Lectures on
Cellular Automata Continued

Modified and upgraded slides of

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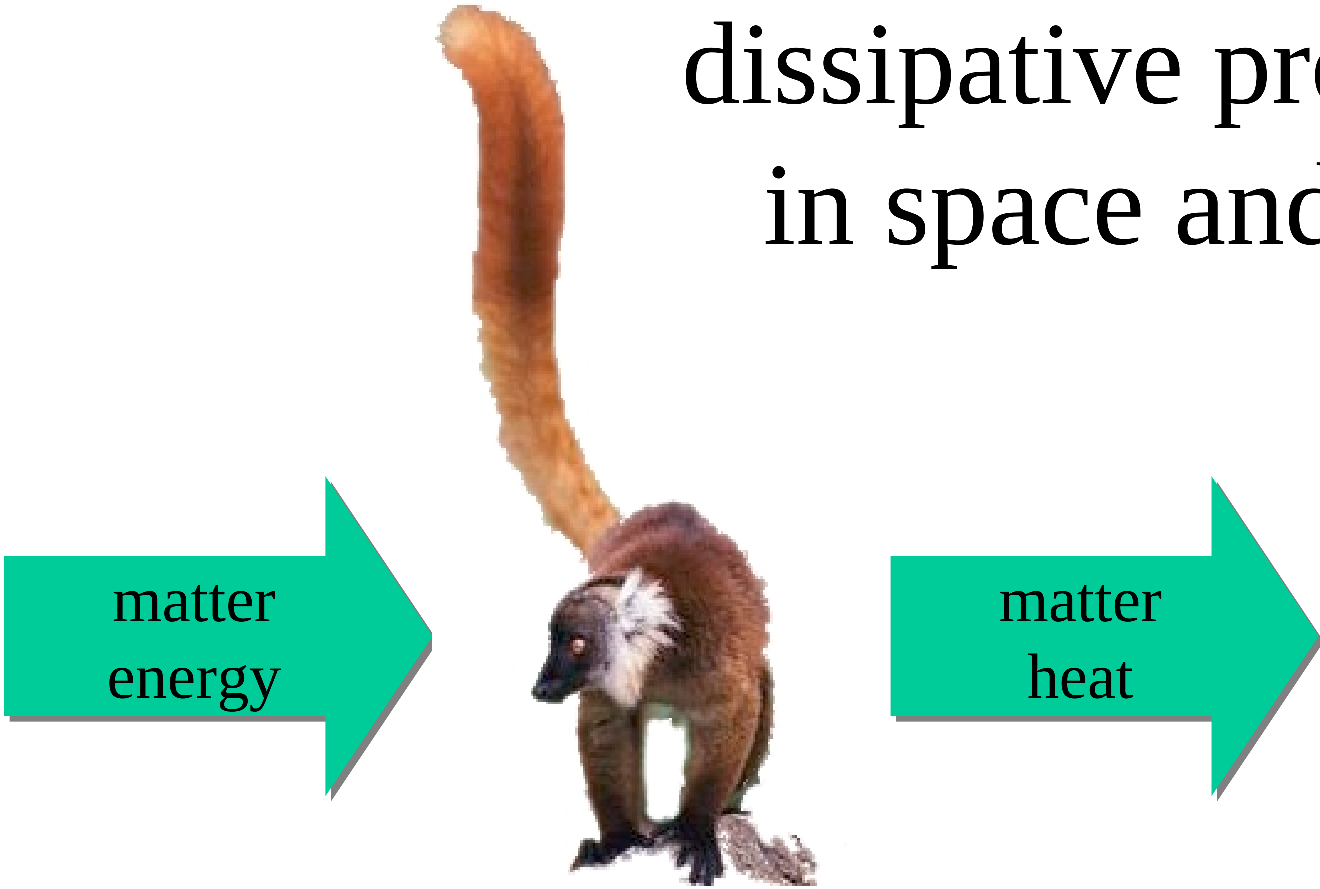
Dynamical Systems and Cellular Automata

