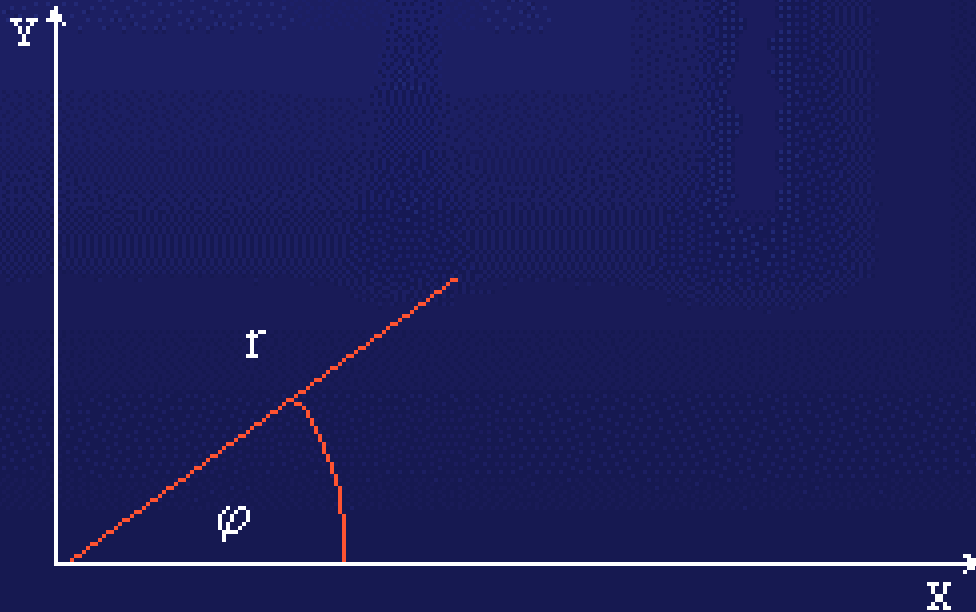


Howgh Transform

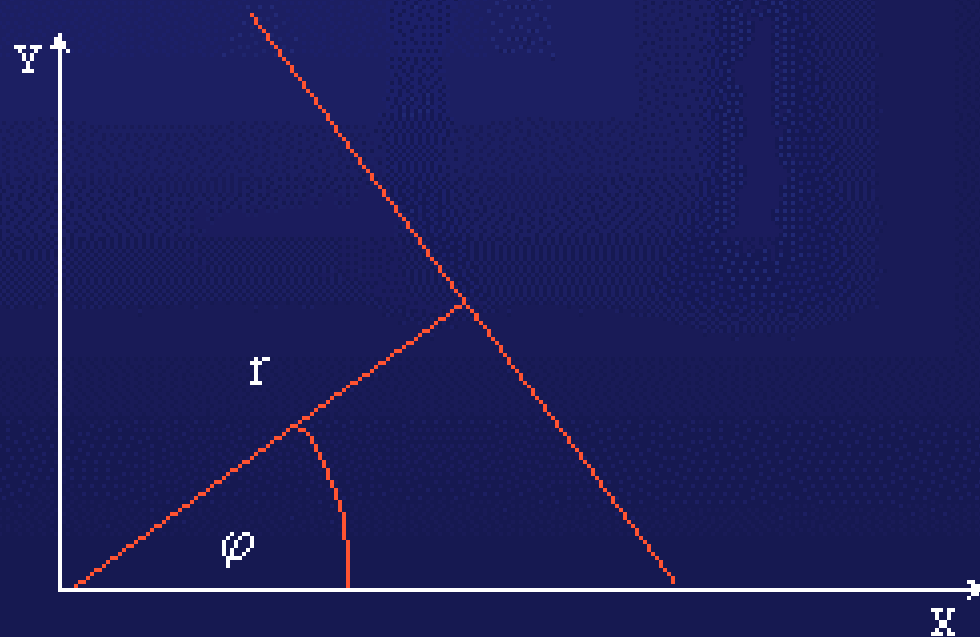
Let us go deeper to understand the principles of creating any kind of Hough Transforms.....

SHT: How it Works

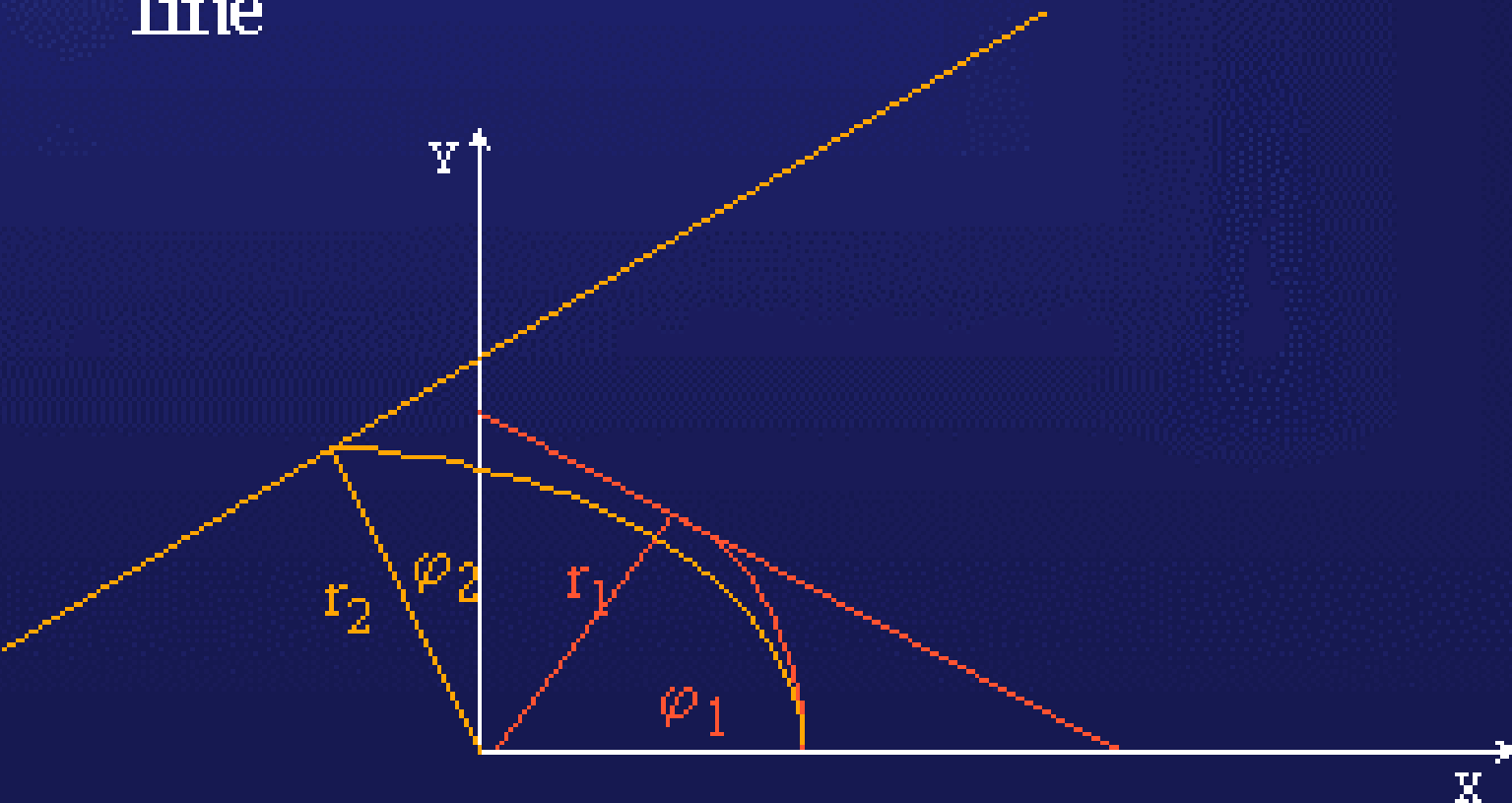
- Usually we are given (r, φ)



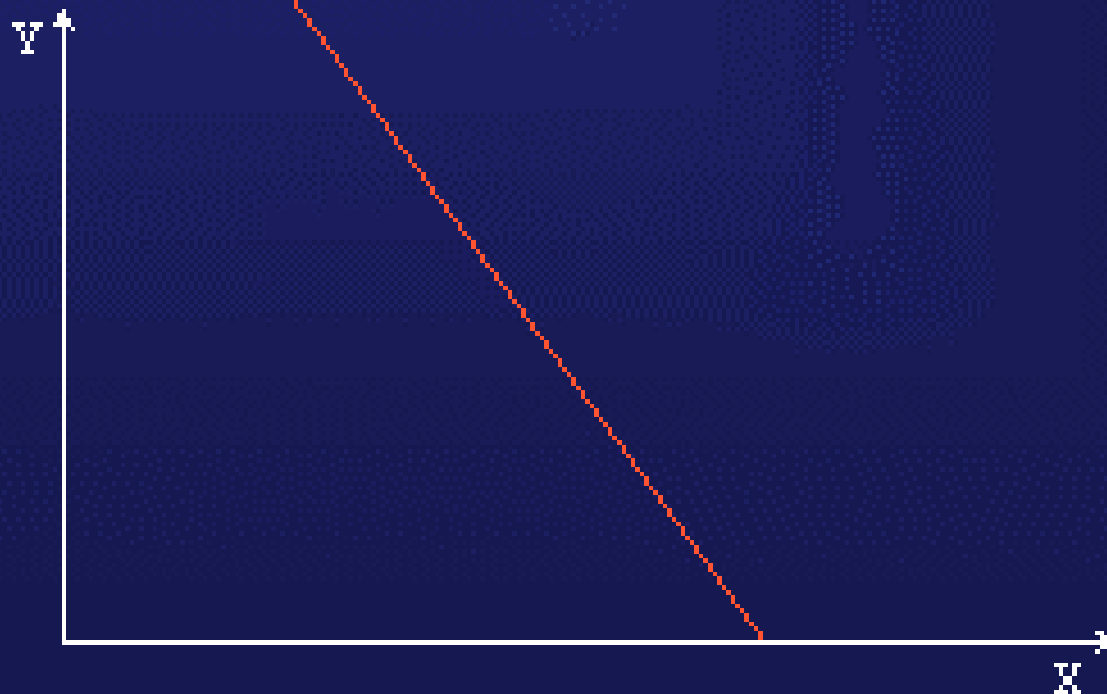
- Usually we are given (r, φ) and must calculate all (x, y) points



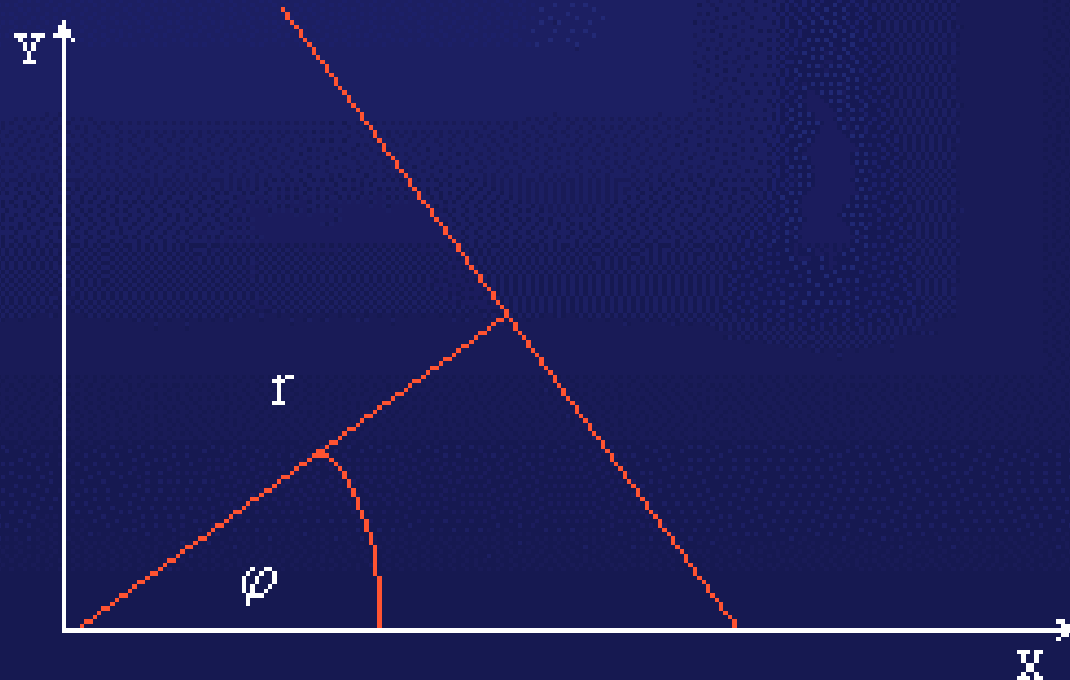
- Therefore given different (r, φ) we get different lines i.e. (x, y) points i.e. (r, φ) parametrise the line



