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Hough Transform for Lines and Curves

CMPE 264: Image Analysis and Computer Vision

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Line and curve detection

- Find lines, curves, or parts of lines or curves in an input image. Such an image might be the output of a edge detector discussed in the previous lectures
- Two problems
  - Grouping – Find the set of points compose each instance of the target curve in the image
  - Model fitting – Given a group, find the curve or line that explains the data points best

Hough transform for lines

- Basic ideas
  - A line is represented as $y = mx + n$. Every line in the image corresponds to a point (m,n) in the parameter space
  - Every point in the image domain corresponds to a line in the parameter space (why ? Fix (x,y), (m,n) can change on the line $n = y - mx$. In other words, for all the lines passing through (x,y) in the image space, their parameters form a line in the parameter space)
  - Points along a line in the space correspond to lines passing through the same point in the parameter space (why ?)

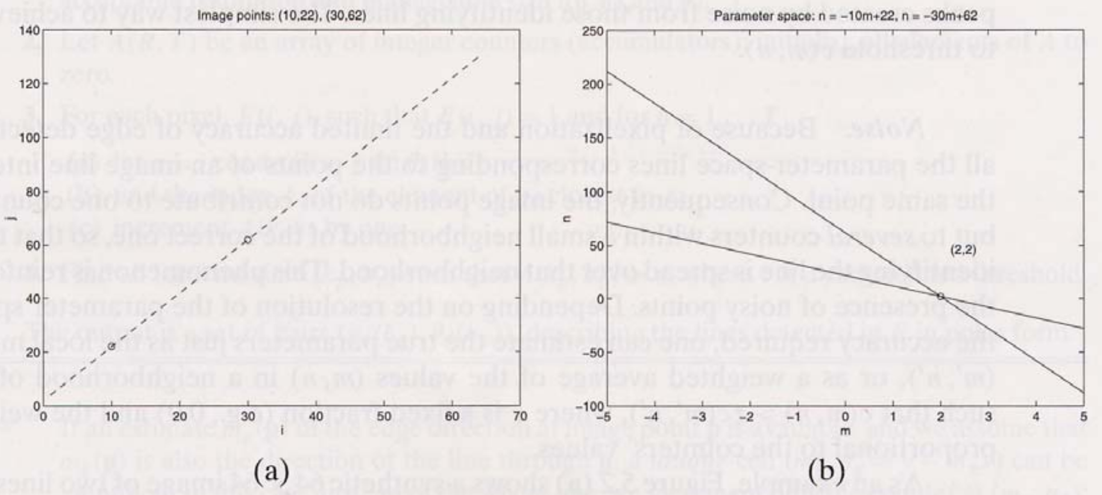
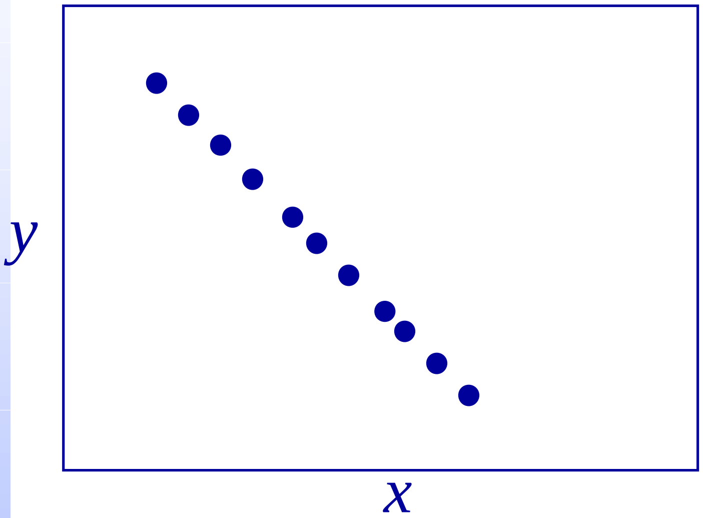
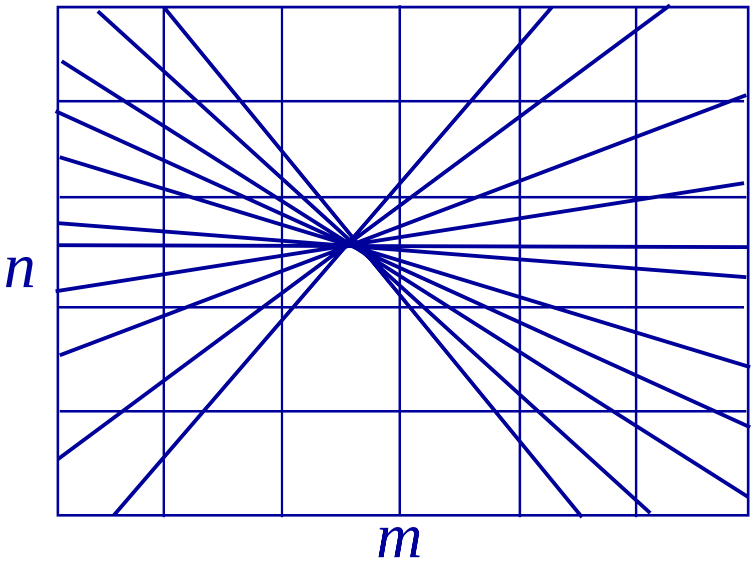
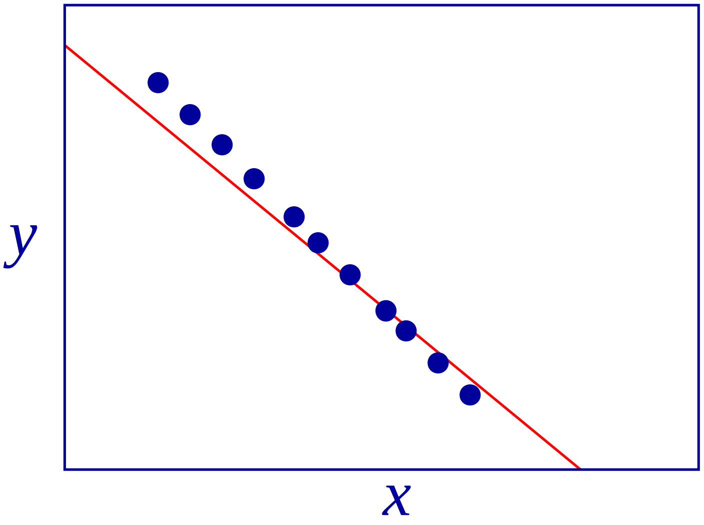


