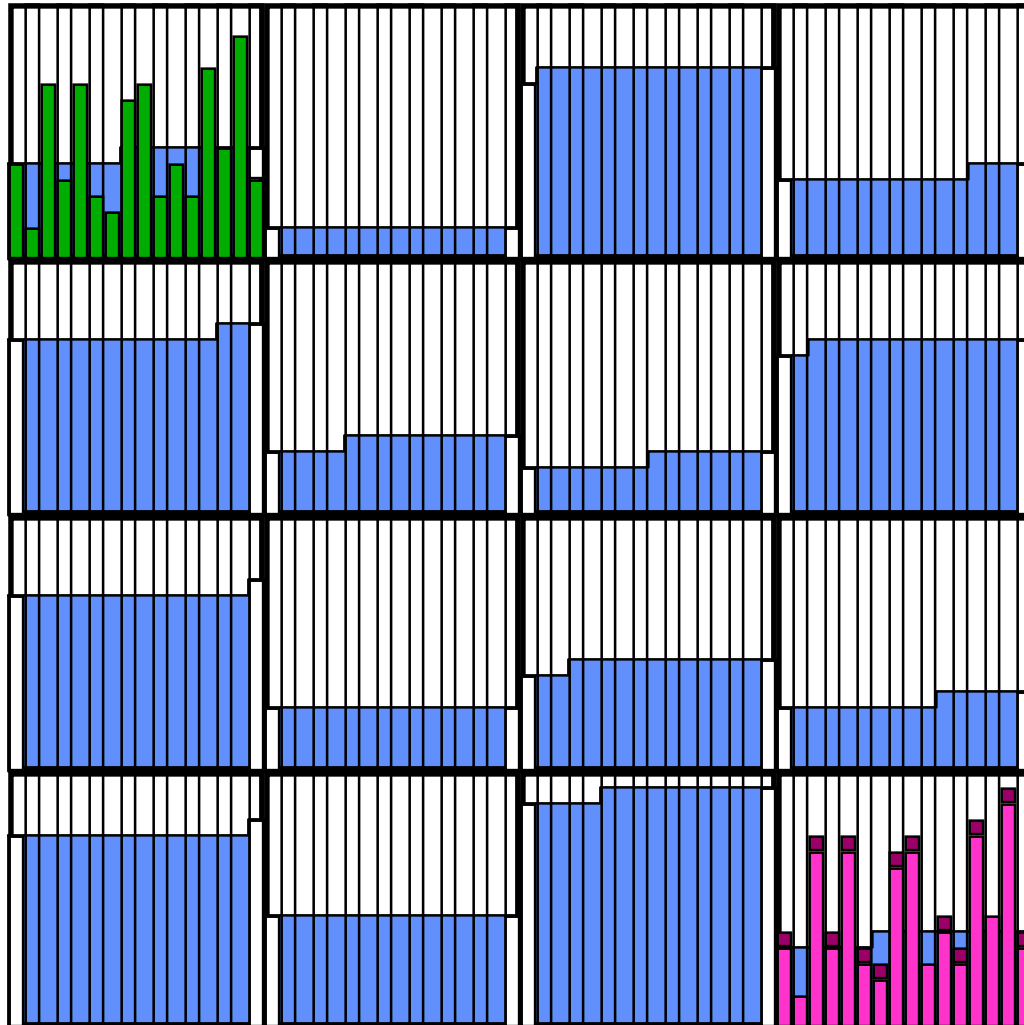
log n on EREW-PRAM



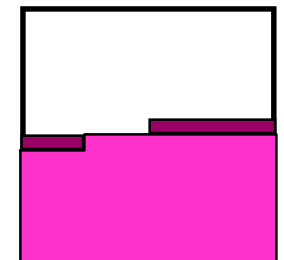
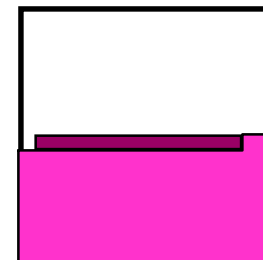
log n / log log n on CRCW-PRAM