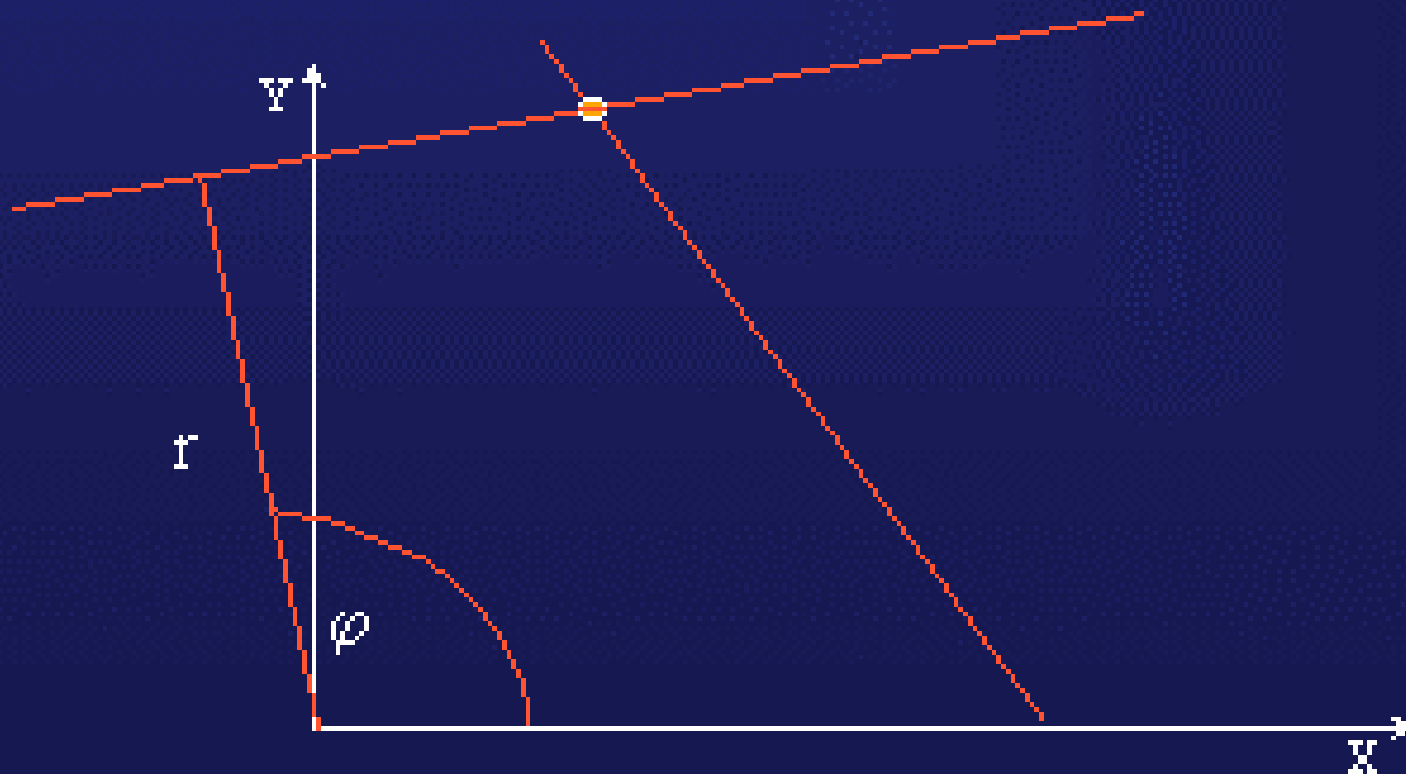
- But from an image analysis point of view we already have (x,y) points



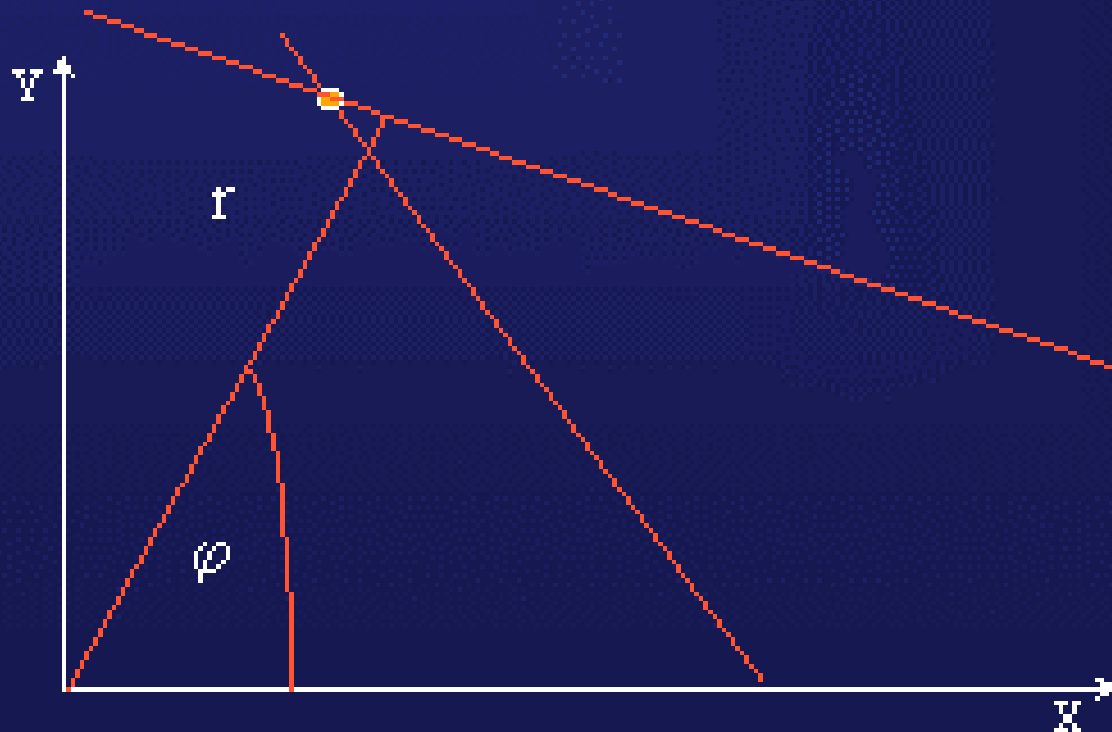
- But from an image analysis point of view we already have (x,y) points and we want (r,φ)



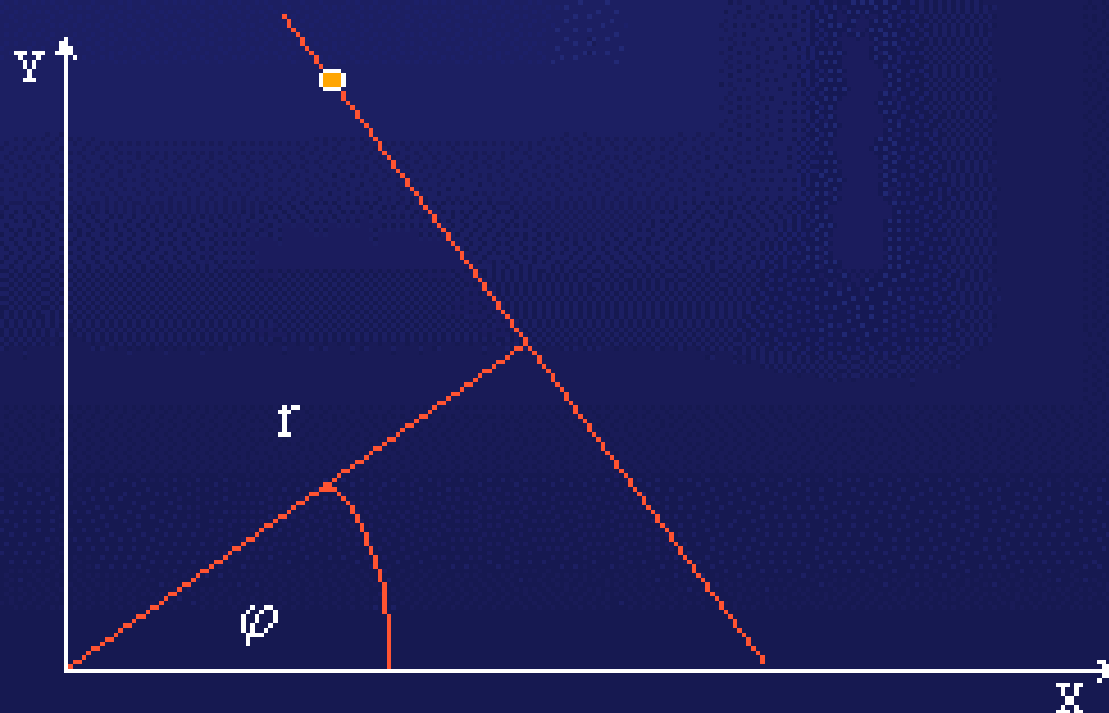
- For each (x,y) point we generate all possible (r,φ) i.e. r and φ are now the variables



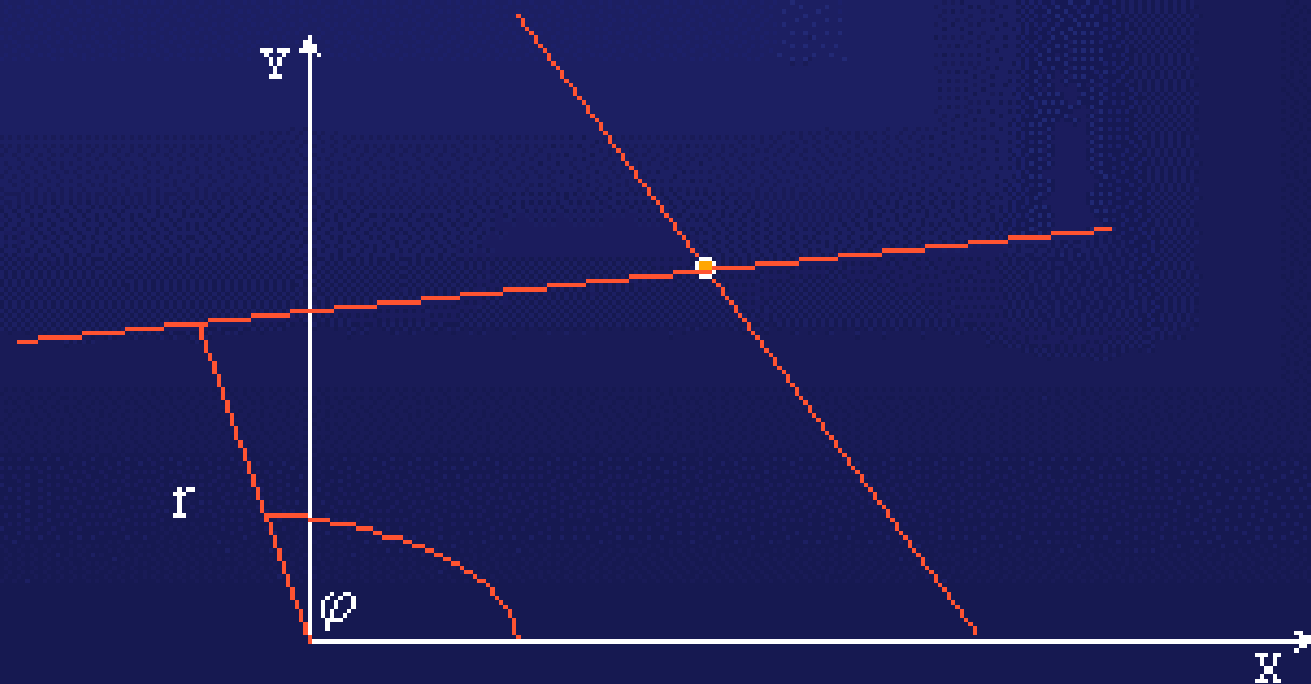
- That is to say that for each point we ask, what are all the possible line passing through this point