Figure 5.1 Illustration of the basic idea of the Hough transform for lines. The two image points (a) are mapped onto two lines in parameter space (b). The coordinates of the intersection of these lines are the parameters (m, n) of the image line through the two points.

Hough transform for lines







n

2	2	0	1	2	2
3	5	3	3	2	2
3	7	11	10	4	3
2	3	1	4	5	2
2	1	0	1	3	3

m

Hough transform for lines

- A real example
  - Two lines correspond to two peaks in the parameter space

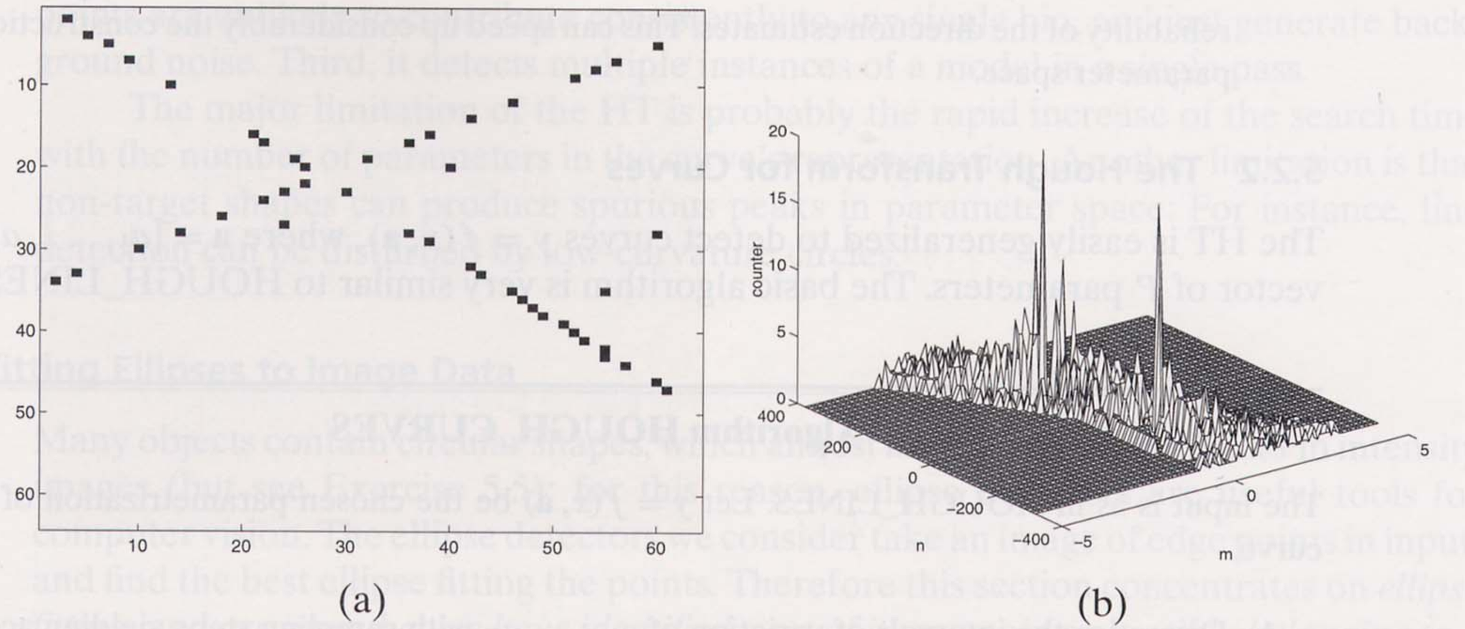
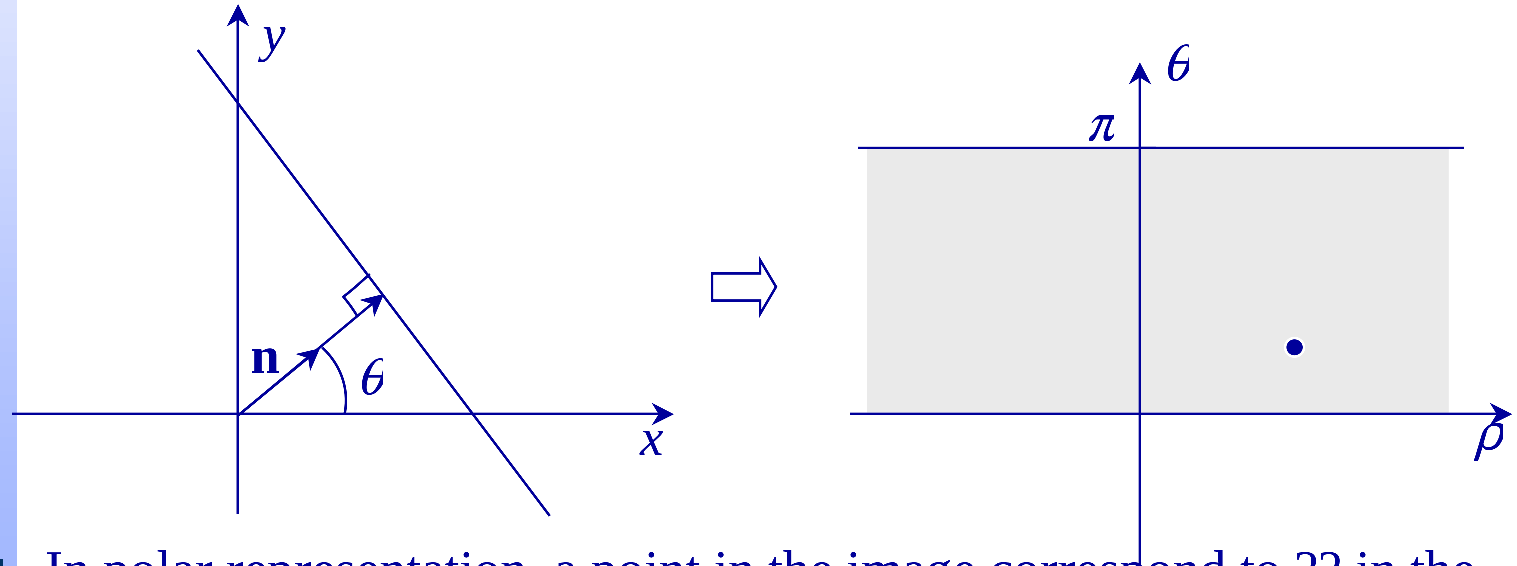