P A R C

sorting with all-to-all mapping

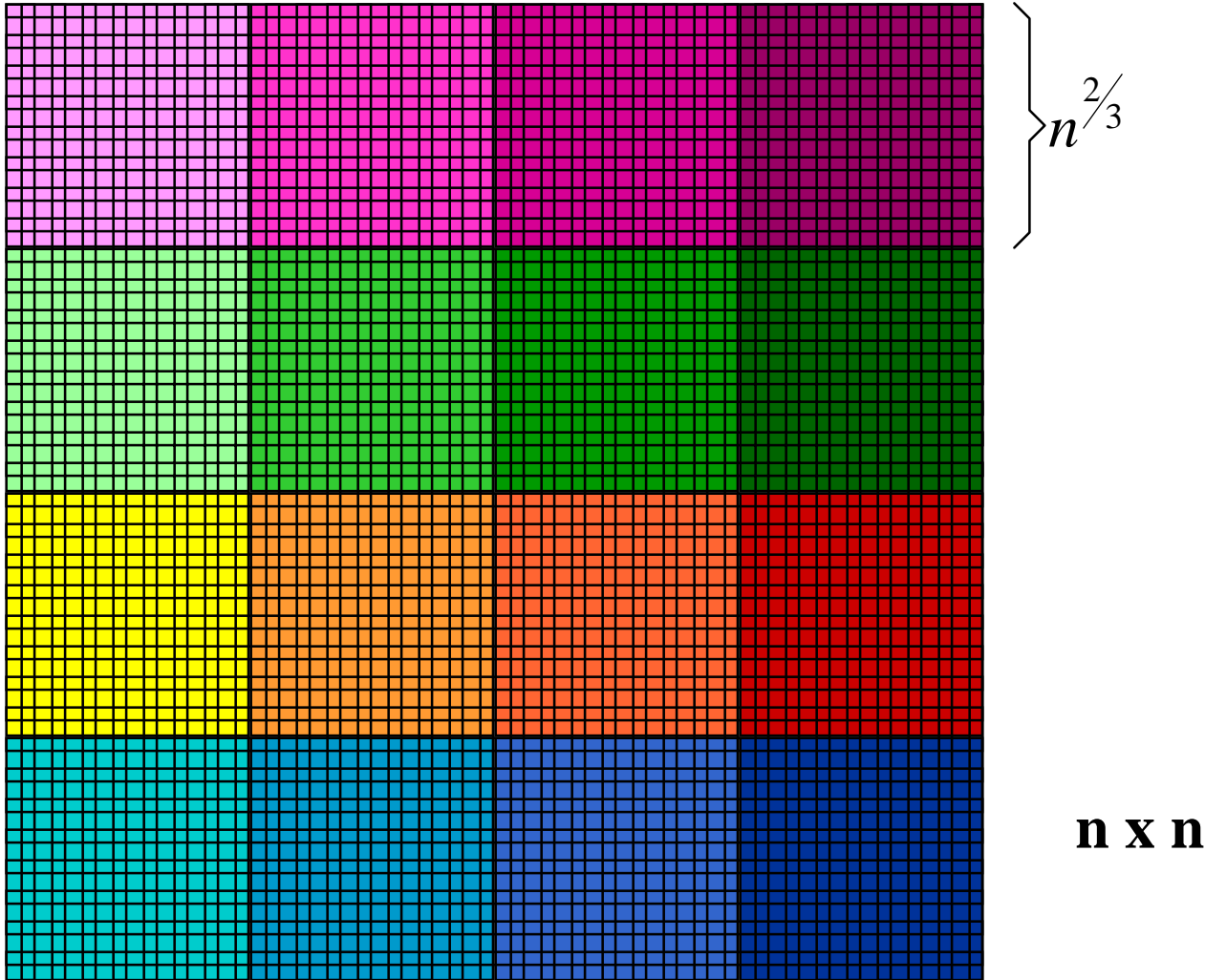


Sorting:
sort blocks
all-to-all (columns)
sort blocks
all-to-all (rows)
o-e-sort blocks