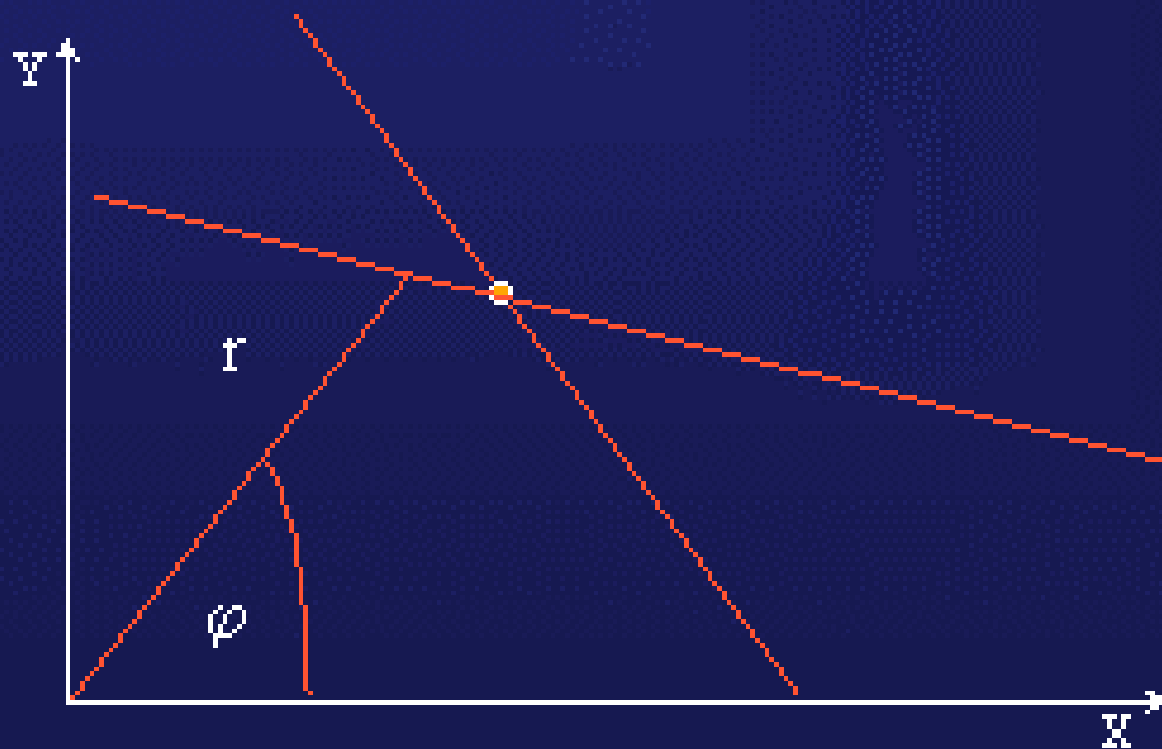
- For each possible line we calculate, we take note of it (i.e its (r, φ) point).



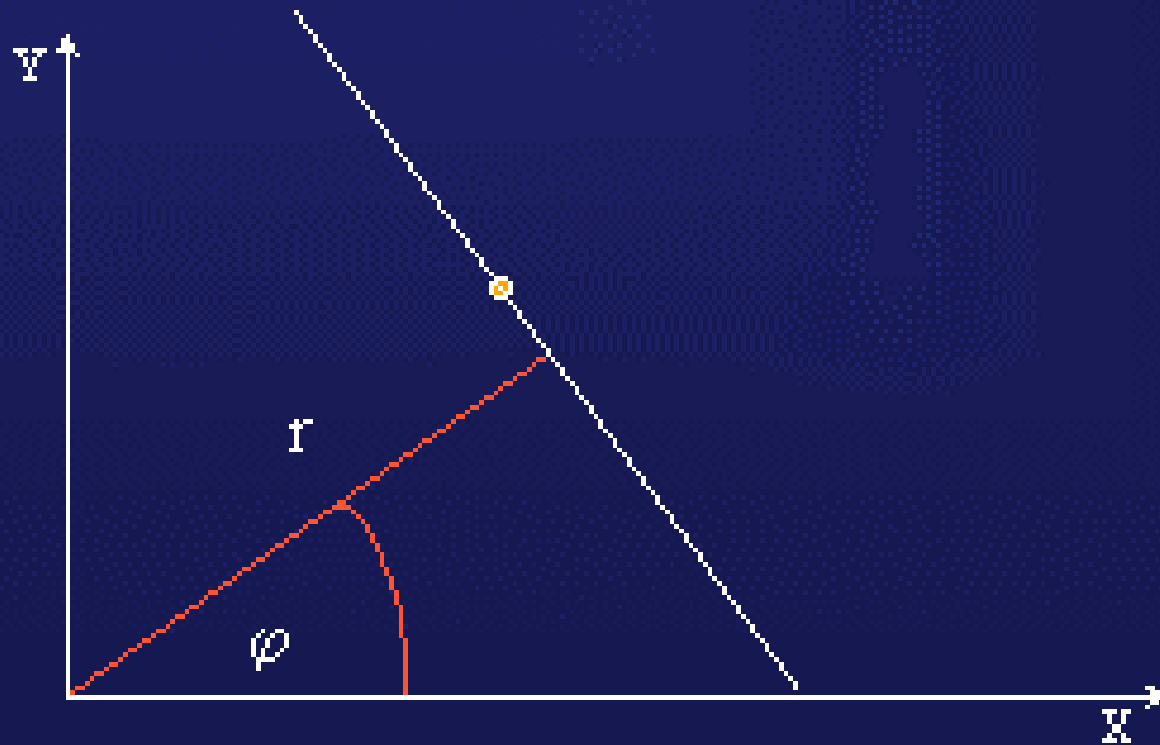
- At the end of this process we have a list of (r, φ) points for each (x, y) point



- If a set of (x,y) points are co-linear there will be specific (r,φ) point common to them all

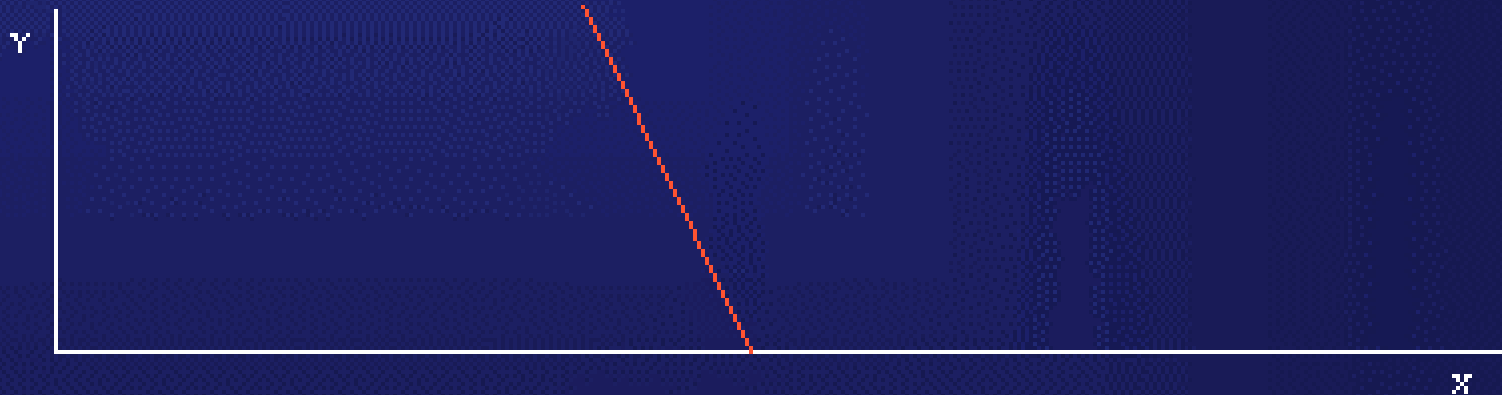


- Isolating these common (r, φ) points then gives us the lines in the image

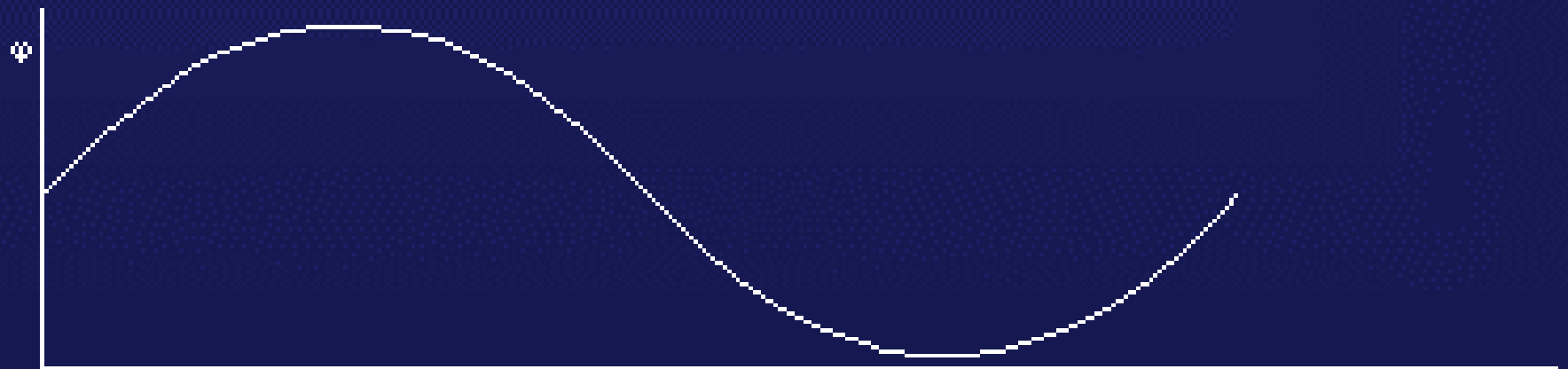
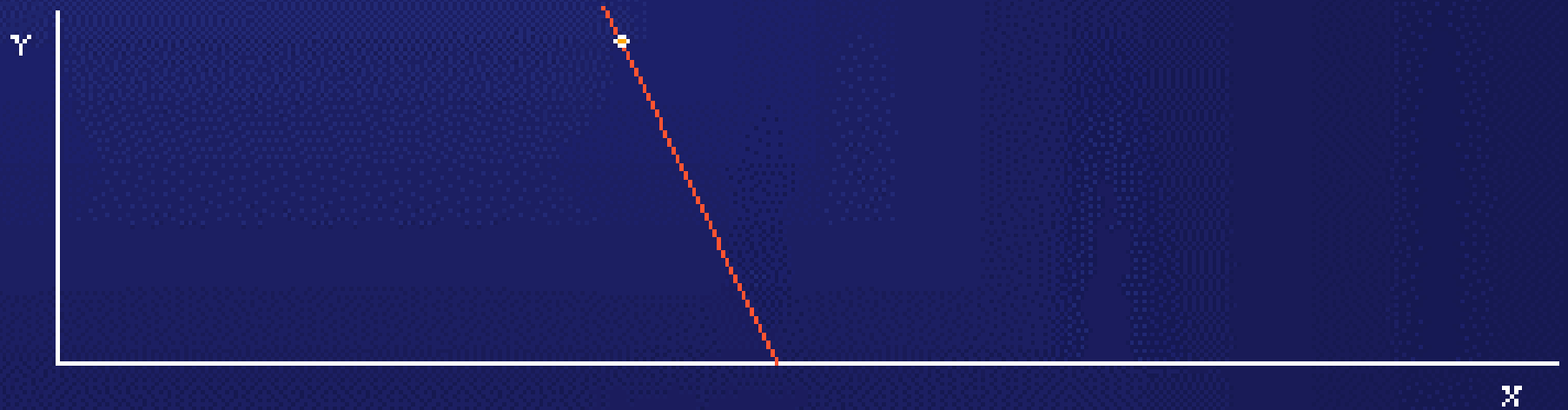