Figure 5.2 (a) An image containing two lines, sampled irregularly, and several random points. (b) Plot of the counters in the corresponding parameter space (how many points contribute to each cell (m, n)). Notice that the main peaks are obvious, but there are many secondary peaks.

Detecting lines using Hough transform

- *m* can be huge for near vertical lines, and there is no representation for a vertical line. Lines are not “evenly” distributed in the parameter space
- Solution – using polar representation for lines, more “evenly” distributed

$$\rho = p\,?\mathbf{n} = x\cos\theta + y\sin\theta$$



- In polar representation, a point in the image correspond to ?? in the parameter space (Homework!)

Detecting lines using Hough transform

- Algorithm HOUGH_LINES
  - The input image *E is* M ? N binary array with edge pixels marked with ones and other pixels marked as zeroes. Let ρ_d, θ_d be the arrays containing the discretized intervals of the parameter space ρ ? $[0, \sqrt{N^2 + M^2}], \theta$? $[0, \pi]$
  - Discretize the parameter spaces of ρ and θ using sampling steps $\delta\rho, \delta\theta$, yielding acceptable and manageable resolution of R,T in the parameter space
  - Let *A(R,T)* be the counter array, initialized as zeroes
  - For each pixel *E(i,j)*=1, and for $h = 1, \ldots T$
    - Let $\rho = i \sin \theta_d(h) + j \cos \theta_d(h)$
    - Find index *k* so that $\rho_d(k)$ is closest to ρ
    - Increment *A(k,h)* by one
  - Find all local maxima (k_p, h_p) such that $(k_p, h_p) > \tau$, where τ is a user defined threshold
  - The output is a set of lines described by $(\rho_d(k_p), \theta_d(h_p))$

Detecting curves using Hough transform

- Algorithm HOUGH_CURVES
  - Let $f(x, y, \mathbf{a}) = 0$ be the parametric form of the curves
  - Discretize the parameters $a_1, \ldots, a_p$ with sampling steps yielding acceptable and manageable resolution of $s_1, \ldots, s_p$
  - Let $A(s_1, \ldots, s_p)$ be the counter array, initialized as zeroes
  - For each pixel $E(i,j)$=1, increment all counters such that $f(i, j, \mathbf{a}) = 0$
  - Find all local maxima $\mathbf{a}_m$ such that $\mathbf{a}_m > \tau$, where τ is a user defined threshold
  - The output is a set of curves described by $\mathbf{a}_m$

Summary

- Can detect lines even when they are partially occluded
- Hough Transform is a “voting” algorithm
- Since each point is handled independently, parallel implementations are possible
- It becomes difficult when the dimension of the parameter space is large

Homework

- In the polar representation, a point in the image correspond to what kind of curve in the parameter space ? (Derivation required !)