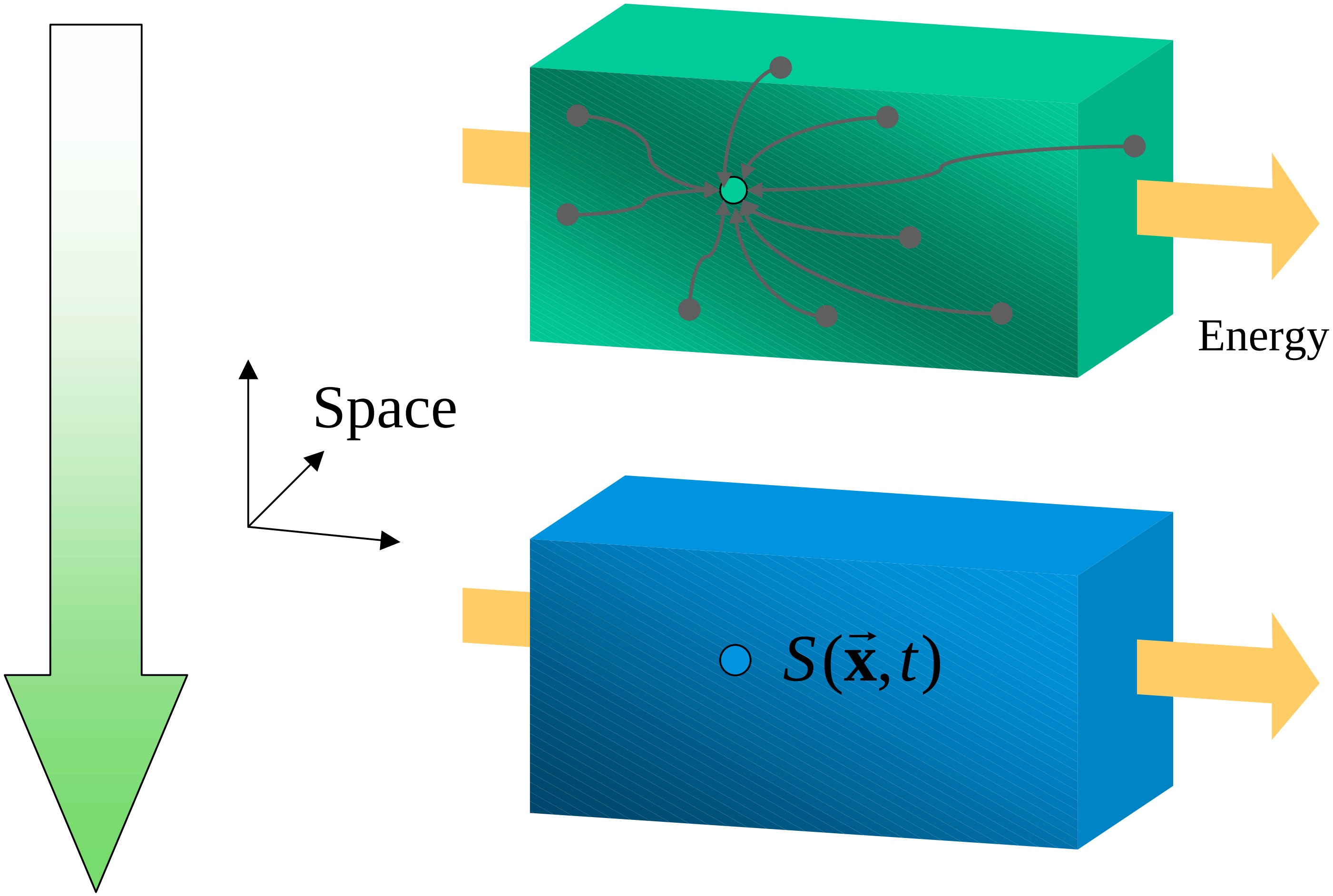
Morphogenetic modeling

Organisms are dissipative processes in space and time



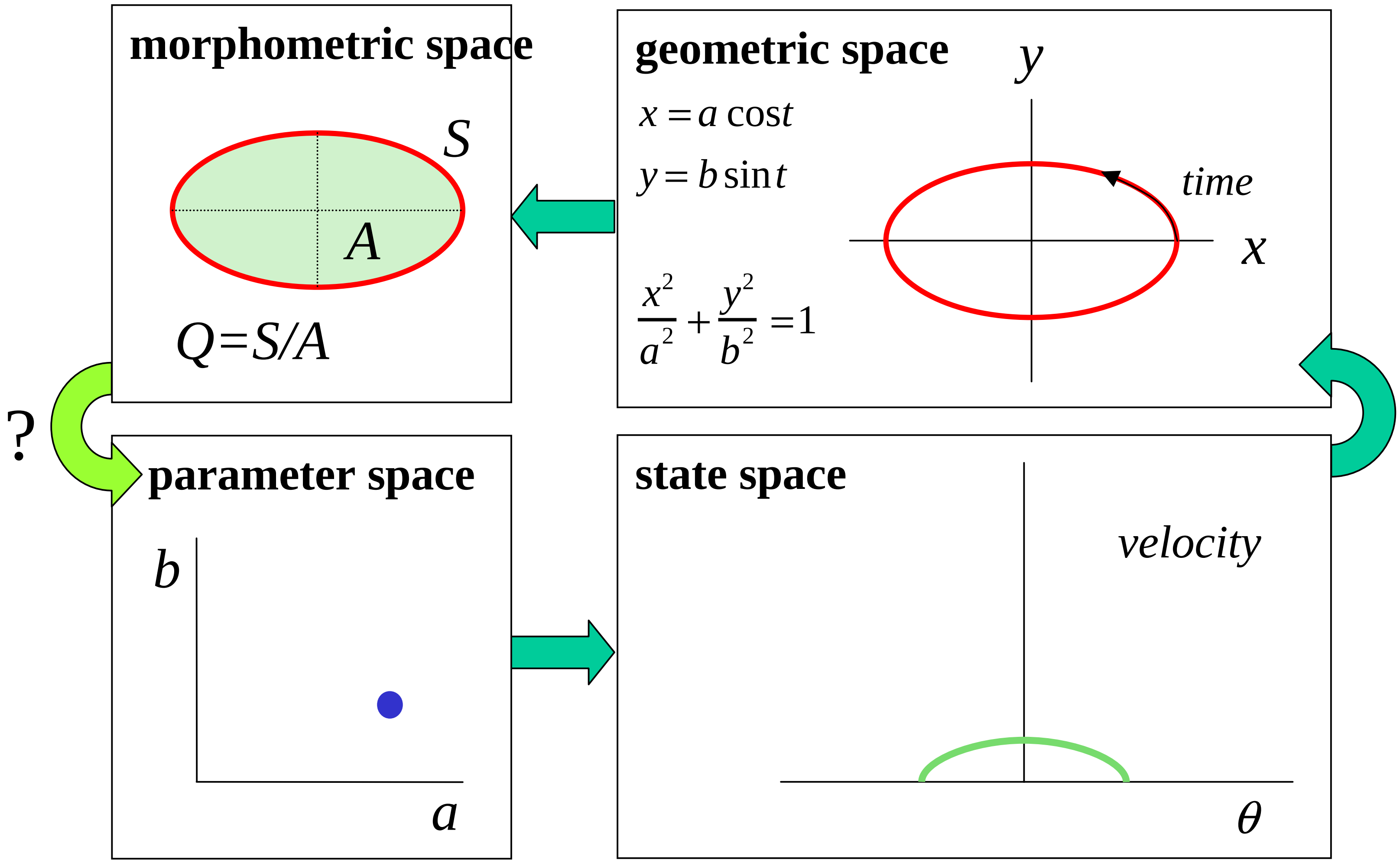
entropy dissipator

Dynamical system

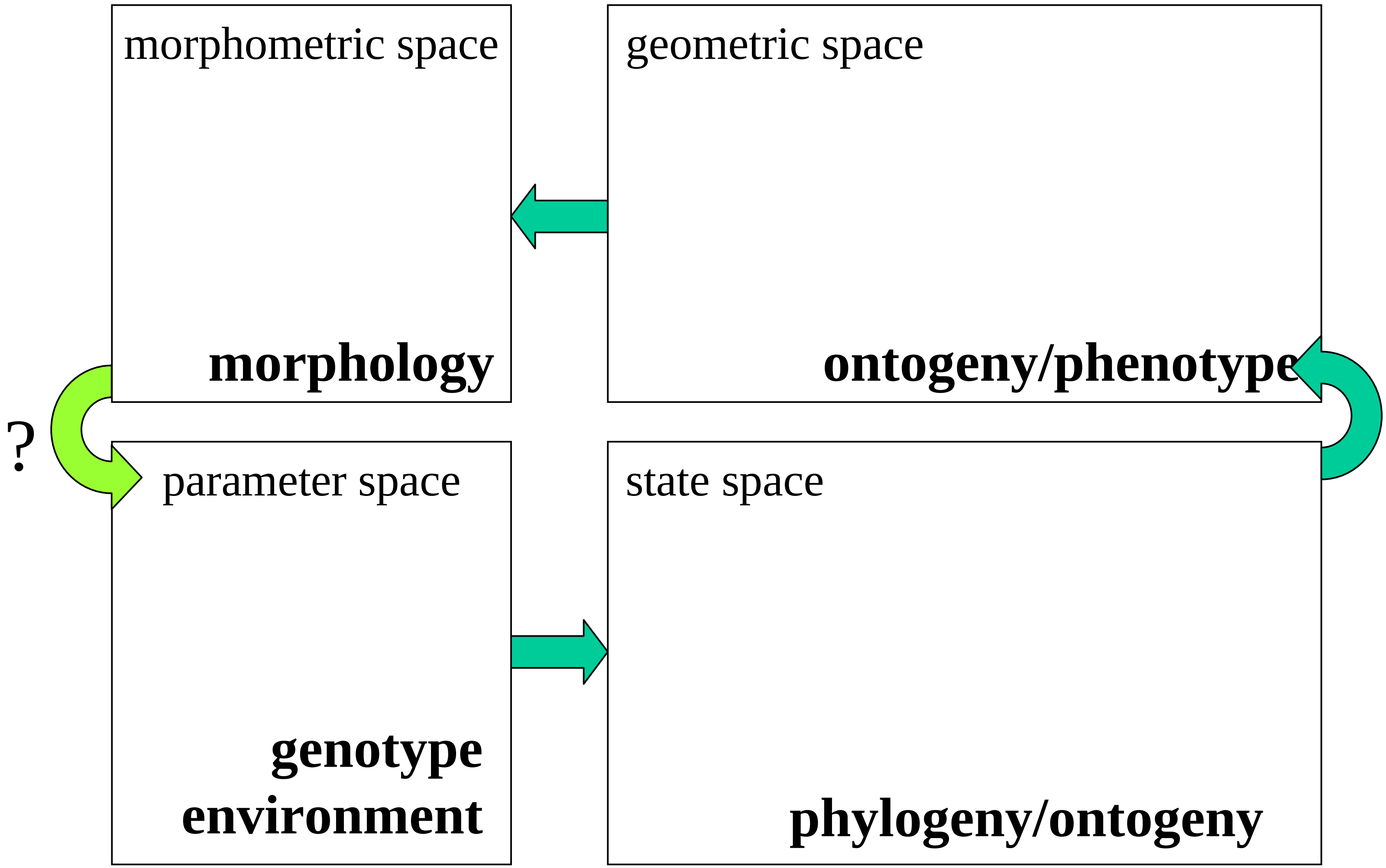


state S at $\vec{\mathbf{x}}$ at time t ?

Which space? - which geometry?