SHT: Another Viewpoint

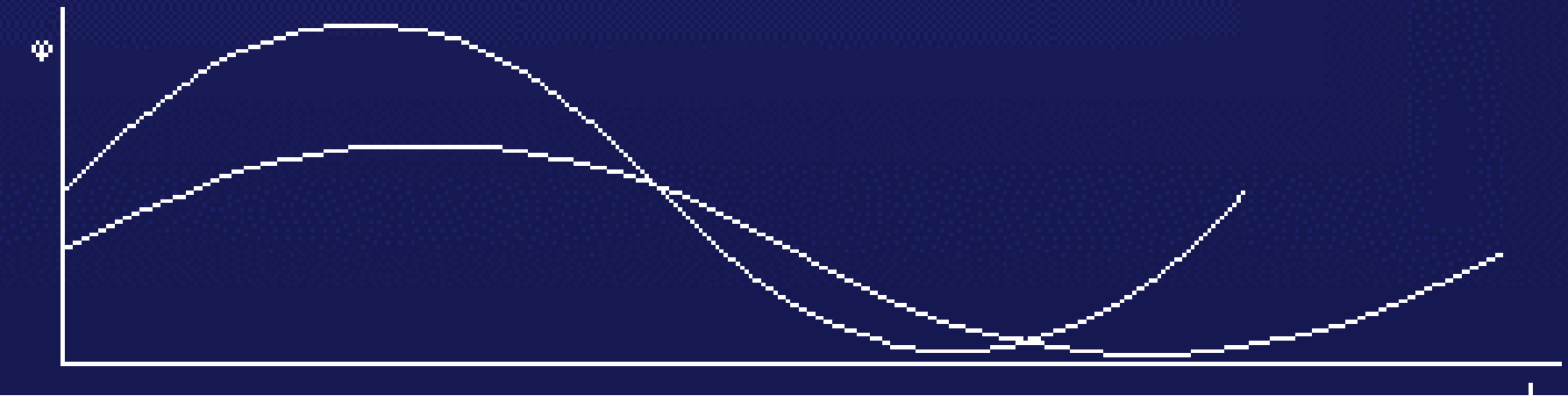
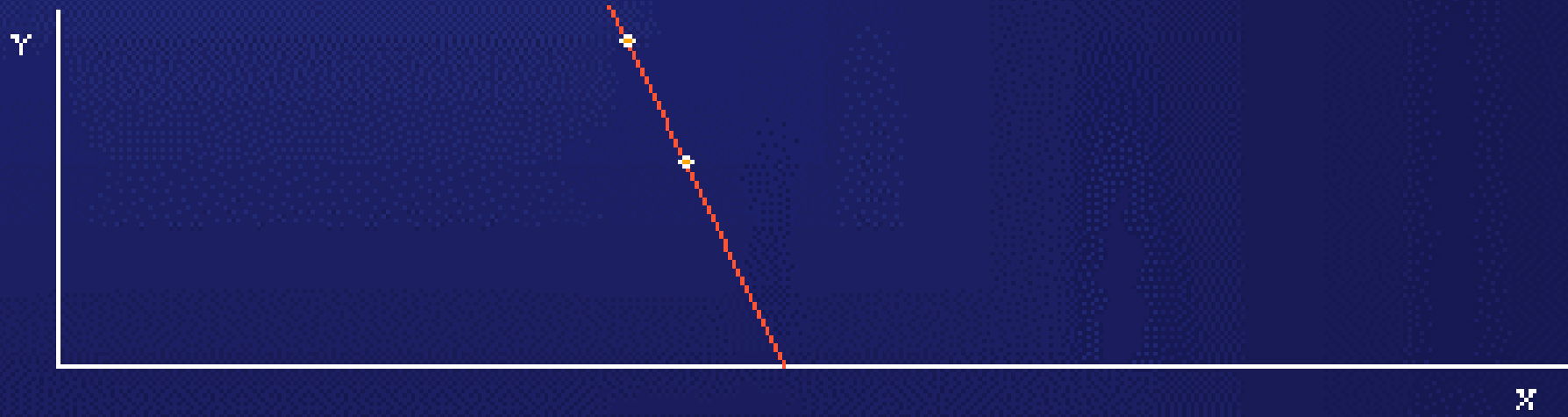
- Since any (r, ϕ) represents a point in parameter space we may plot all possible (r, ϕ) points for each (x, y) point



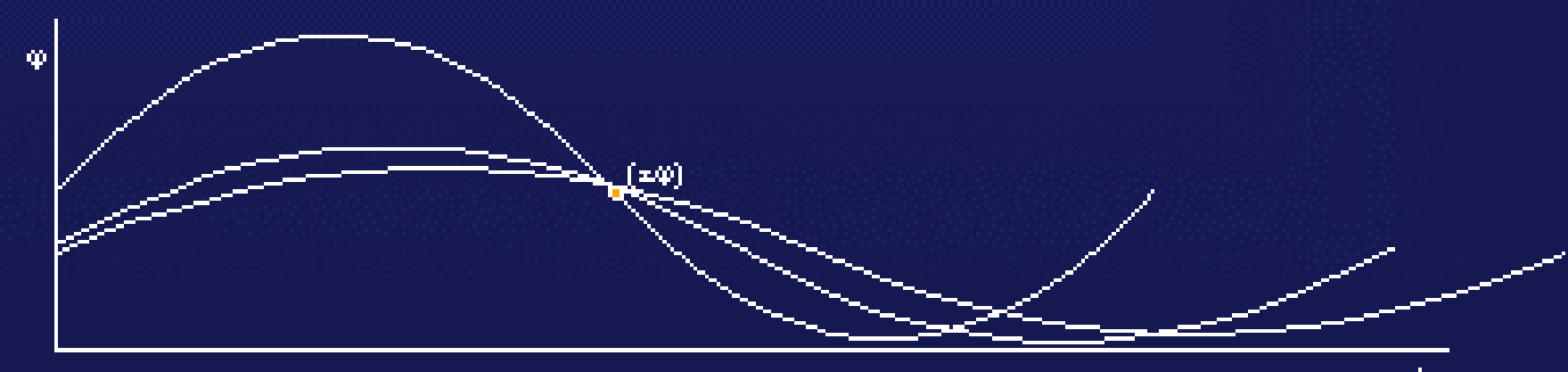
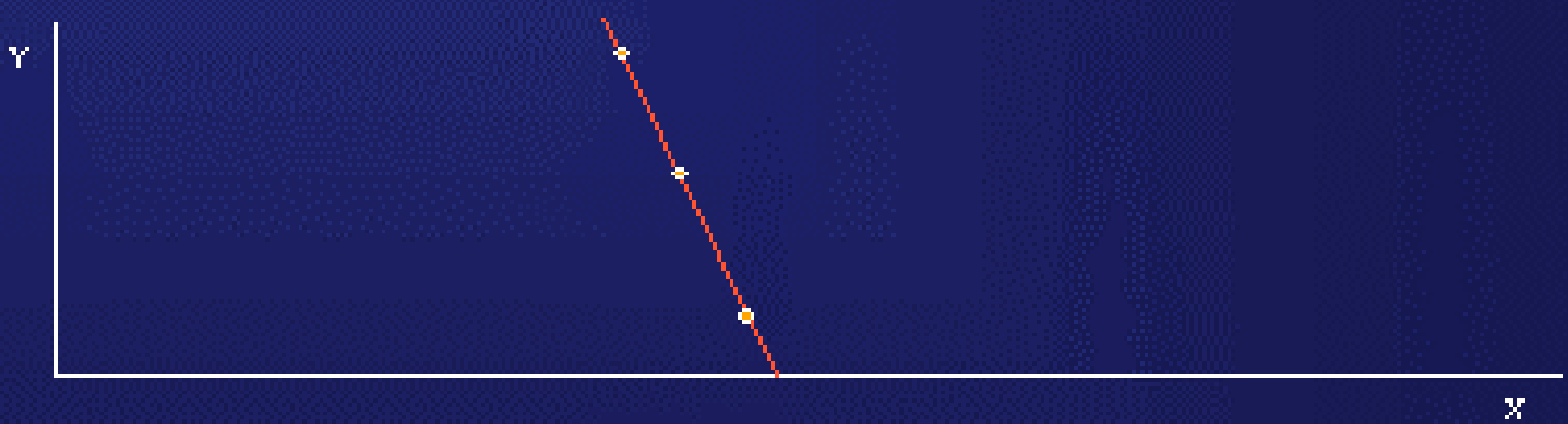
- Taking any particular (x,y) point its set of (r,ϕ) points is a sinusoid through parameter space



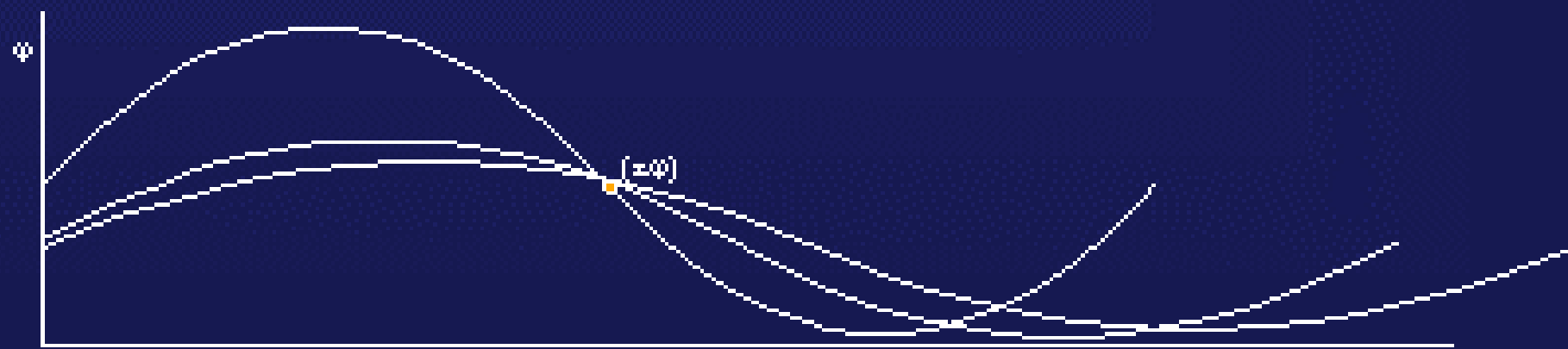
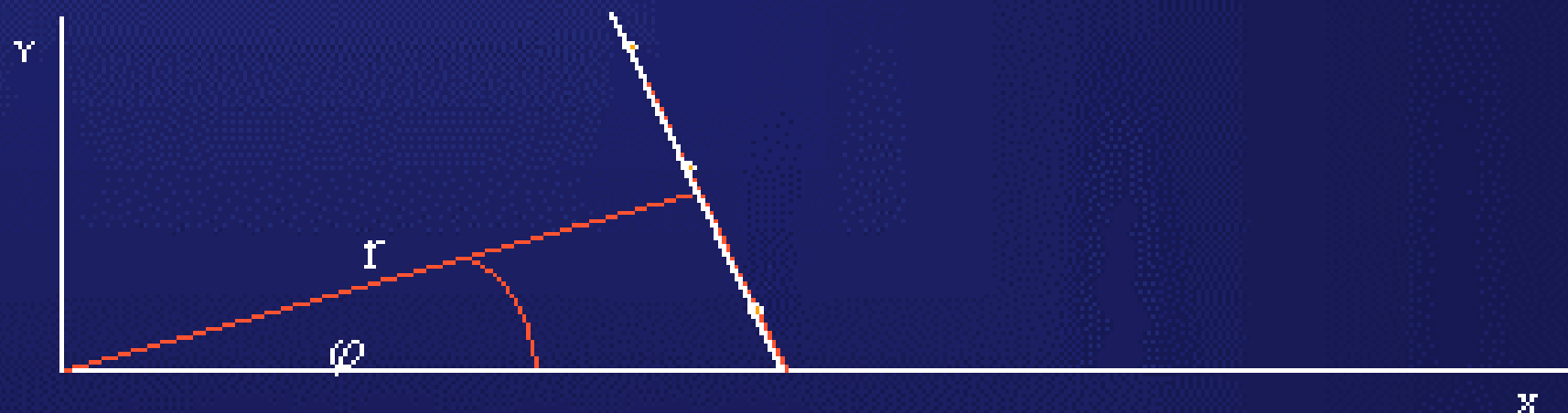
- Each sinusoid tells us every possible line that can pass through the corresponding (x,y) point



- The line upon which the (x,y) points lie is then given by the (r,φ) pair on which all the sinusoids agree (i.e. intersect)