P A R C

all-to-all mapping



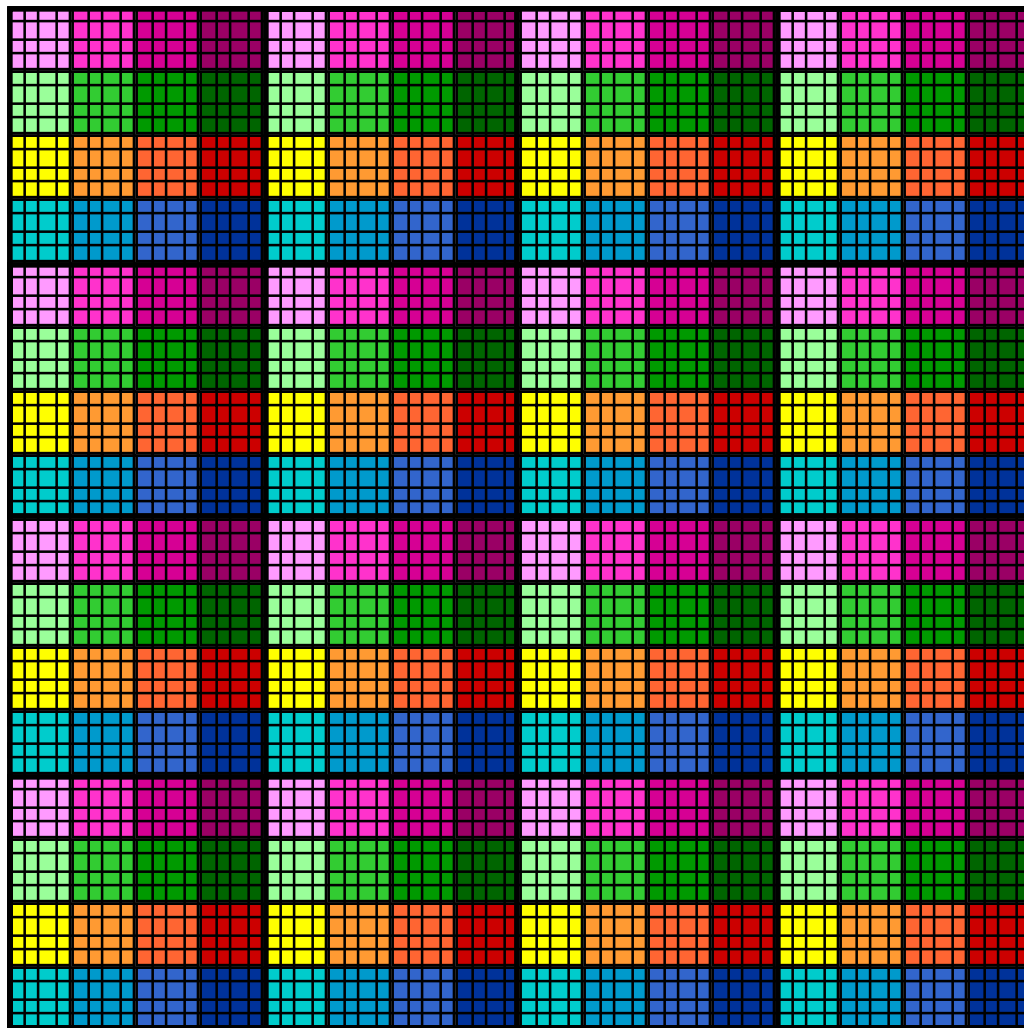
P Å R C

vertical all-to-all

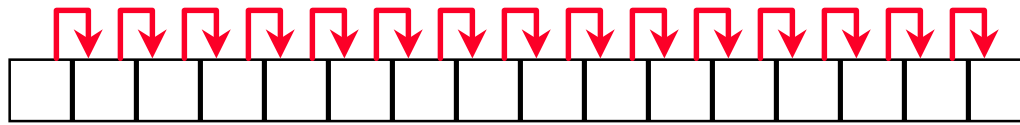


P Å R C

horizontal all-to-all

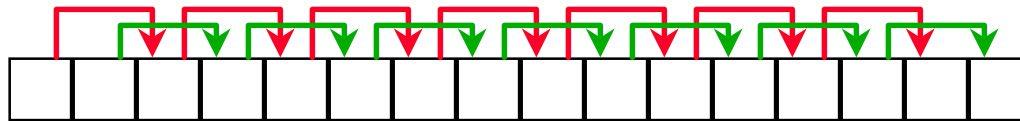


PÄRC

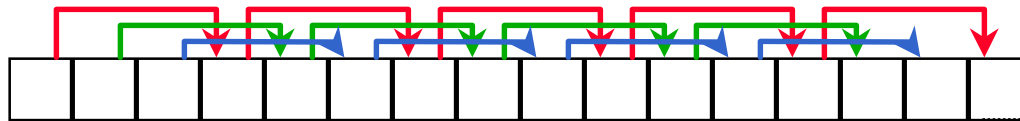


1 step

$(k/2)^2$ steps



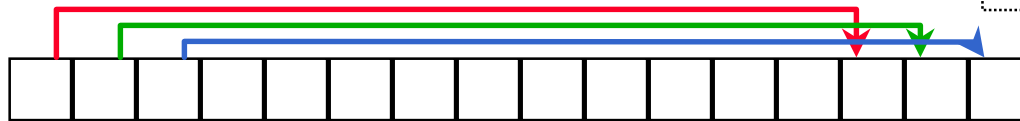
2 steps



3 steps

...

$k/2$ steps



3 steps



2 steps



1 step

P A R C

sorting in optimal time

$(k/2)^2$ steps

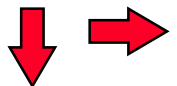
$k=n^{1/3}$

each step takes $n^{1/3}$ time

--> $T = n/4$

 x 2

 x 2

 /2

$T_{\text{all-to-all}} = n/2$

Sorting:

sort blocks ($O(n^{2/3})$)

all-to-all ($n/2$)

sort blocks ($O(n^{2/3})$)

all-to-all ($n/2$)

sort blocks ($O(n^{2/3})$)

time: $n + o(n)$



Reconfigurable mesh

Special features

SIMD

constant diameter

faster than PRAM ?

Suitable applications

routing/sorting/load balancing

sparse matrix multiplication

segmentation / component labeling

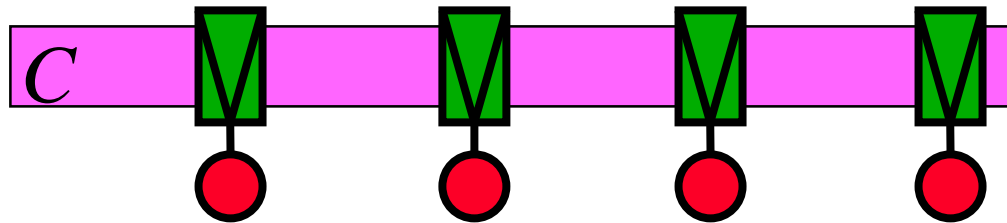
feature extraction

image database ?

1. Kapoor, H. Schröder, B. Beresford-Smith, “Connected Component Labelling on 3D Reconfigurable Mesh Architecture”, *Parallel Processing*, 1994