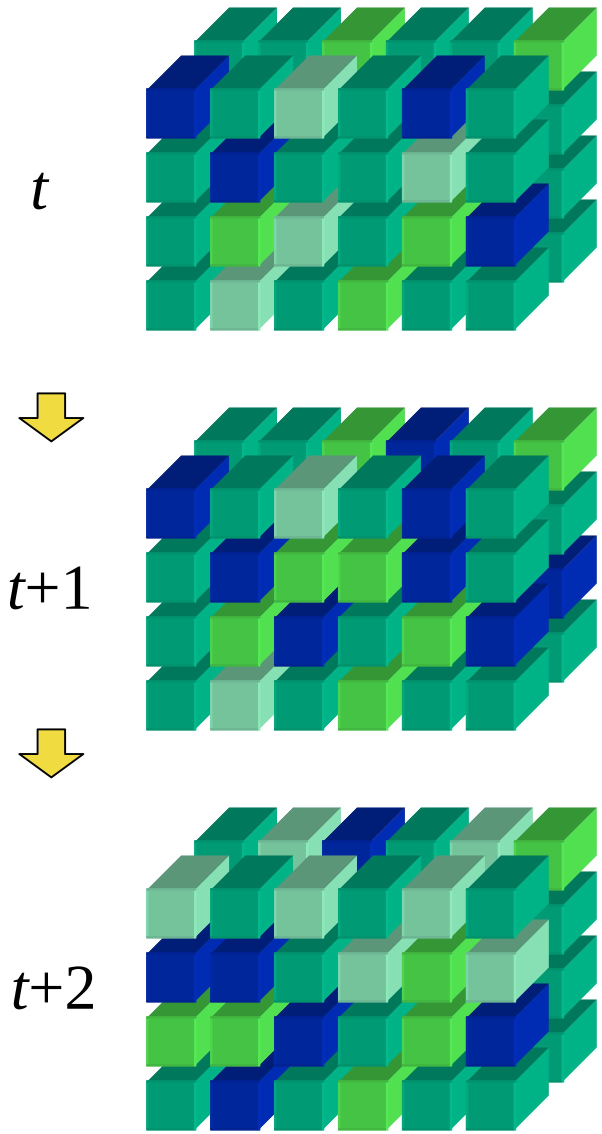
Which space? - which geometry?



Discretization has many aspects

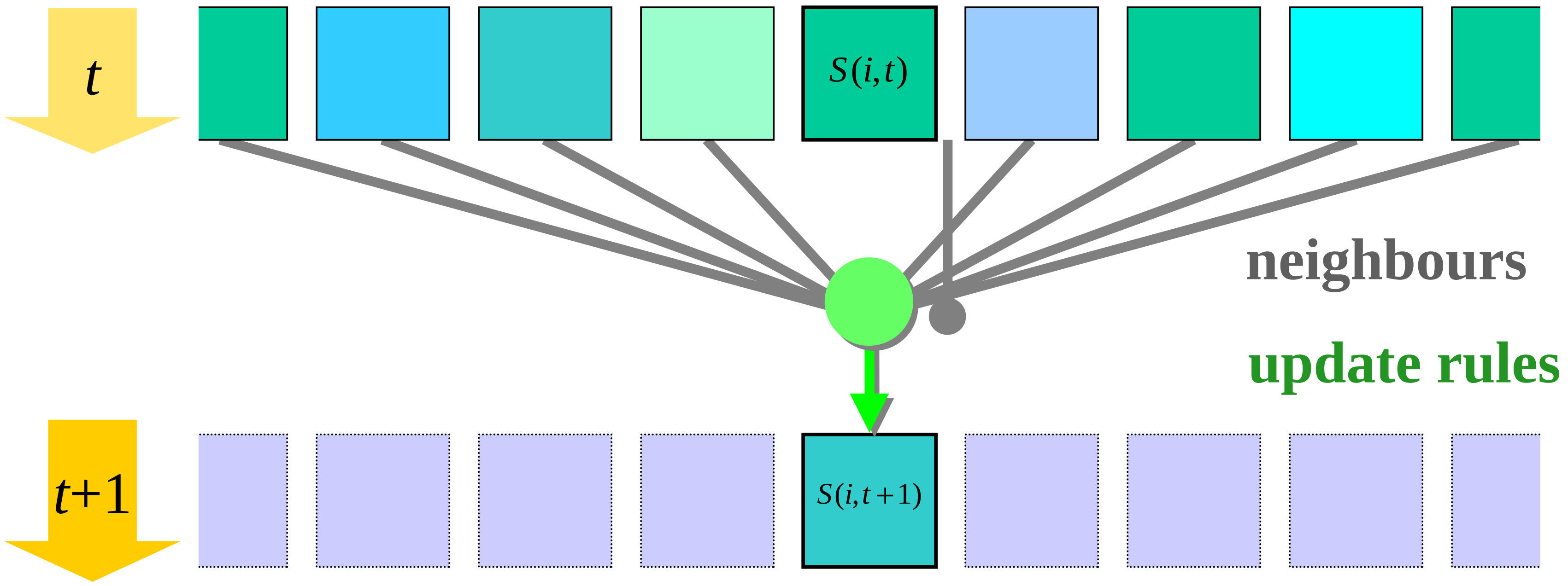
- discrete time steps
  T=0,1,2,3,…
- discrete cells at
  X,Y,Z=0,1,2,3,...
- discretize cell states
  S=0,1,2,3,...

We can mix discrete and continuous values in some models



Cellular Automaton

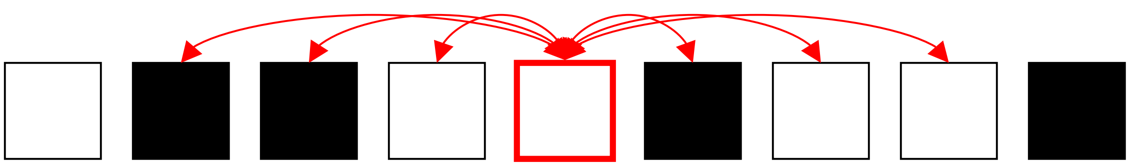
In standard CA the values of cells are discretized



$$S(i,t+1) = f(S(neighbours,t), \text{update rules})$$

A simple Cellular Automaton

- Reduce dimensions: D=1, i.e. array of cells
- Reduce # of cell states: binary, i.e. 0 or 1
- Simplify interactions: nearest neighbours
- Simplify update rules: deterministic, static



Effects of simplification

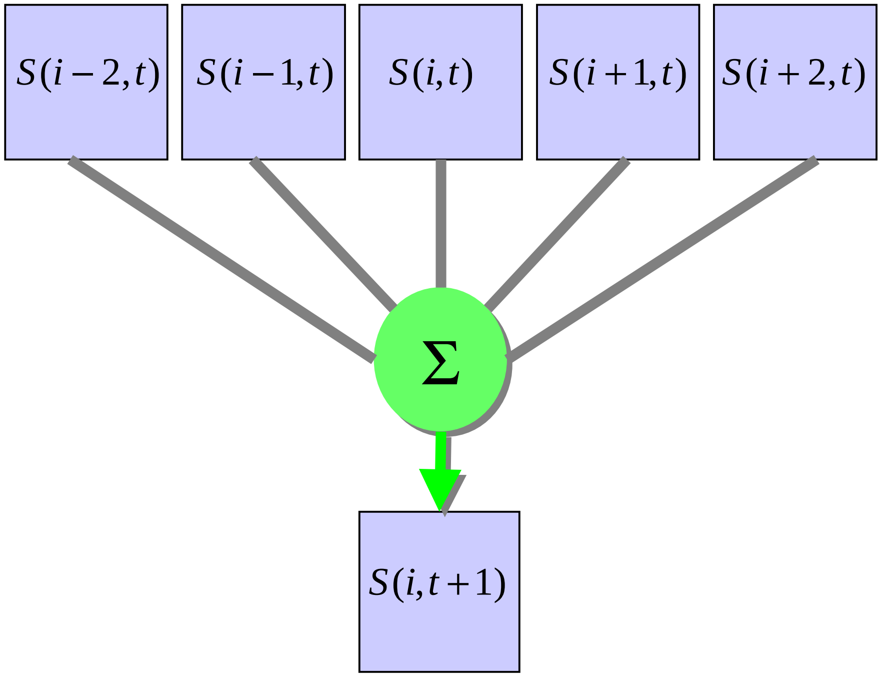
CA model	**Morphogenesis**
Spatial discretization	~ level of detail (organism, cell, molecule)
Temporal discretization	~ level of detail (cell cycle, reaction rate)
Reduction of dimension	→ profound effects (2D “Game of Life”)
Binarization	computational convenience (01→A, 10→T, 00→G, 11→C)
nearest-neighbour interactions	~ spatial restrictions
simple update rules; simultaneity, ubiquity	~ non-linearity → profound effects