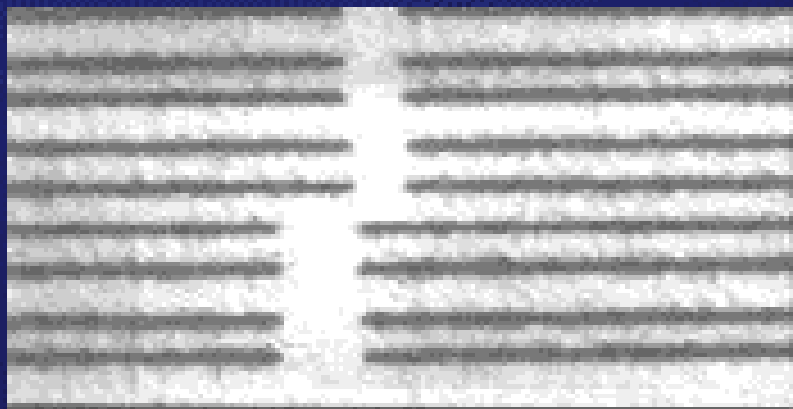


- Finding this intersection we then have the required (r, φ) parameters and therefore the line in the image

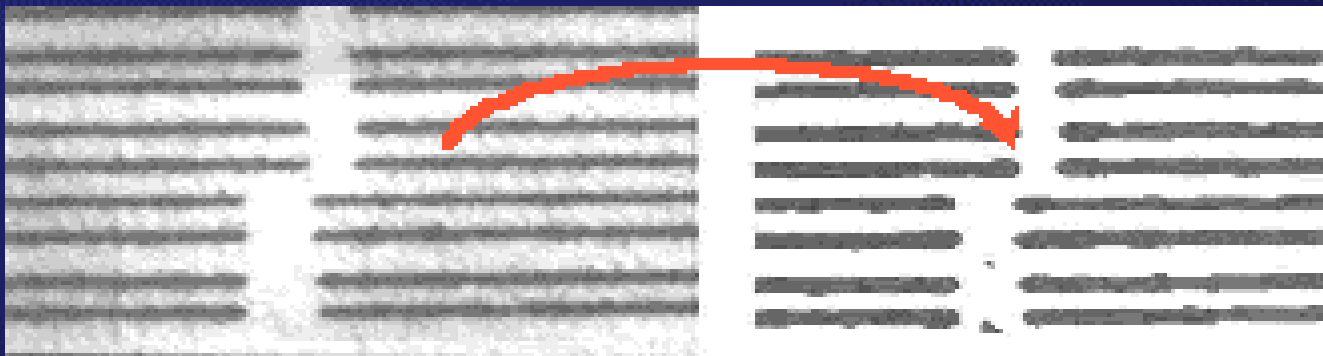


SHT : Example

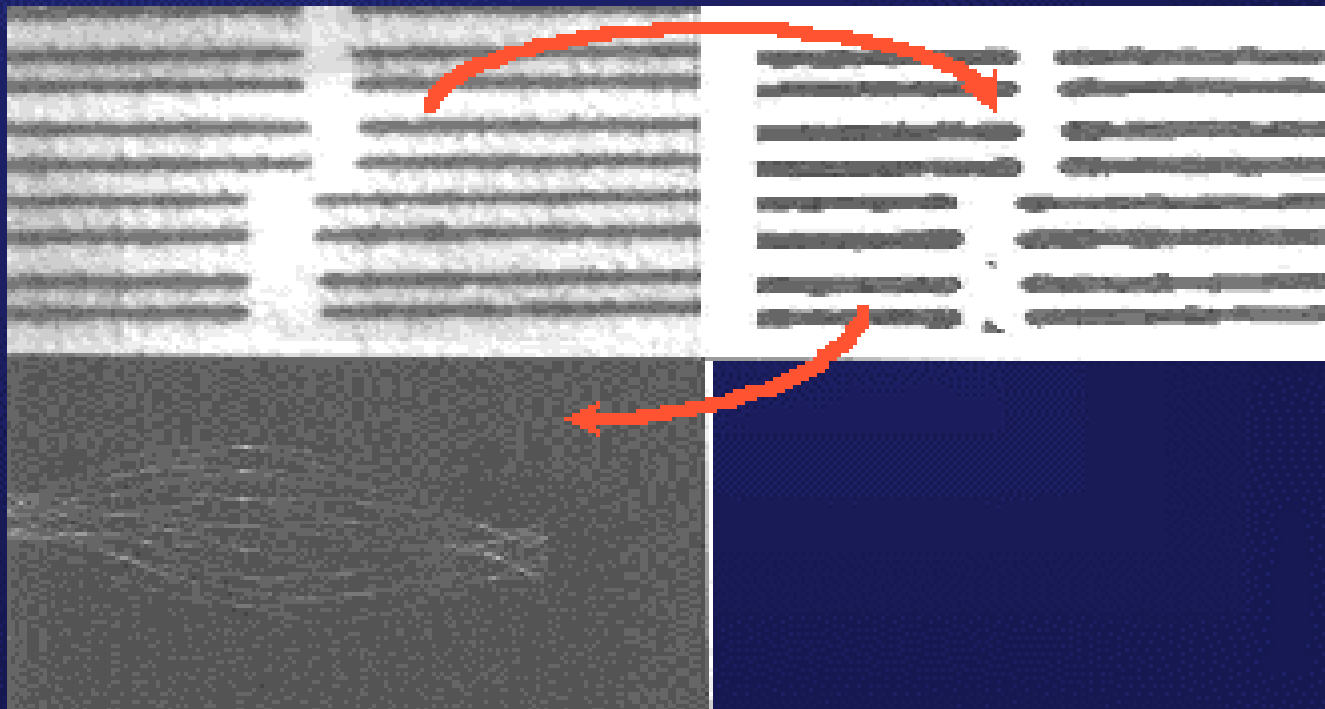
- Given a greyscale image we wish to find all occurrences of lines in it



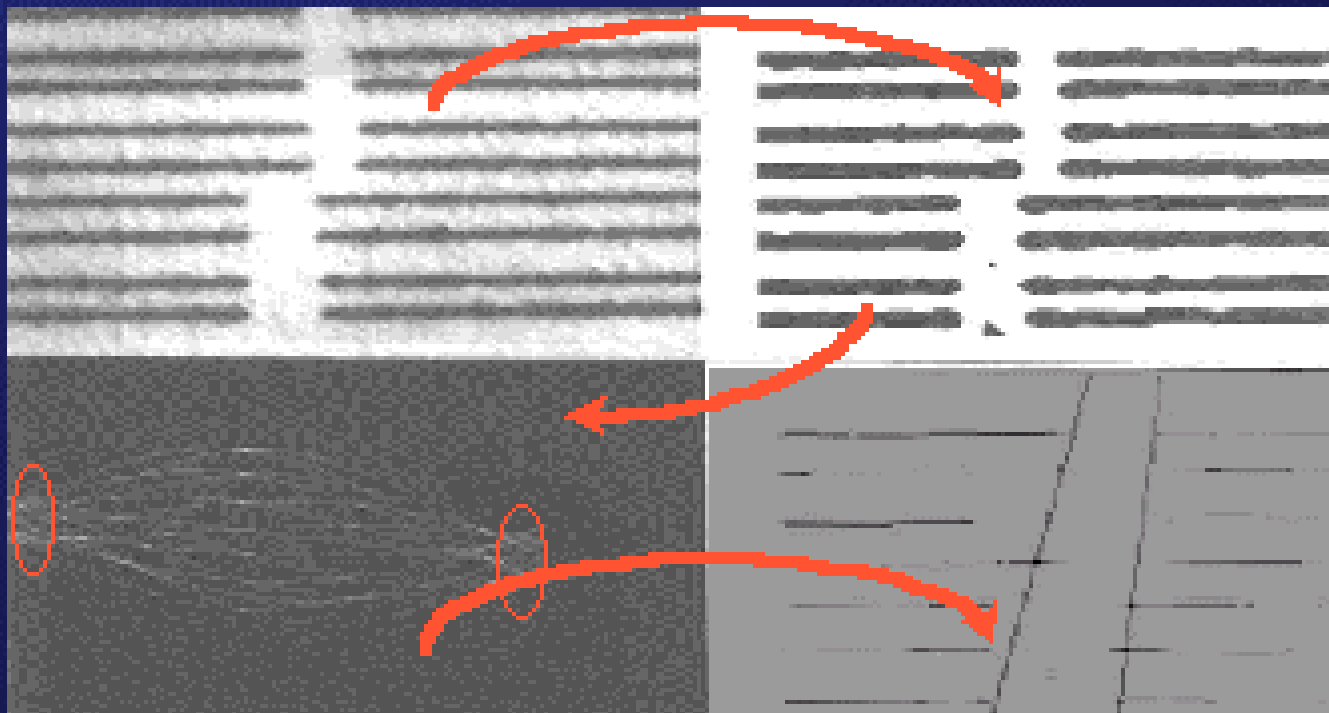
- First we threshold the image and to ease the extraction of all the edges points



- Next we calculate each edge point's corresponding sinusoid in Parameter/Hough Space



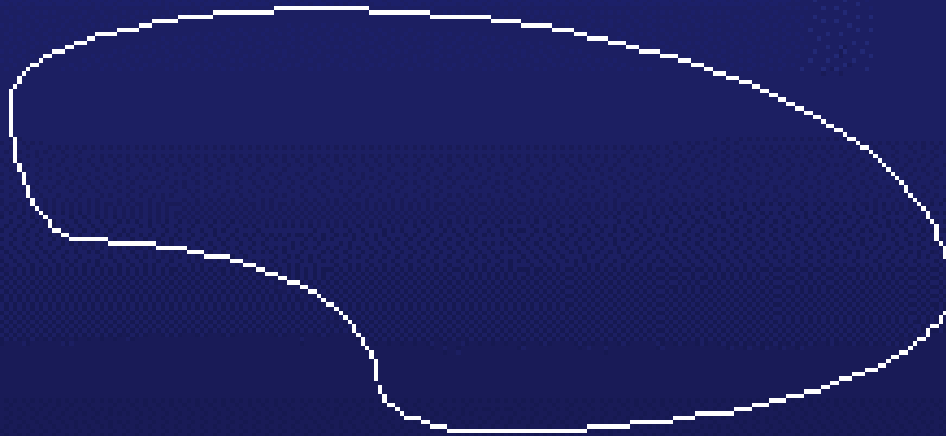
- Isolation of the intersections of the sinusoids, based on a given threshold, then yields the sought (r, ϕ) pairs



*Generalized
Hough
Transform*

SNHT: Building the R-Table

- Given a shape we wish to localise the first step is to build a look-up table, known as a R-table, which will replace the need for a parametric equation in the transform stage



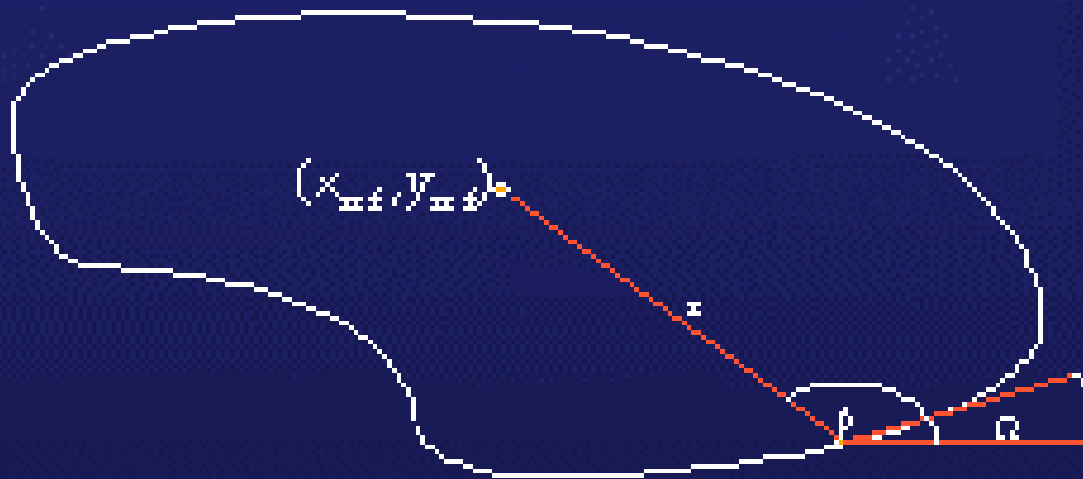
Template Object



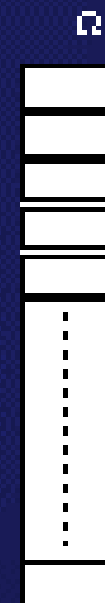
R-Table

GHT: Building the R-Table

- For each object *feature point* we calculate
 - the orientation, Ω , of the tangential line at that point and
 - the length, r , and orientation, β , of the radial vector joining the reference point and the feature point