2. M. Kaufmann, H. Schröder, J. Sibeyn, “Routing and Sorting on Reconfigurable Meshes”, *Parallel Processing Letters*, Vol 5, Nr 1, pp 81-96, 1995
3. G. Turner, H. Schröder, “Token Distribution and Load Balancing on reconfigurable, d-D. Meshes”, *Journal of Parallel Architectures and Algorithms*, 1996
4. M. Middendorf, H. Schmeck, H. Schröder, G. Turner. “Multiplikation of matrices with different sparseness properties on dynamically reconfigurable meshes”, to appear in VLSI Design.
5. Kapoor A., Schröder H., Beresford-Smith B., “Constant Time sorting on a Reconfigurable Mesh “, Australian Computer Science Conference, February 1993.
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7. Kapoor, H. Schröder, B. Beresford-Smith, “Deterministic Permutation Routing on the Reconfigurable Mesh”, Proceedings 8th IPPS, pp 536-540, Mexico, 1994.
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9. H. Schmeck, H. Schröder, G. Turner, “Efficient Sparse Matrix Multiplication on a Reconfigurable Mesh”, PARS-Workshop, Stuttgart, Germany, October 1995.
10. H. Schröder, “Mathematical Morphology for Robot Vision on a Reconfigurable Mesh Architecture”, ISCA Special Purpose Architectures Workshop, May 1992.
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P A R C