Wolfram's 1D binary CA

- cell array
- binary states
- 5 nearest neighbours
- „sum-of-states“ update rule:

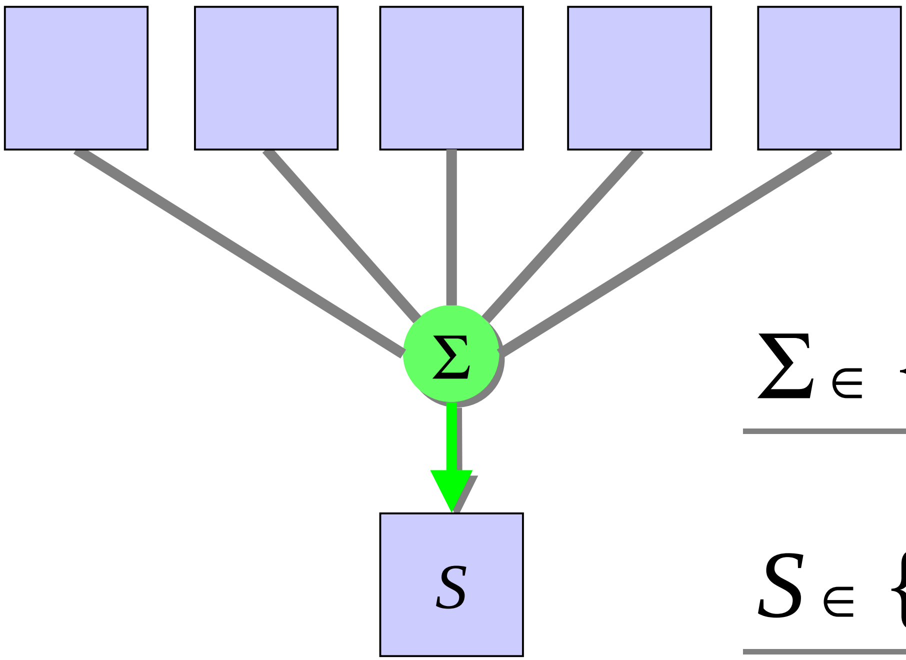
$$S(i,t+1) = f\left(\sum_{j=-2}^{j=2} S(i+j,t)\right)$$