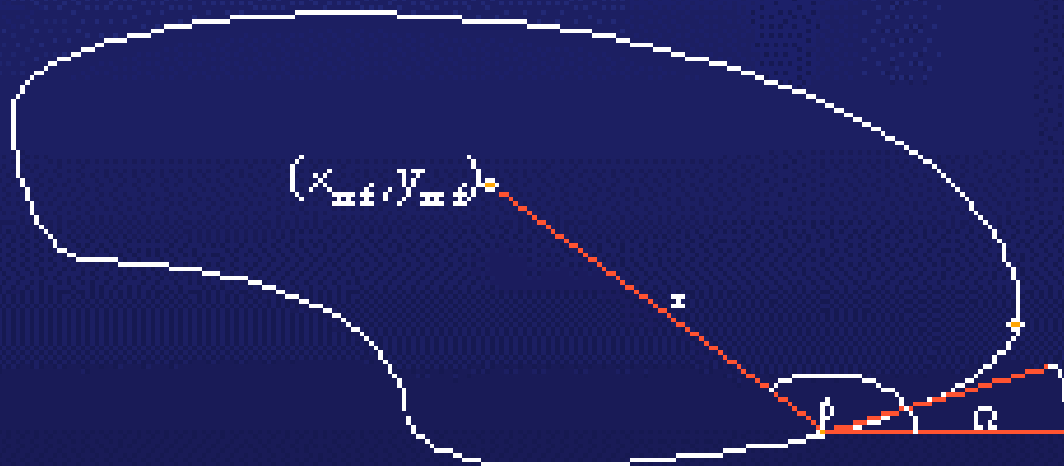
Template Object



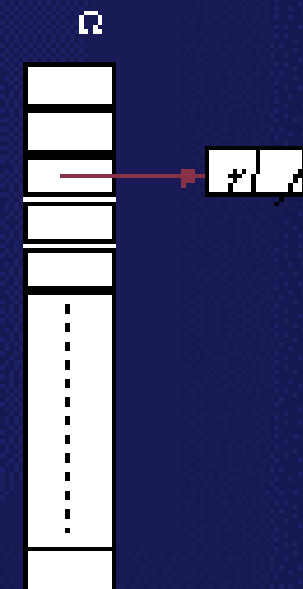
R-Table

GHT: Building the R-Table

- Using Ω as an index into the R-table we place a tuple storing r and β at the indexed position



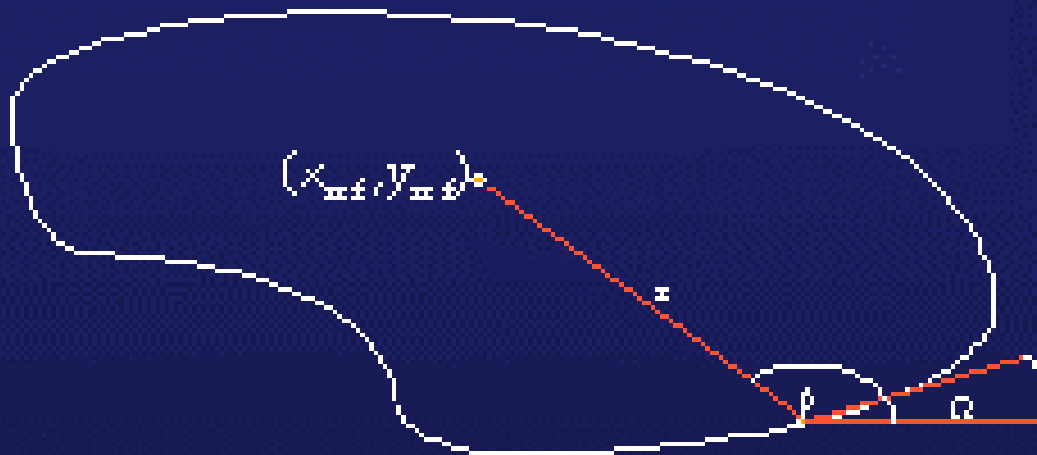
Template Object



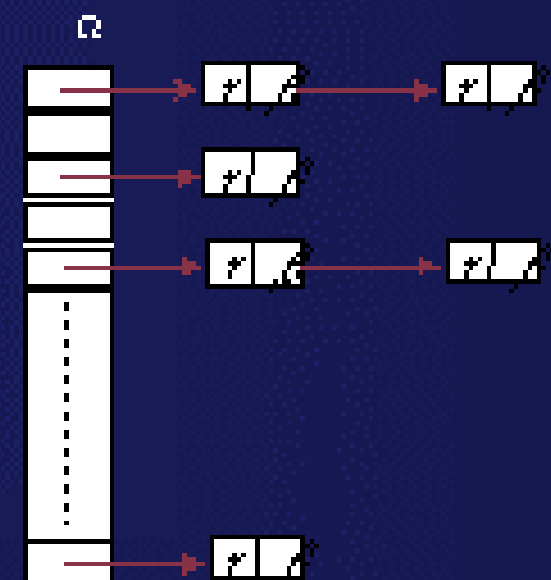
R-Table

GHT: Building the R-Table

- Having done this for each feature point the R-table will fully represent the template object. Also, since the generation phase is invertible we may use it to localise occurrences of the object elsewhere



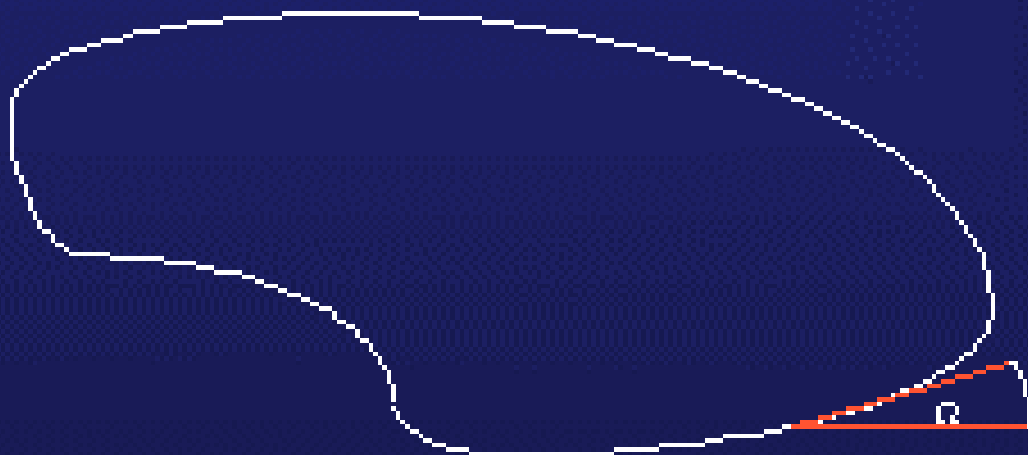
Template Object



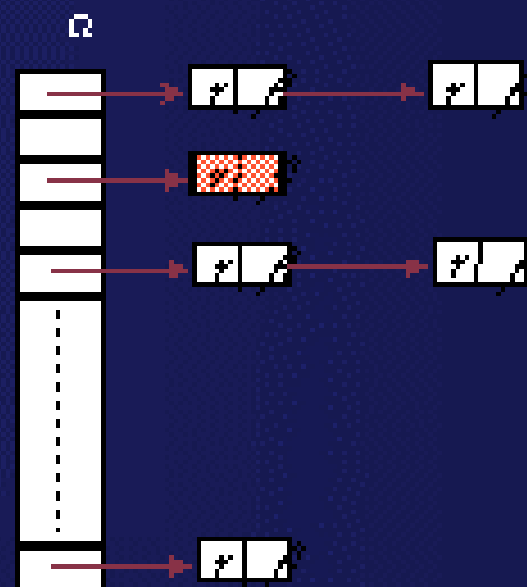
R-Table

GHT: Object Localization

- Given an unknown image each edge point is segmented and its orientation, Ω , is calculated. Again, using Ω as an index into the R-table each (r, δ) pair at this location is extracted



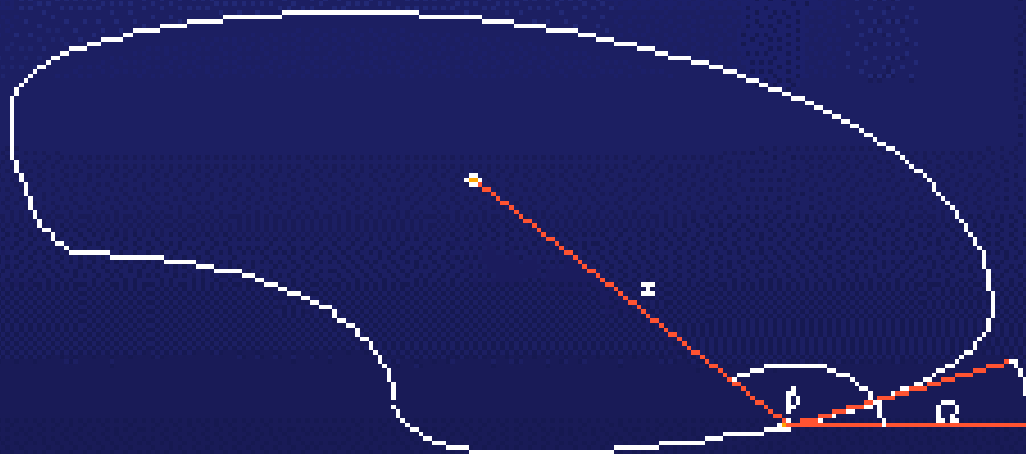
Template Object



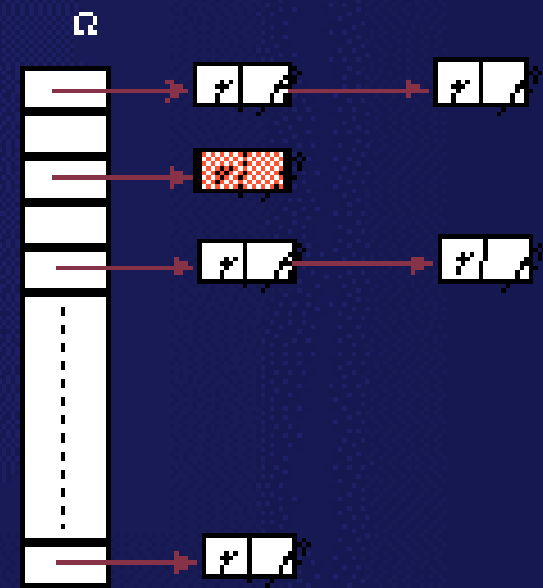
R-Table

GHT: Object Localization

- Using these (r, β) pairs we calculate each possible position for the reference point.



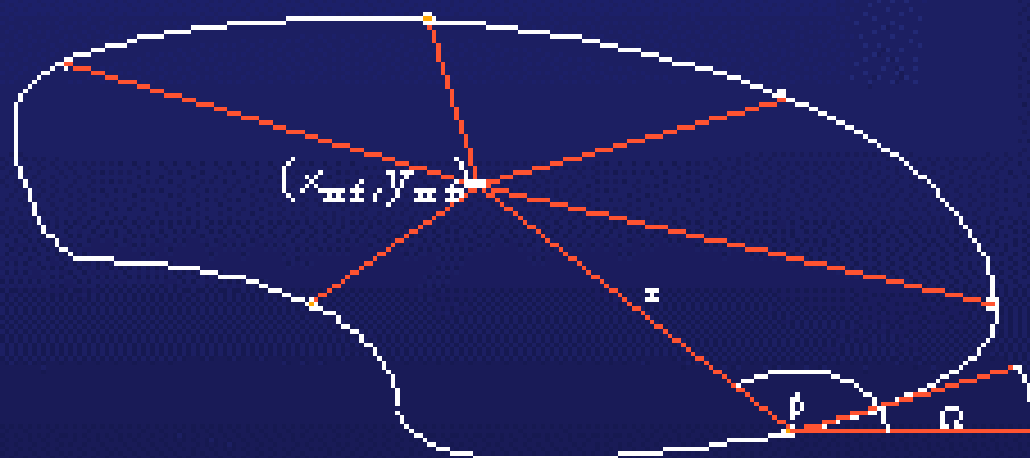
Template Object



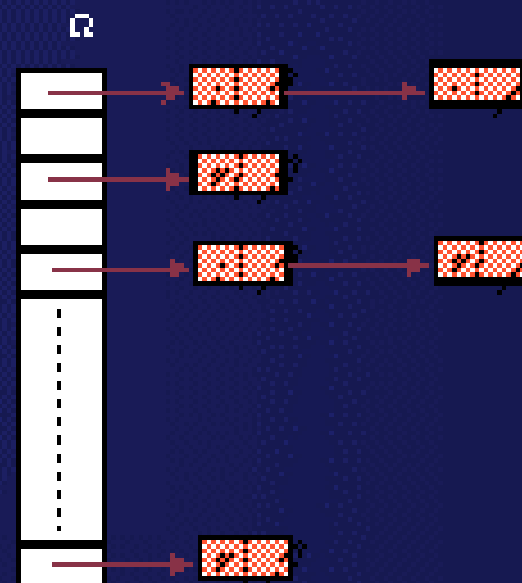
R-Table

GHT: Object Localization

- Although some bogus reference points may be calculated, given that the object exists in the image, a maximum will occur at the reference point.