Optical Highway



All-to-all connection

$$P^3W = C$$

$P = \#(\text{processors})$

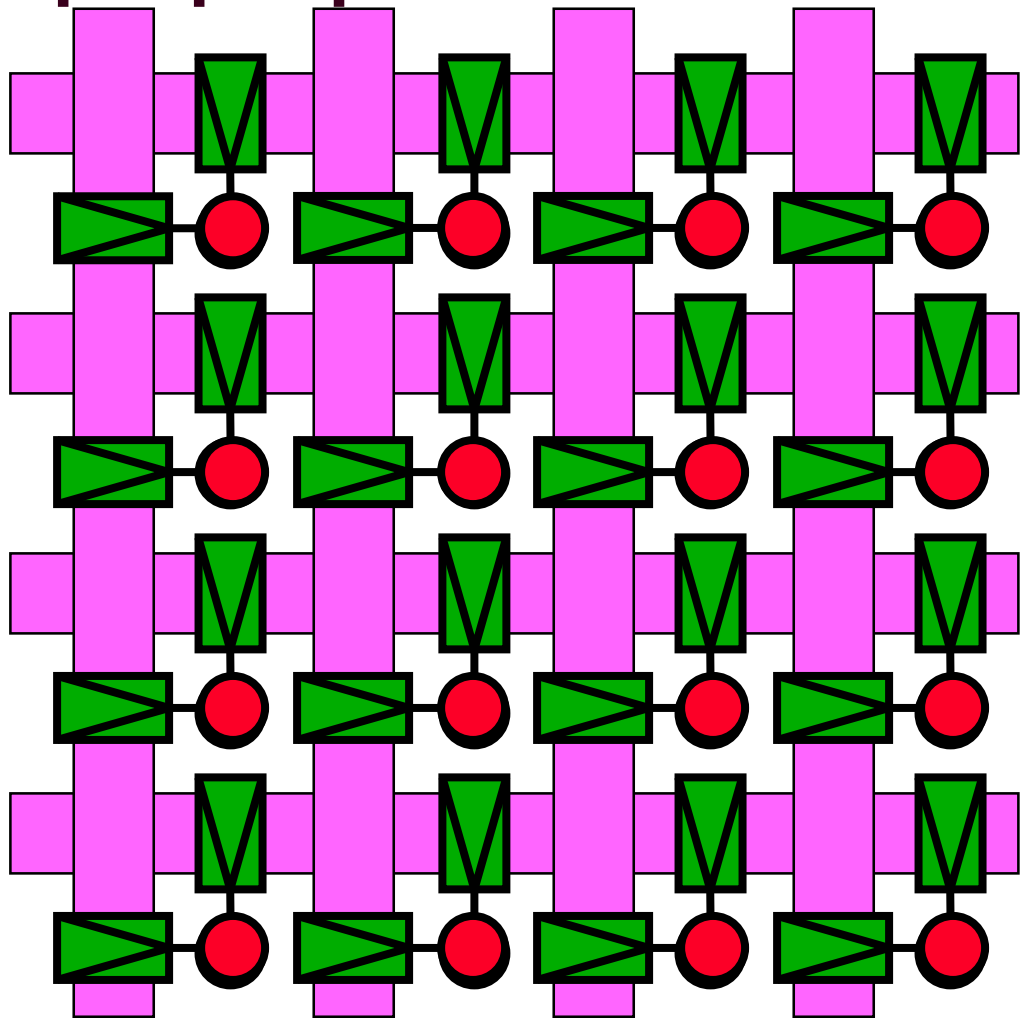
$W = \text{width}$

$$C = 10^6$$

$$W=1; \quad P=100$$

$$W=100; \quad P=22$$

P A R C



**Horizontal
all-to-all**

**Vertical
all-to-all**



Features of optically connected meshes

SIMD/SPMD/MIMD

implement all major architectures

all-to-all communication in 2 steps

Bulk synchronous processing (BSP)

no latency hiding

no pin-limitation

Applications

coarse grain parallel computing only?

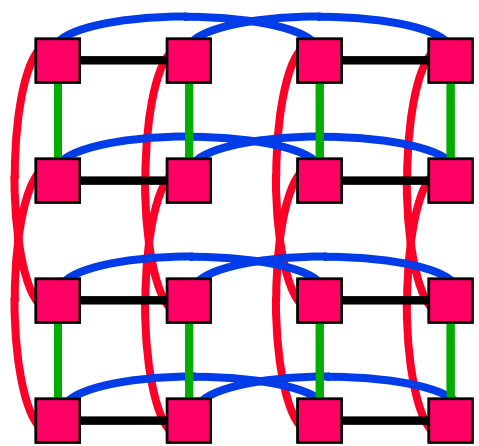
ray-tracing ?