Wolfram, S., *Physica* **10D** (1984), 1-35



Observe importance of symmetry of logic function

CA rules

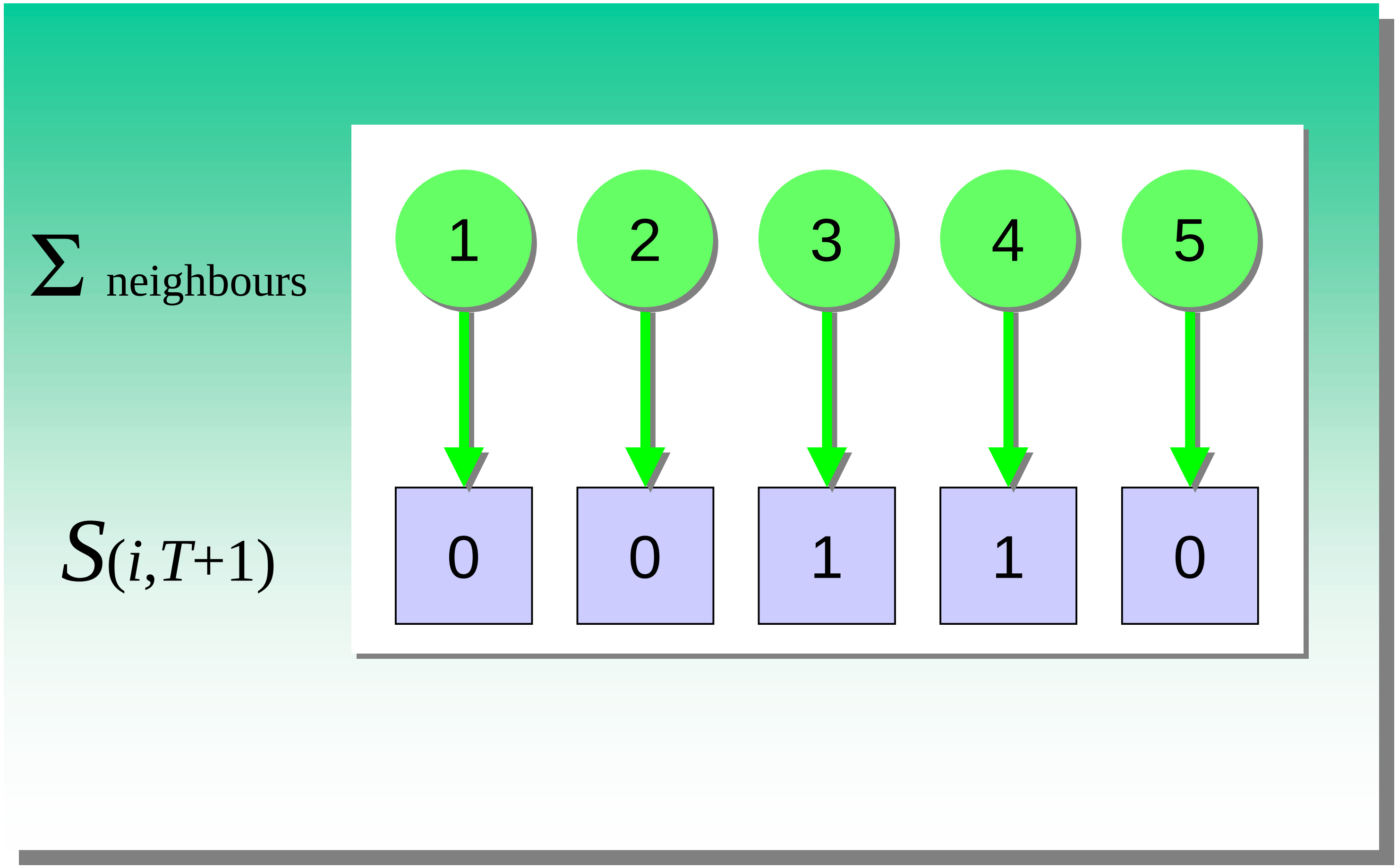


$\Sigma \in \{1,2,3,4,5\}$

$S \in \{0,1\}$

→ $2^5 = 32$ different rules

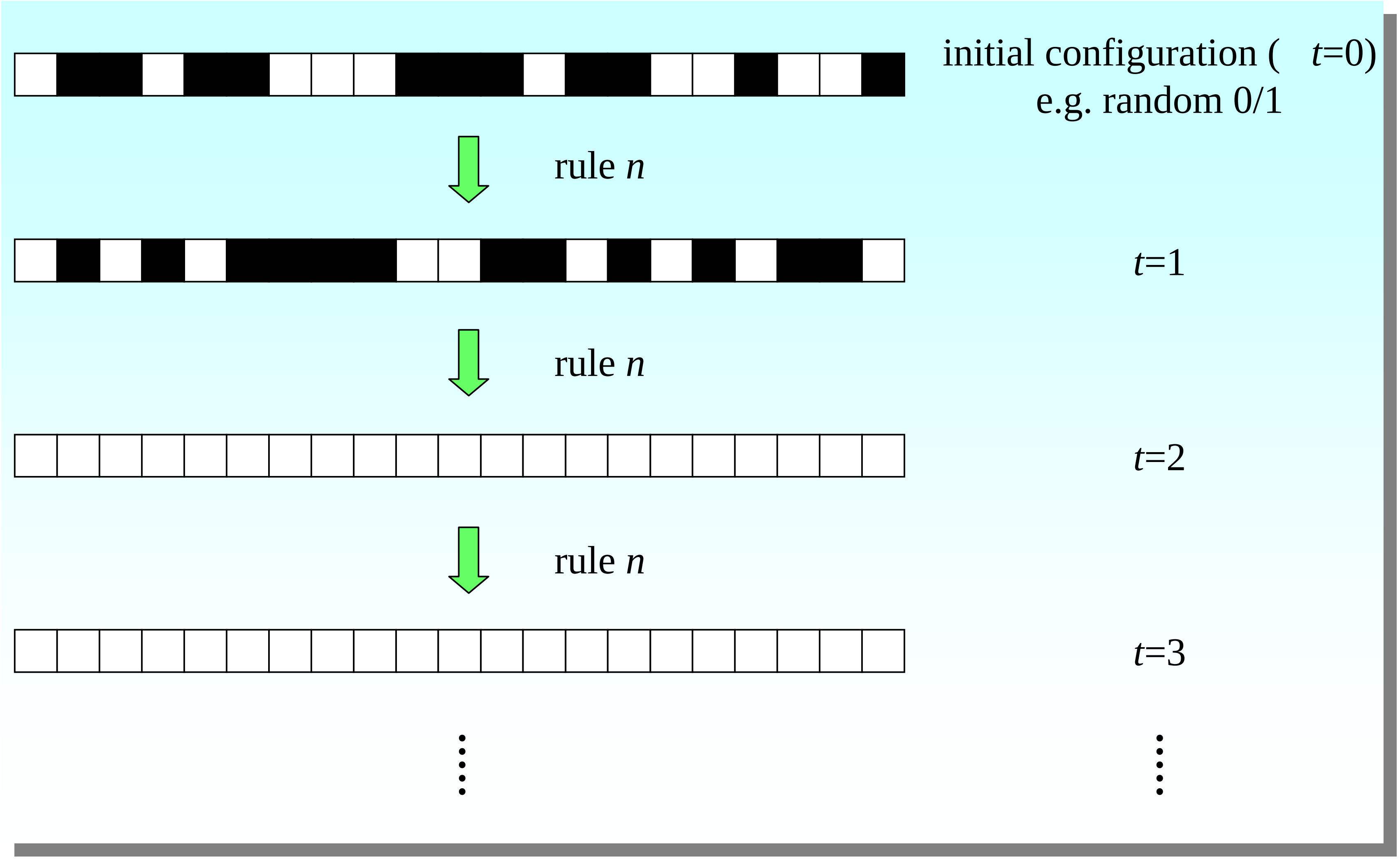
Code for rule 6 (00110)



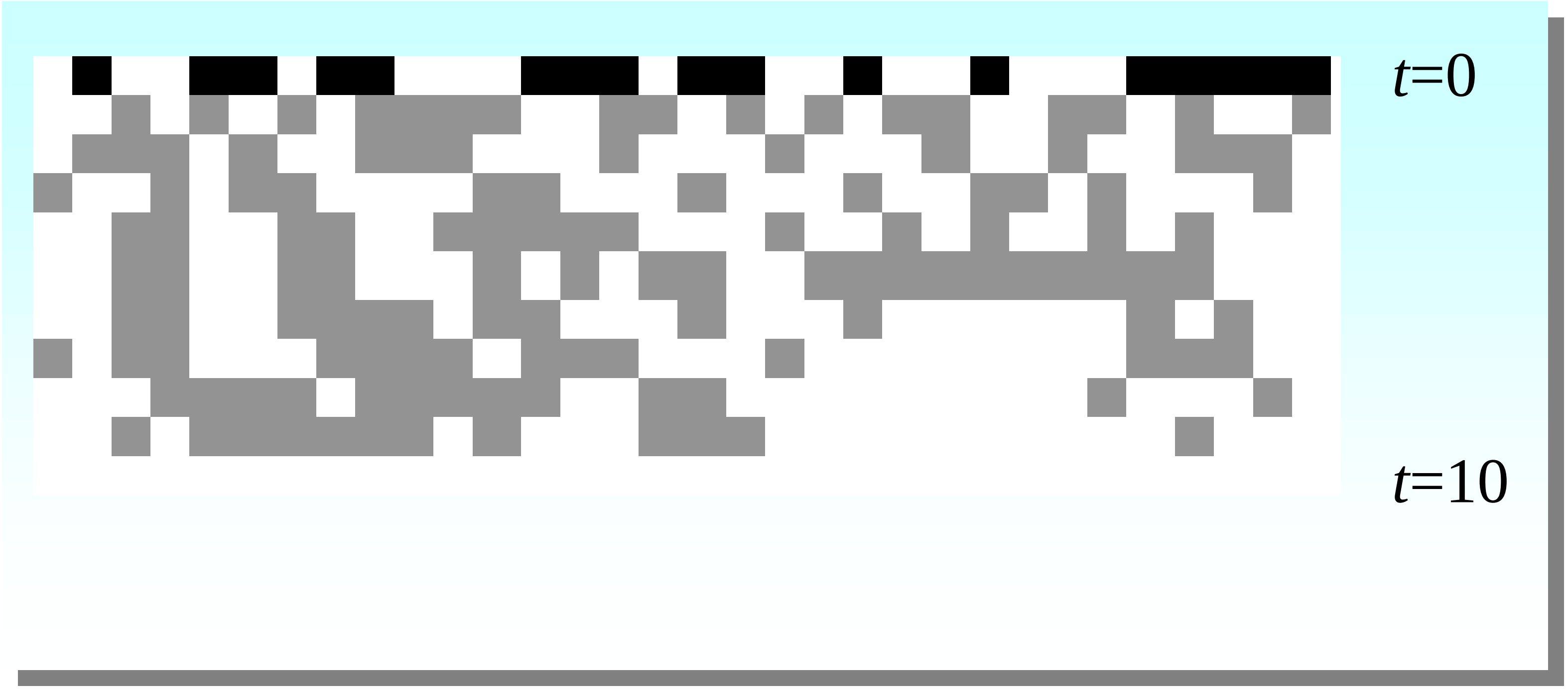
Rule codes

0	0	0	0	0	rule 0
0	0	0	0	1	rule 1
0	0	0	1	0	rule 2
0	0	0	1	1	rule 3
0	0	1	0	0	rule 4
0	0	1	0	1	rule 5
⋮					⋮
1	1	1	1	1	rule 32