Template Object



R-Table

Conclusions on GHT

- Standard Techniques allow for **invariance** to **scale and rotation** in the **plane**
- In general, objects in the real world are 3-dimensional
- Hence a single silhouette provides no invariance to pose (i.e. rotation out of the plane).
- No pose estimation.
- Roll on the SNHT....!

The Surface

Normal Hough

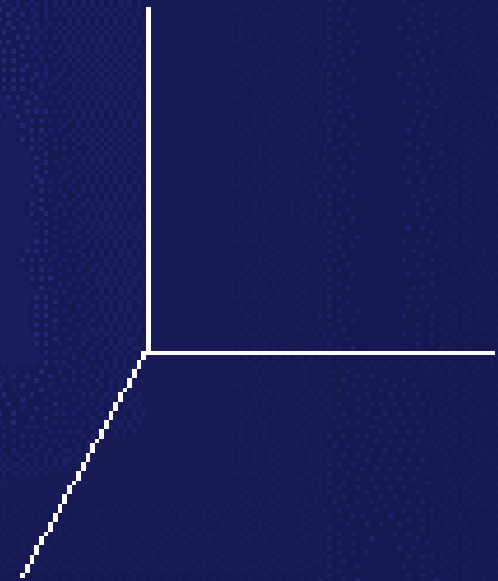
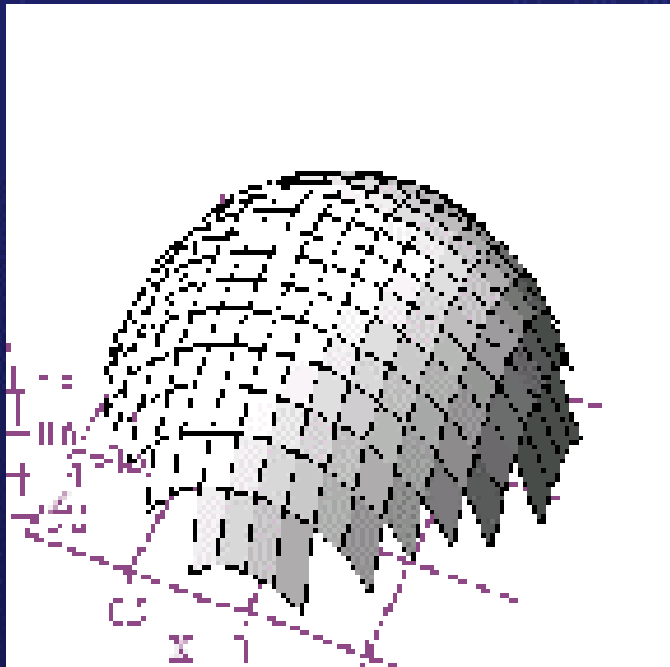
Transform

The Surface Normal Hough Transform

- A technique for computing the **3-D position of a surface** having a specific pose
- A technique for computing the **3-D position and orientation (pose)** of a surface with respect to the pose of a prototypical exemplar of that surface
- Hence a technique for directly registering two **similar surfaces**

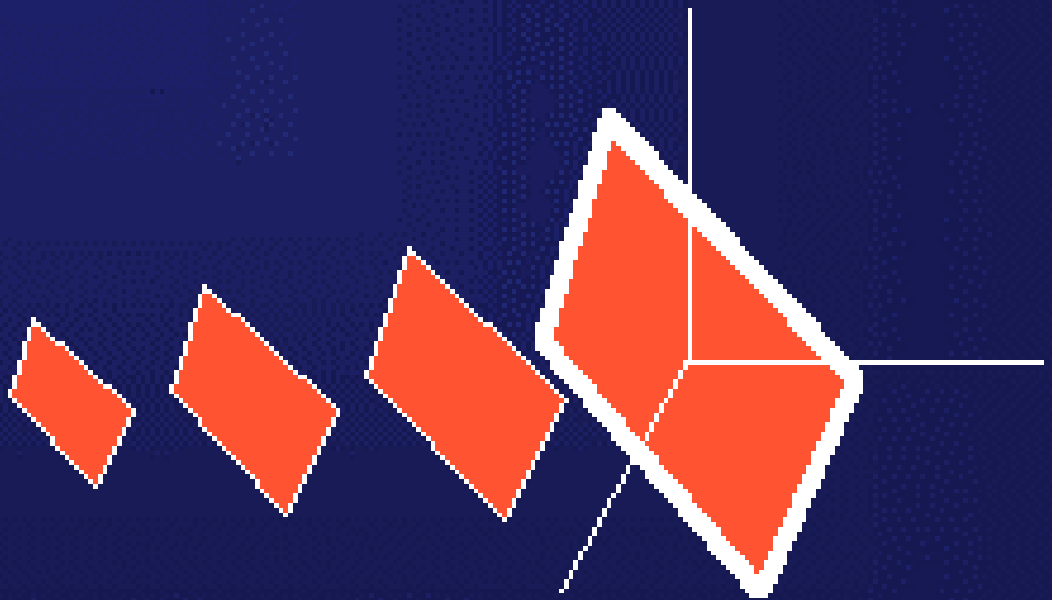
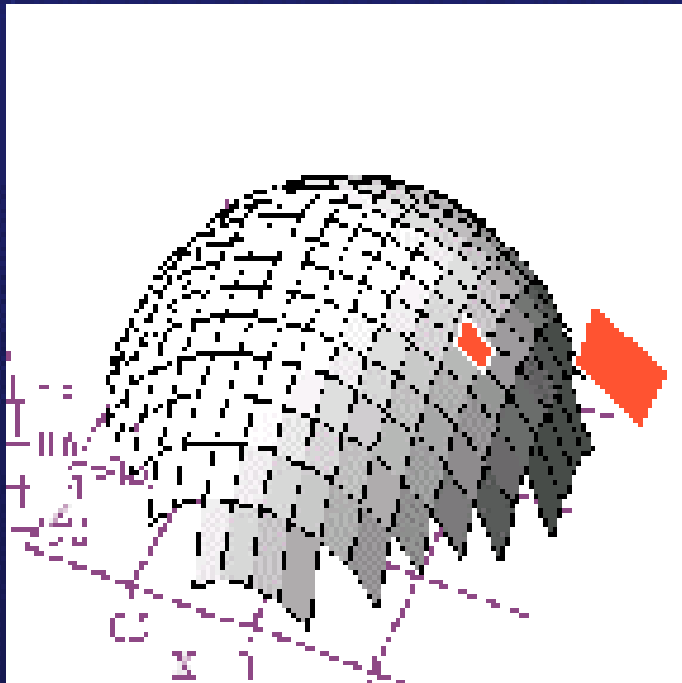
SNHT: Calculating Orientation

- Suppose we wish to register the surface below with a similar surface



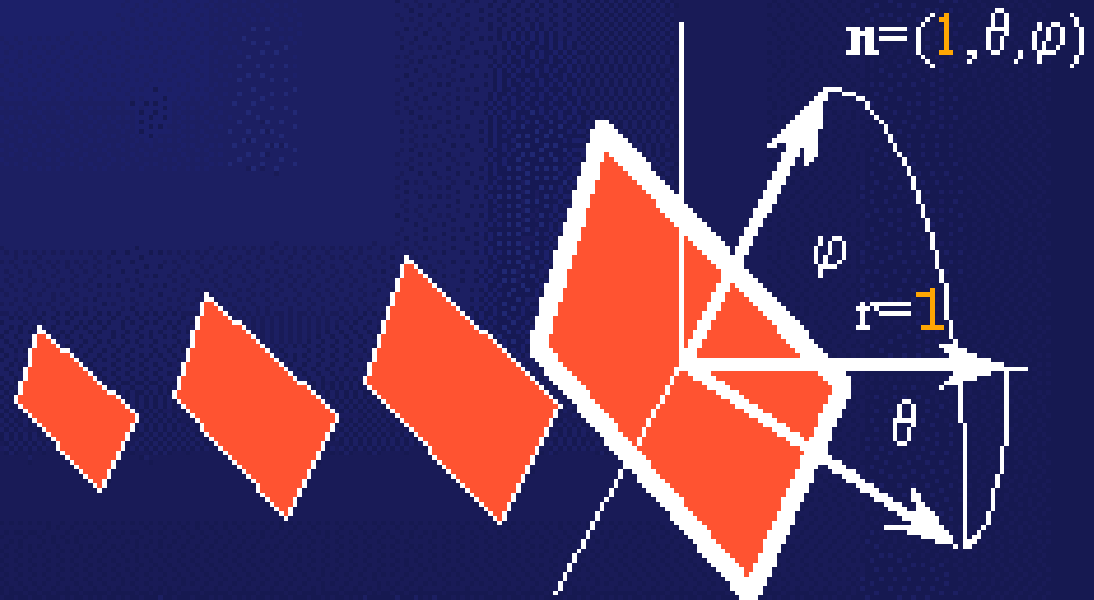
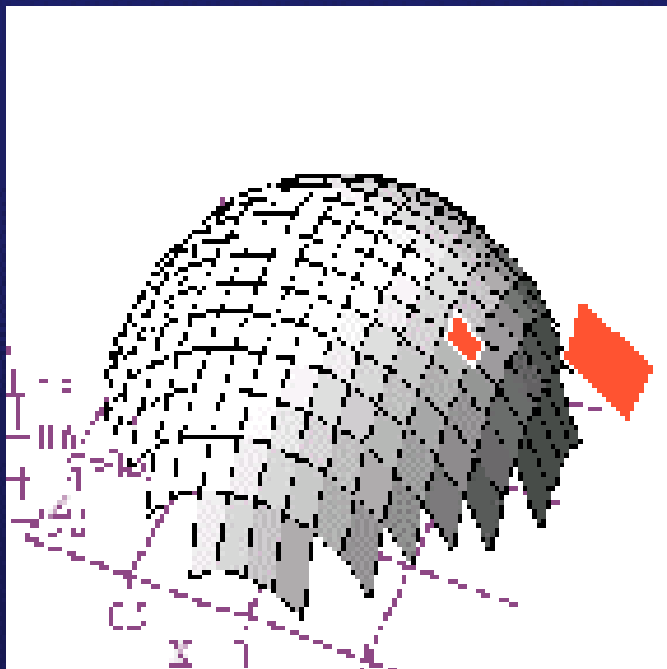
SNHT: Calculating Orientation

- To use the Hough Transform we must again talk about the orientation at each point