???

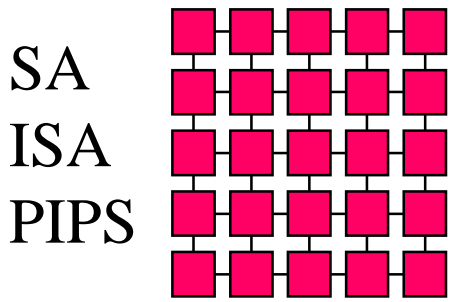
1. H. Schröder et al, “RMB --- A Reconfigurable Multiple Bus Network”, HPCA 96, San Jose, 1996
2. H. Schröder, O. Sykora, I. Vrto, “Optical All-to-All Communication for some Product Graphs”, SOFSEM '97, Milovy, Czech Republic, 1997

P A R C

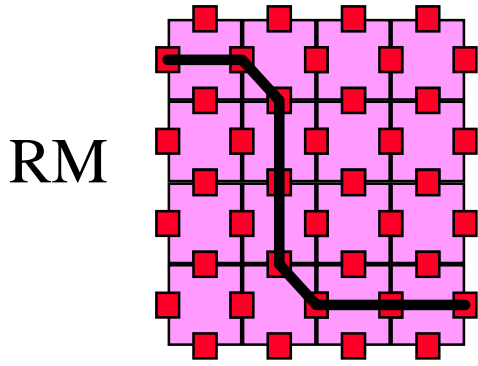
Bisection-width / Diameter



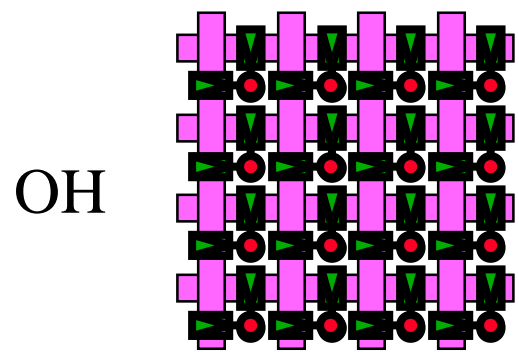
Diameter $\log n$
 bisection width n



diameter \sqrt{n}
 bisection width \sqrt{n}



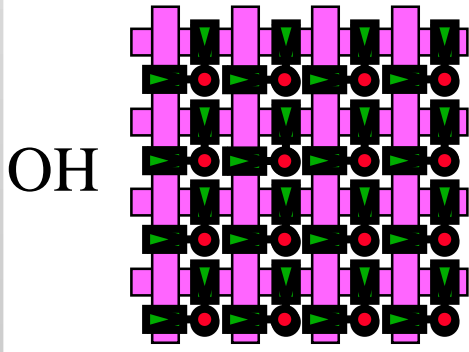
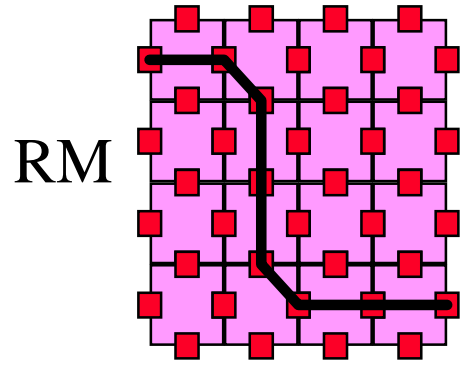
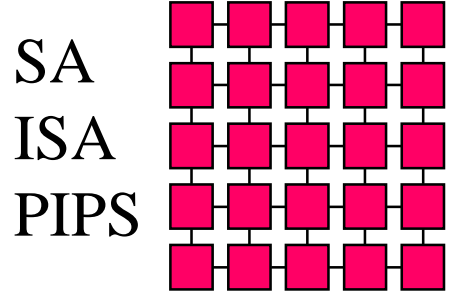
Diameter 1
 bisection width \sqrt{n}



Diameter 1
 bisection width n



Suitable problems ?



SA: suitable applications?
 ISA: 2D-problems, aggregate functions
 local communication
 PIPS: 3D-problems, local communication

RM: diameter-bound > bisection-width-bound

OH: PRAM equivalent?