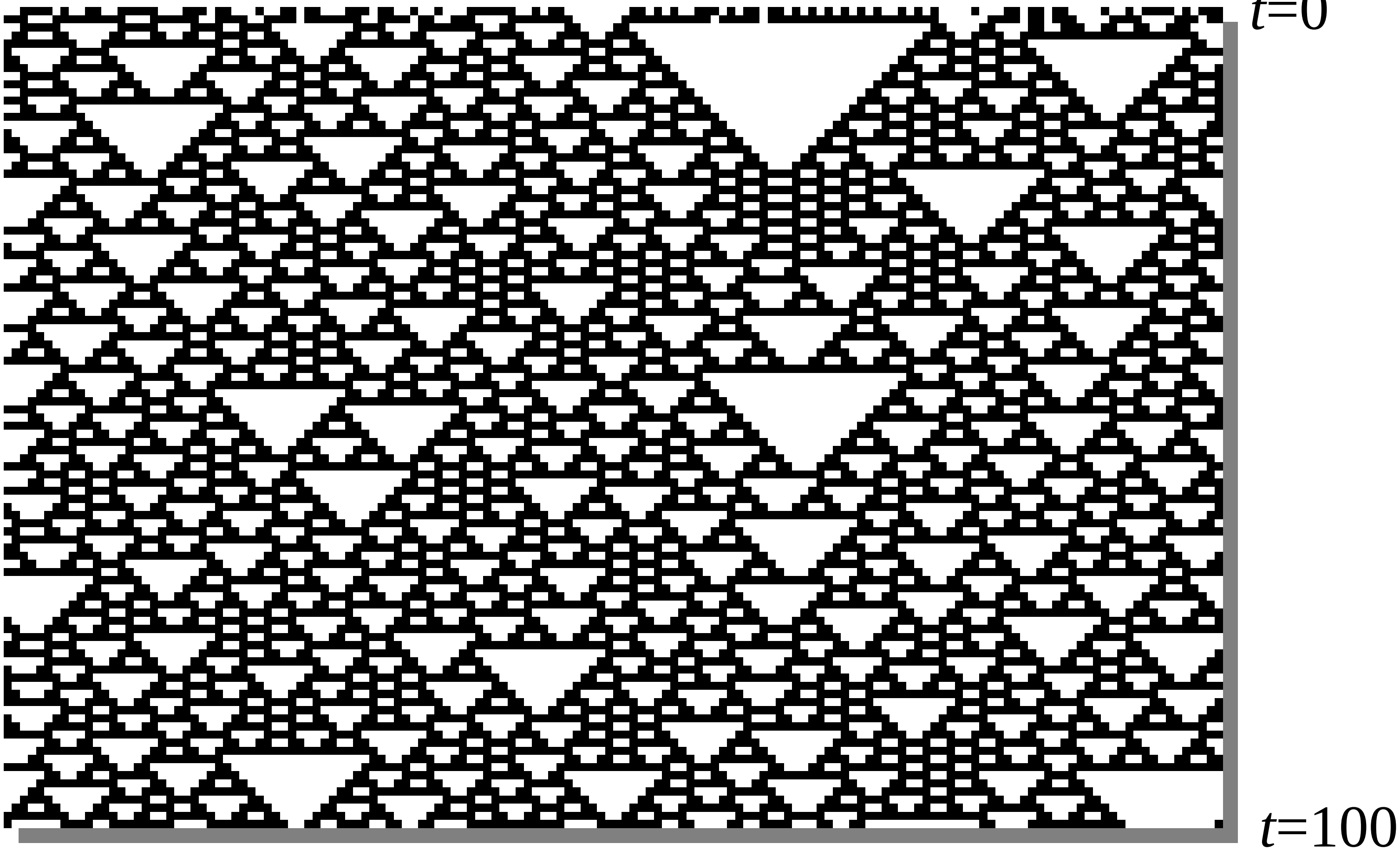
Temporal evolution



Temporal evolution

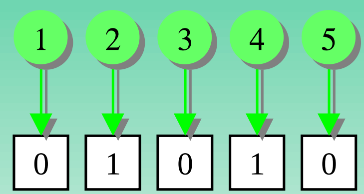


Rule 6: 00110



Rule 10: 01010

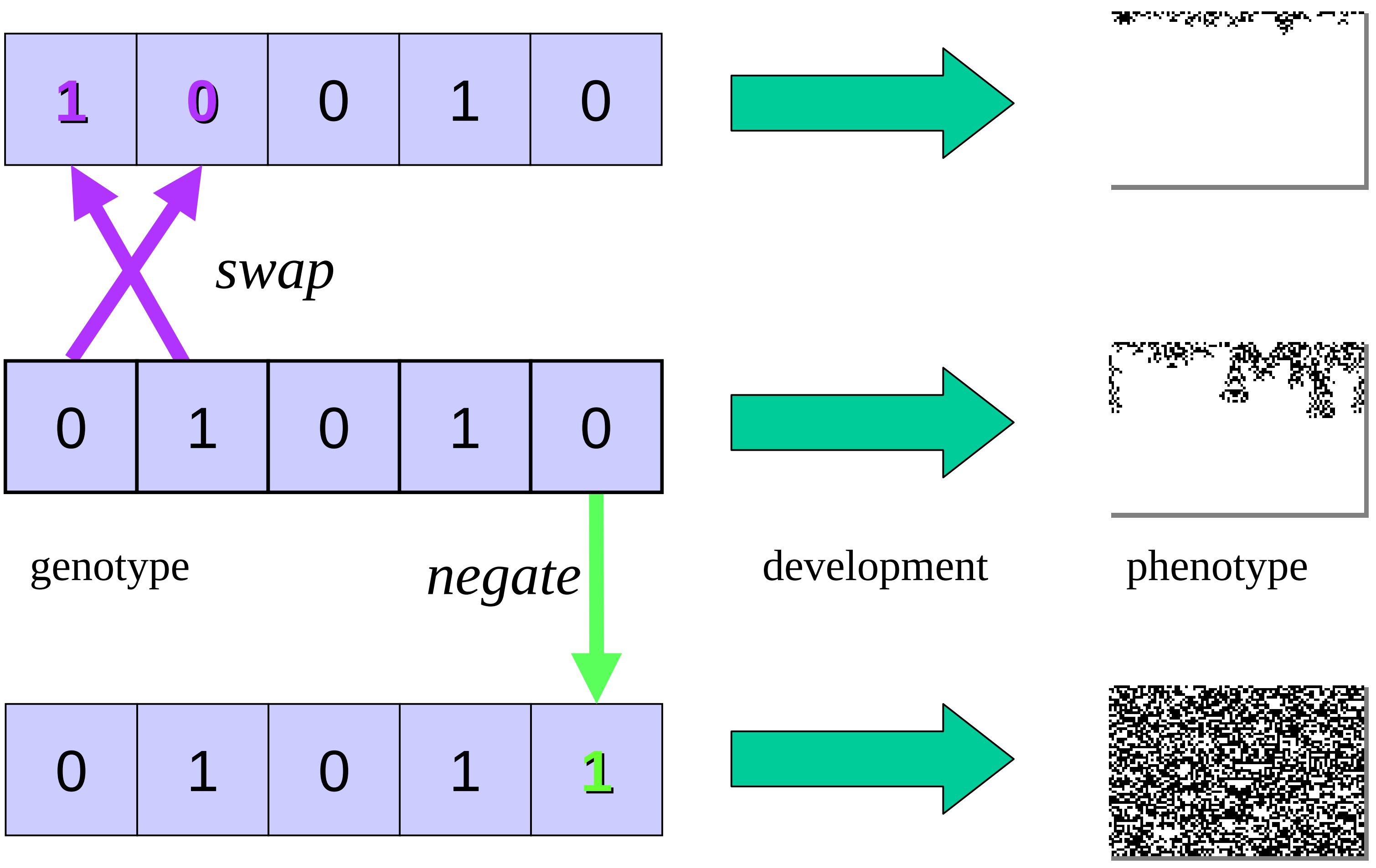
CA: a metaphor for morphogenesis

static binary pattern	0 1 0 1 0	genotype
translated into state transition rule		translation, epigenetic rules
rule iteration in configuration space		morphogenesis of the phenotype