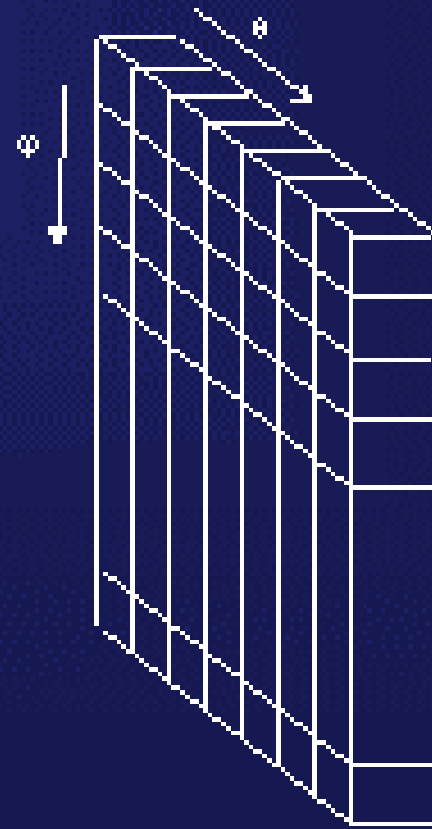
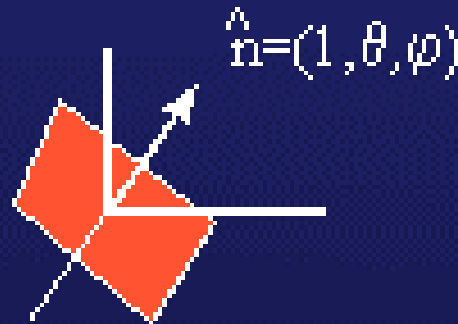
SNHT: Calculating Orientation

- To specify the orientation at a surface point we use the unit normal vector to the tangential plane at that point



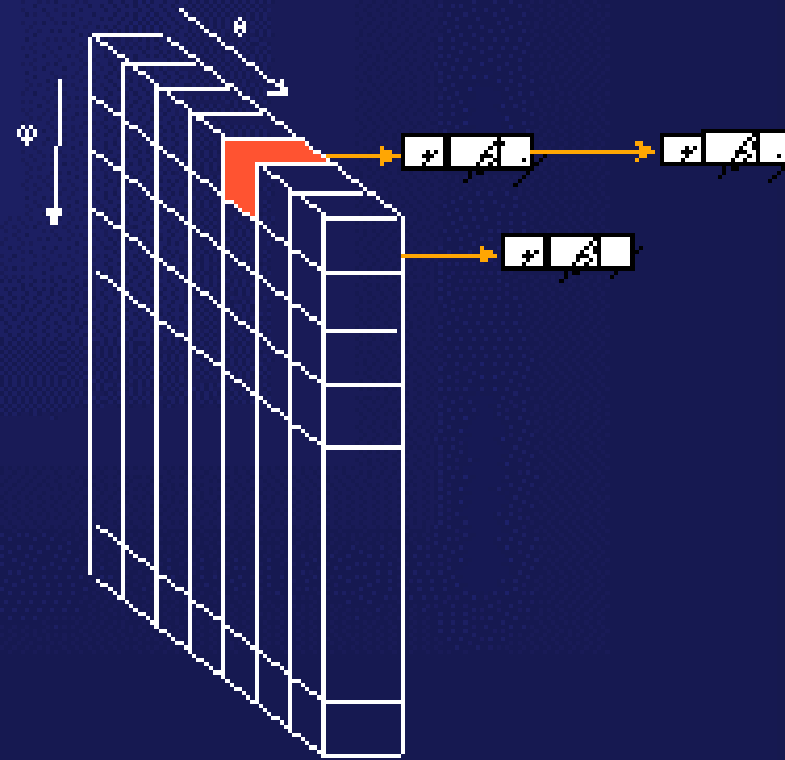
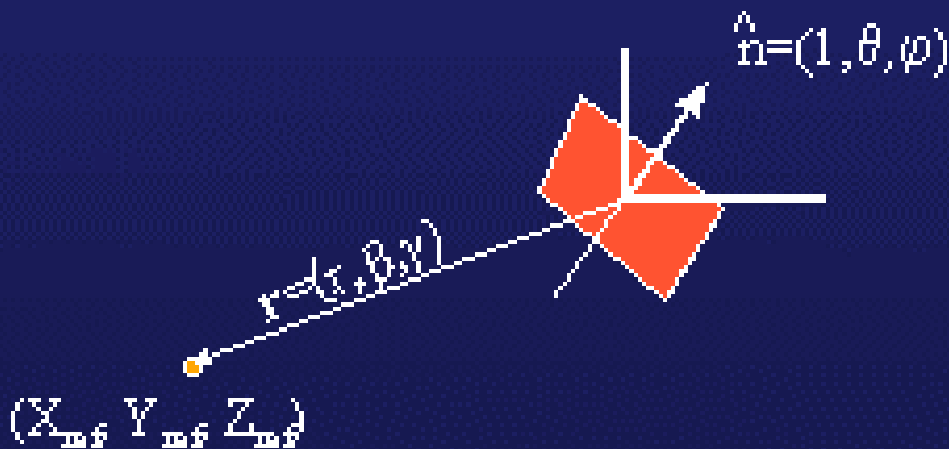
SNHT: Building the R-Table

- Specifying the orientation of the unit normal vector requires two variables, hence the R-table becomes a 2-dimensional array of linked lists



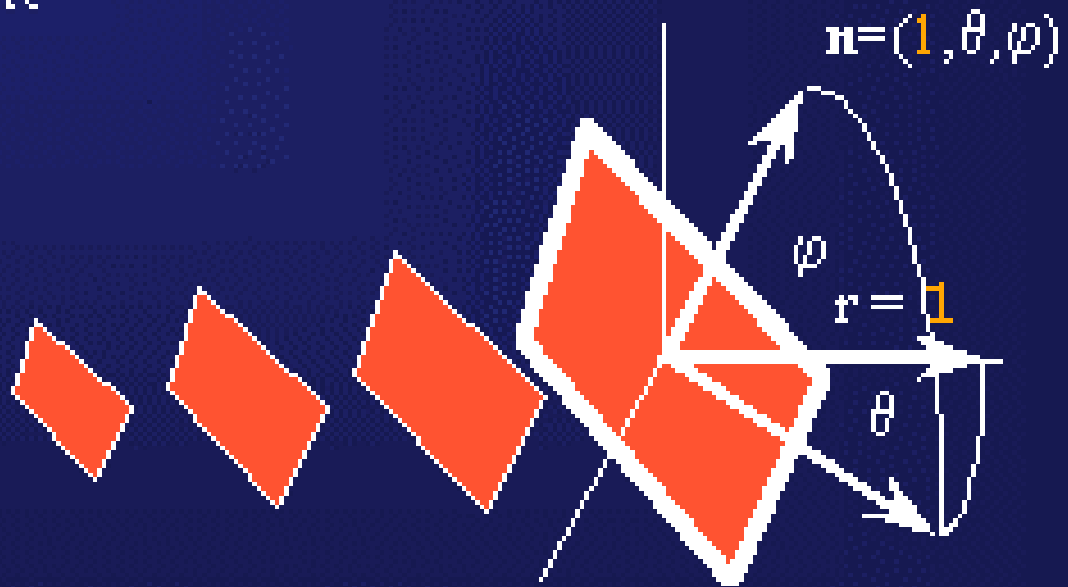
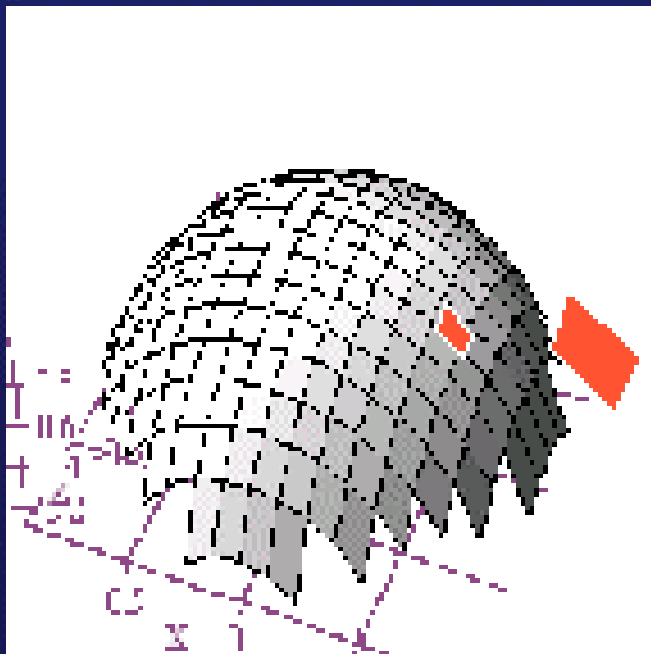
SNHT: Building the R-Table

- Indexing in at this location in the R-table we now place a triple (r, β, γ) representing the length and direction of the vector joining the surface point to a chosen reference point



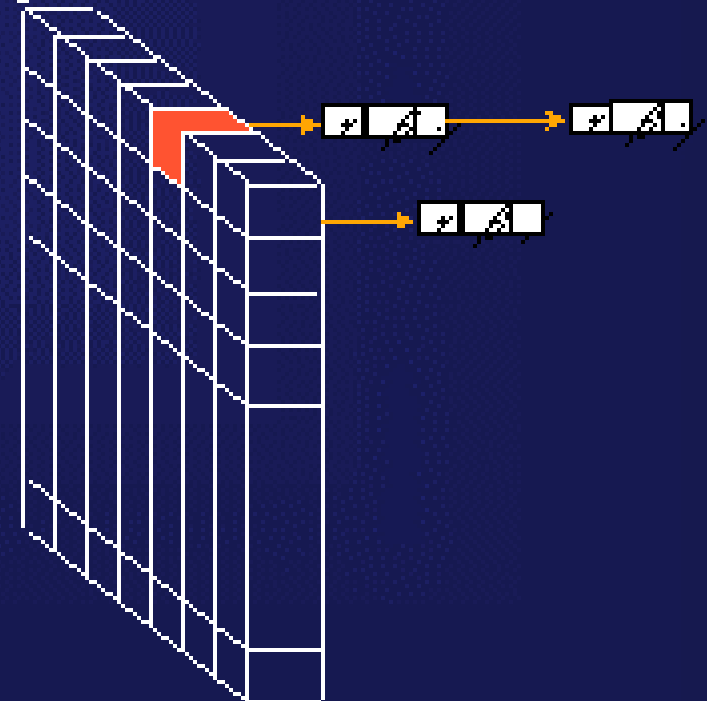
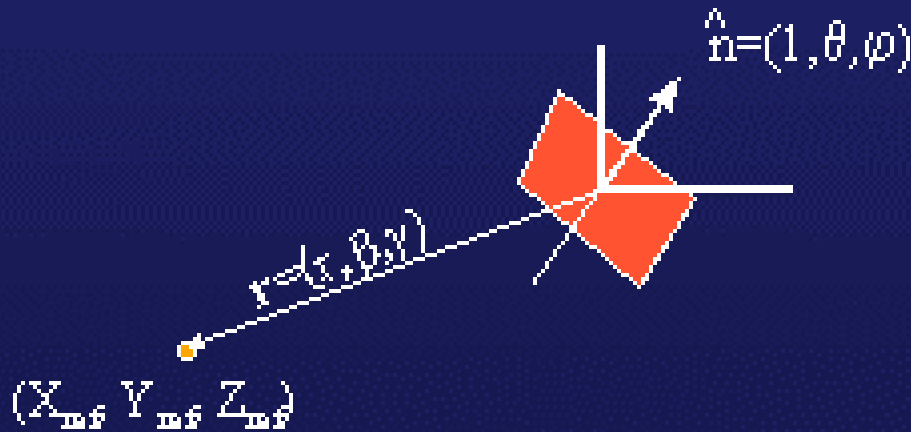
SNHT: Localizing the Object

- Given an surface which we wish to register with the template surface we calculate the unit surface normal at each surface point



SNHT: Localizing the Object

- Using the orientation of this surface normal as an index we extract the (r, β, γ) triples at this location in the R-table and, subsequently calculate the possible positions of the surface reference point



Conclusions and Research Directions

- Technique for registration of surfaces
- Invariant to size and orientation in 3-D
- Can be extended to localization in 2-D
- Implementation
- Validation of simple surfaces
- Face Database for testing of human head pose estimation

Performance Issues

Advantages:

- Works for broken curves
- Uses gradient for speed and further noise removal
- Robust to noise
- Can be extended to a General Hough Transform

Disadvantages

- Expensive when number of parameters is large
- Gradient information can have errors

Gradient Information

- Edge gradient in image space can be used in Hough Transform to reduce one dimension in incrementing the accumulator array
- For line detection the gradient is θ , and so need only to vote for one cell (p, θ) where p is
 - $p = x_i \cos \theta + y_i \sin \theta$
- For circle detection the gradient is θ , and so need only to vote along a line given by the equations
 - $a = x + r \cos \theta, \quad b = y + r \sin \theta$

Sources

- John Mc. Donald
- D. Vernon