CA: a model for morphogenesis

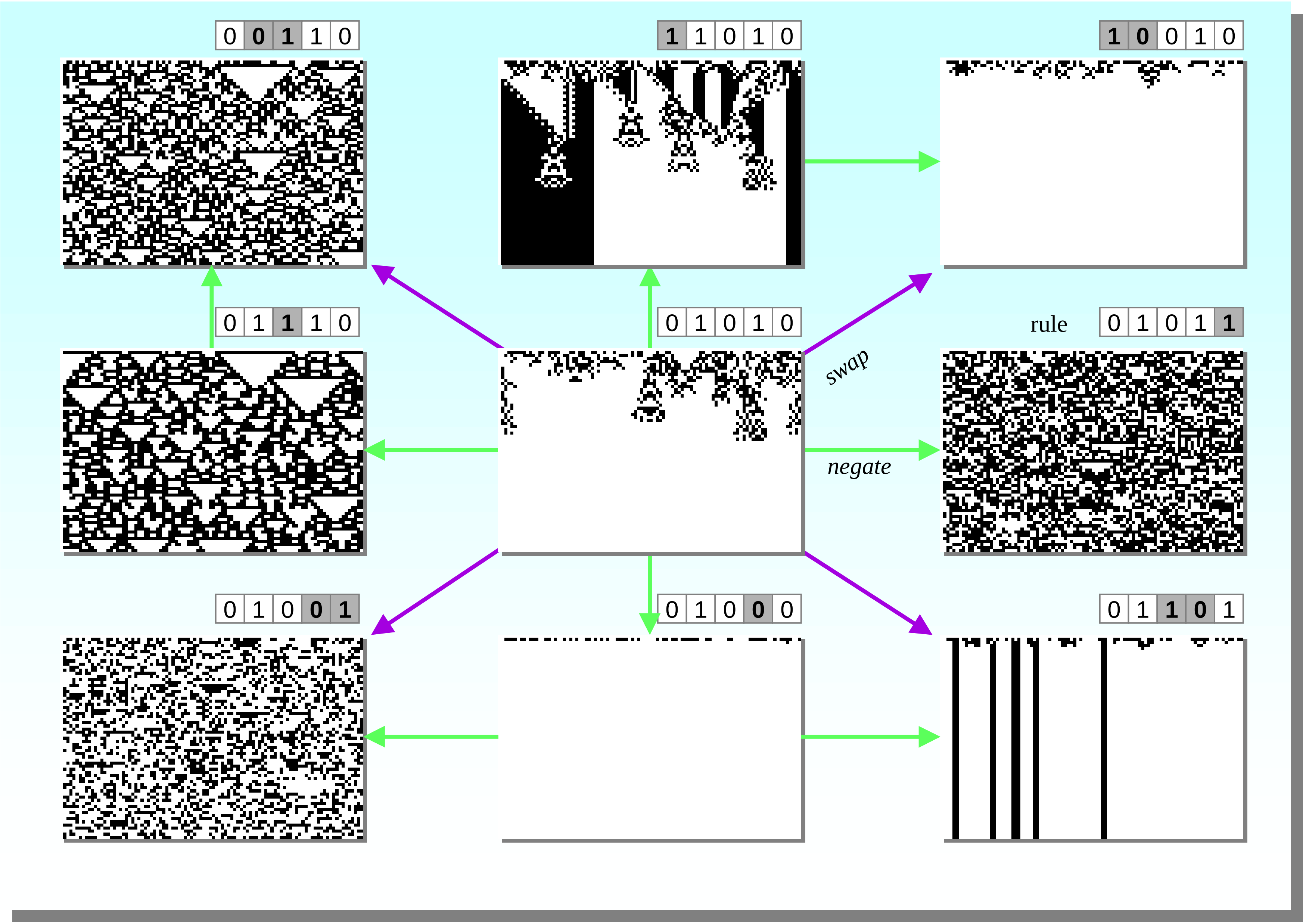
update rule	genotype
iterative application of the update rule	epigenetic interpretation of the genotype, morphogenesis
CA state pattern	phenotype
state space	morphospace

CA rule mutations

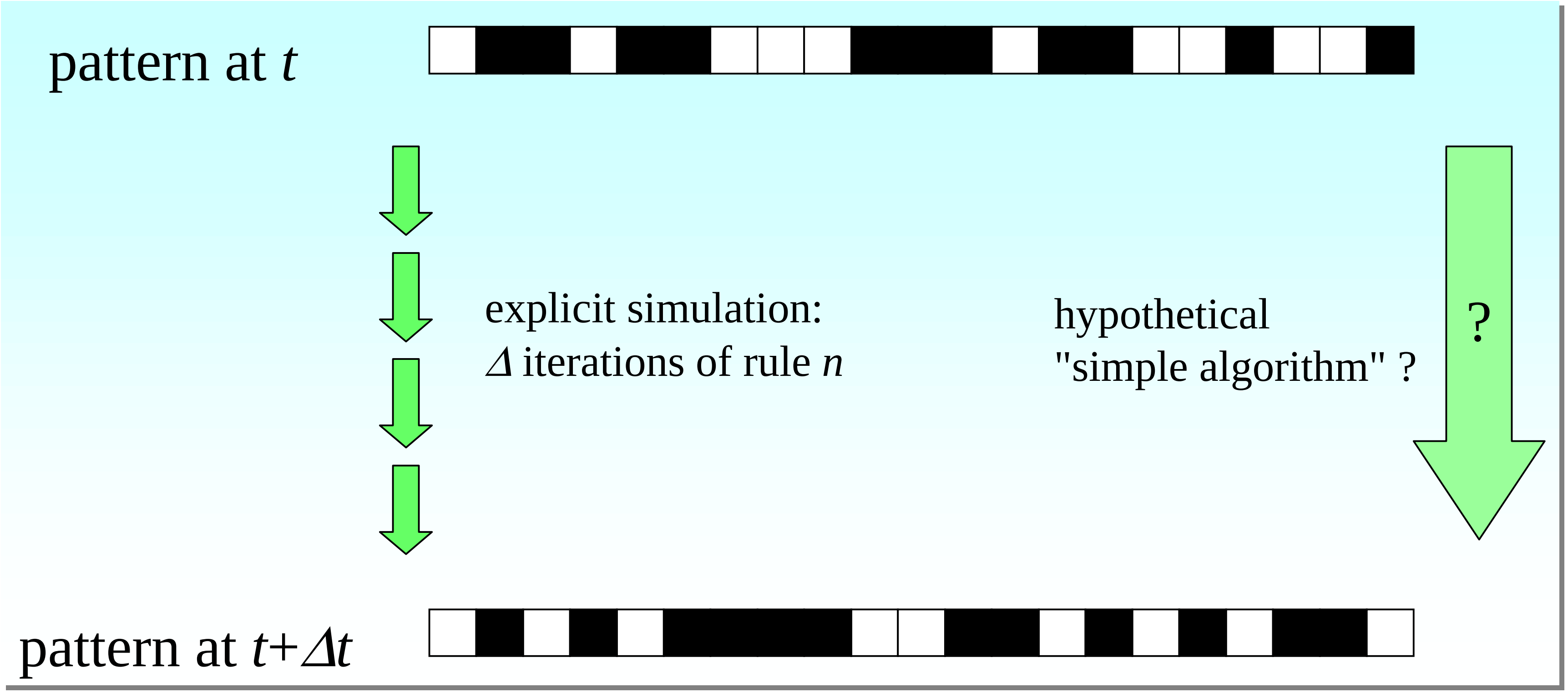


How genotype mutations can change phenotypes?

CA rule mutations



Predictability?



How to predict a next element in sequence? **Tough!**

Predictability

Sum of natural numbers 1… *N*

$$\sum_{1}^{N} n$$

algorithm 1:

1+2+…+N

algorithm2:

$N(N+1)/2$

(Gaussian formula)

***N*th prime number**

$$p_N \in \{p_n\}$$

$$= \{p \mid p/i \notin \mathbf{N}; i \in]1, p[\}$$

algorithm 1:

trial and error

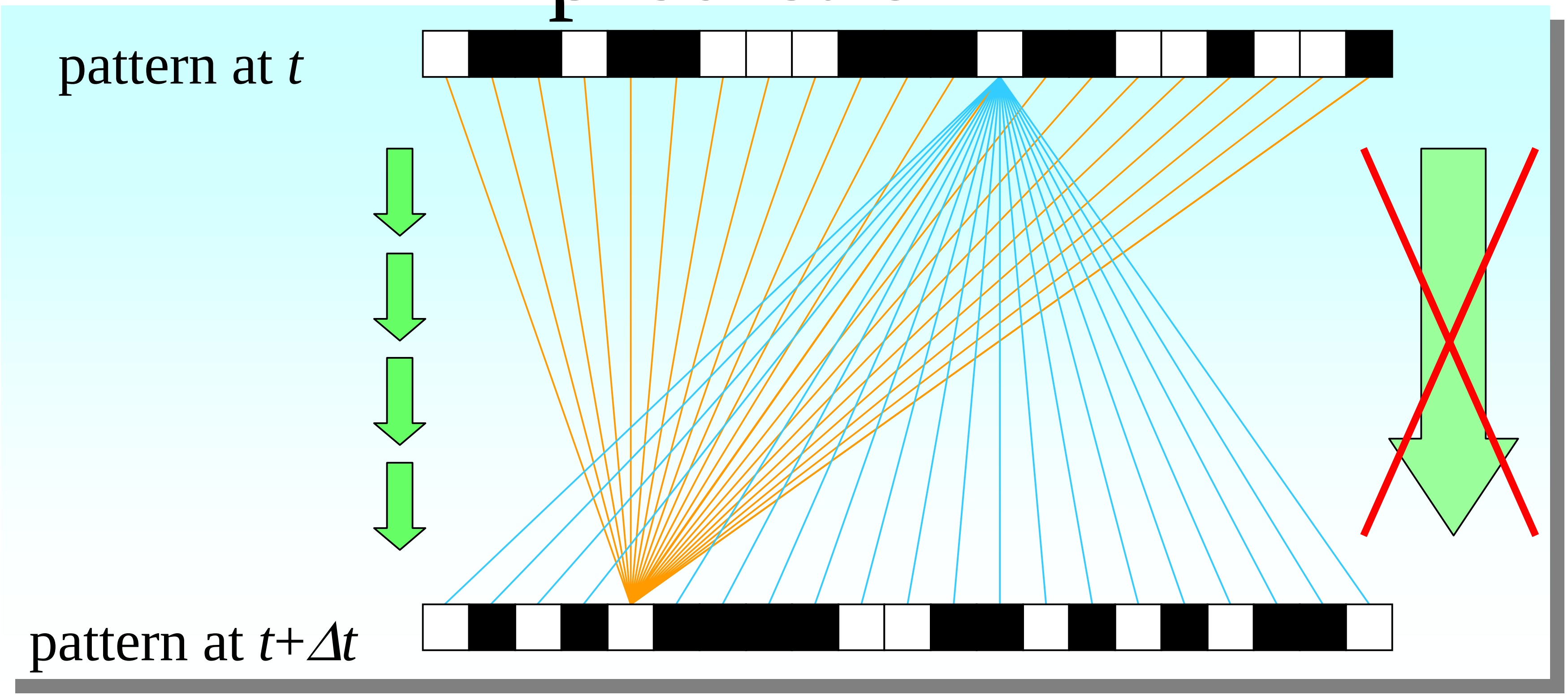
algorithm2: ?

no general formula

easy

Very tough!

CA: no effective pattern prediction



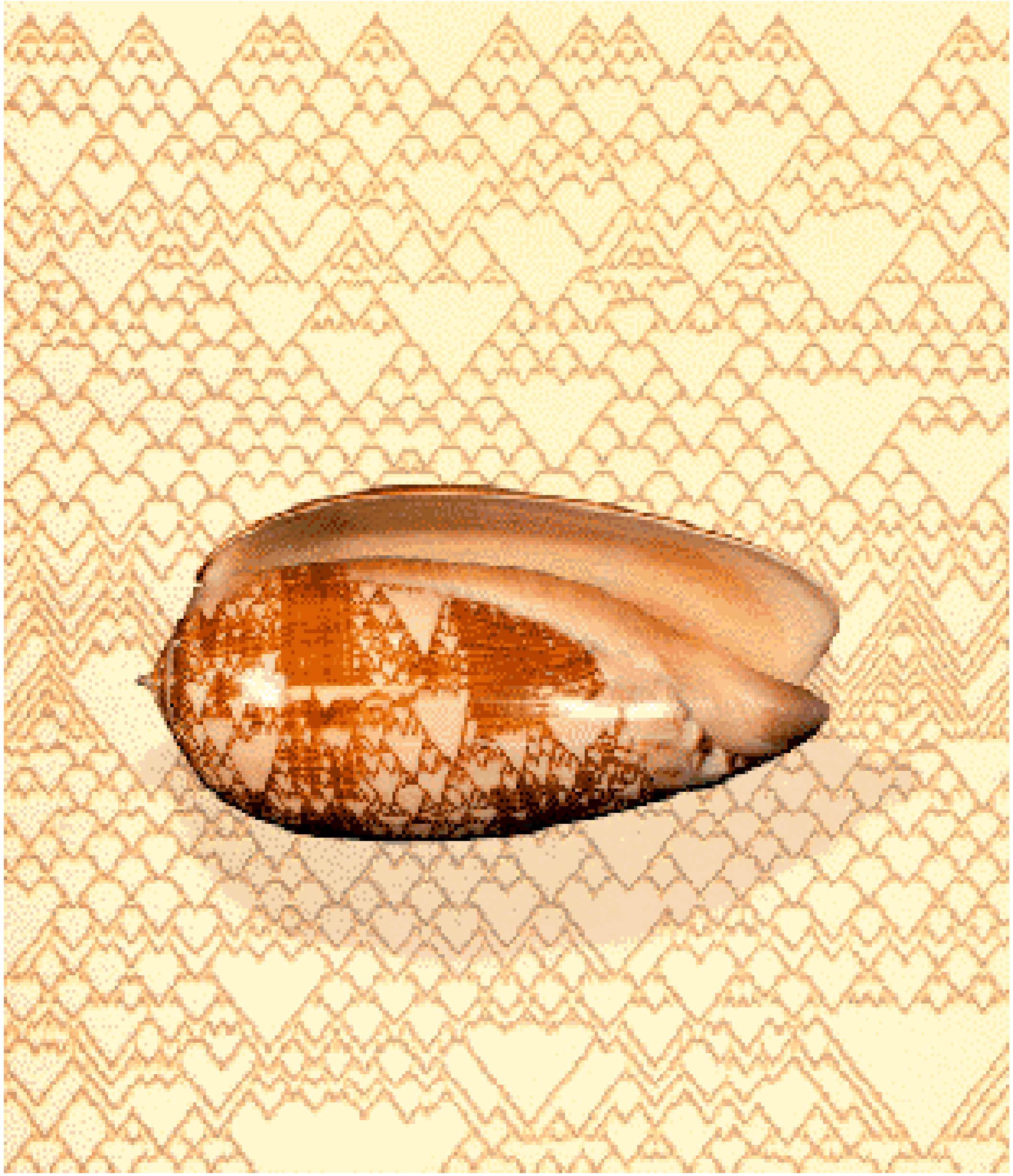
behaviour may be determined only by explicit simulation

CA: deterministic, unpredictable, irreversible

- Simple rules generate complex spatio-temporal behavior
- For non-trivial rules, the spatio-temporal behaviour is computable but not predictable
- The behavior of the system is irreversible
- Similarity of rules does *not* imply similarity of patterns

Aspects of CA morphogenesis

- complex relationship between „genotype“ and „phenotype
- effects of „genes“ are not localizable in specific phenes (pleiotropy)
- phenes cannot be traced back to specific single genes (**epistasis**)
- phenetic effects of "mutations" are not predictable



Meinhardt, H. (1995). *The Algorithmic Beauty of Sea Shells*. Berlin: Springer

Patterns and morphology

- Pattern: a spatially and/or temporally ordered distribution of a physical or chemical parameter
- Pattern formation
- Form (Size and Shape)
- Morphogenesis: The spatiotemporal processes by which an organism changes is size (growth) and shape (development)

Where is the phene?

- Typification?
- Comparative measures?
  - length
  - density
  - fractal dimension
- Spatio-temporal development?

phenotype A

phenotype B