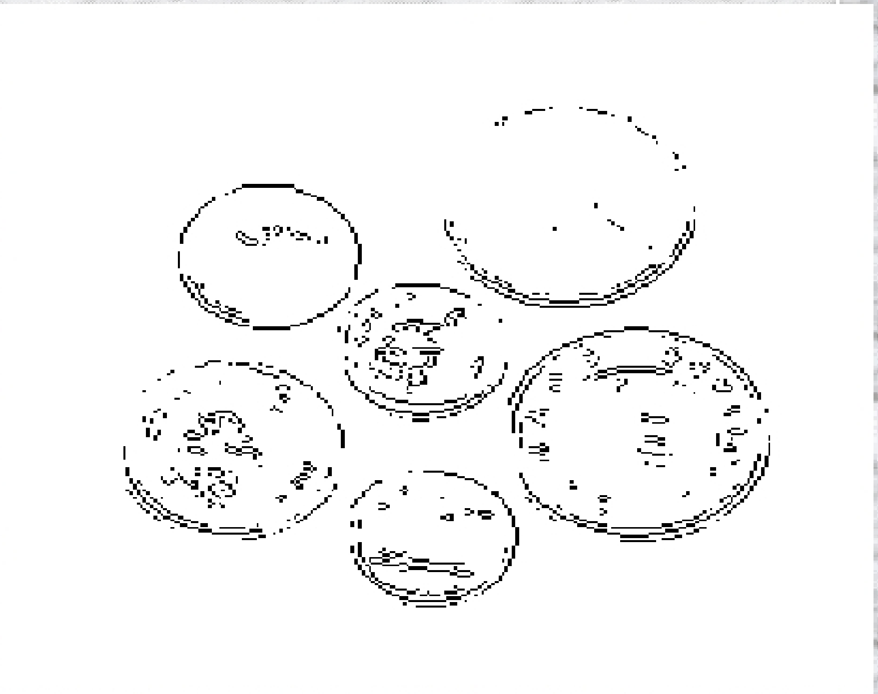
Hough Transform

Motivations for Hough Transform (example)



Hough Transform

- One of the most popular utilizations of a **voting** mechanism

- A kind of ***structured*** Neural Network

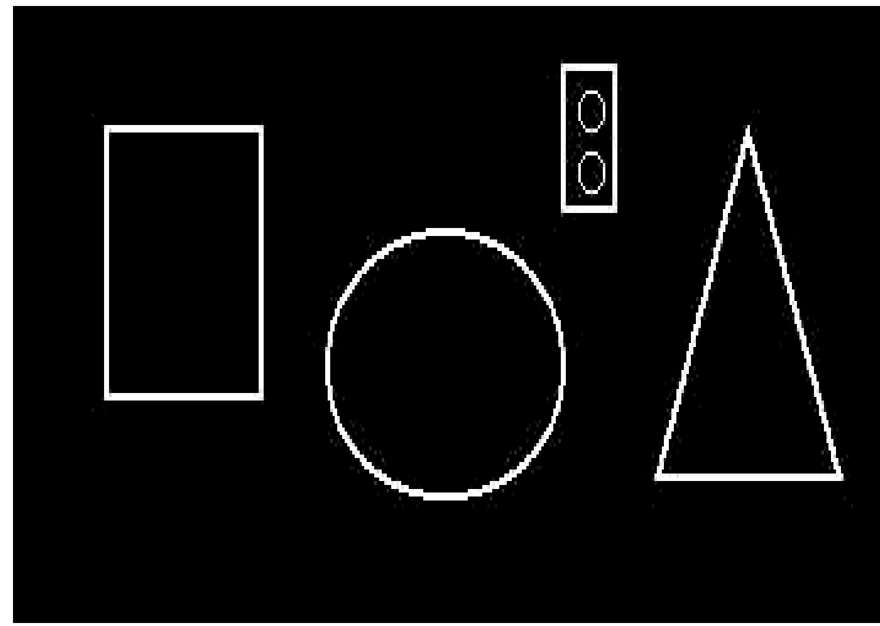
- A transformation from an **image space** to a **parameter space (vote space, Hough space).**

- Voting is performed in the parameter space

- This transform can be also treated as ***template matching***

Hough Transform

- It locates straight lines (SHT) - standard, simple HT
- It locates straight line intervals
- It locates circles
- It locates algebraic curves
- It locates arbitrary specific shapes in an image
  - **But you pay progressively for complexity of shapes by time and memory usage**



Hough Transform idea

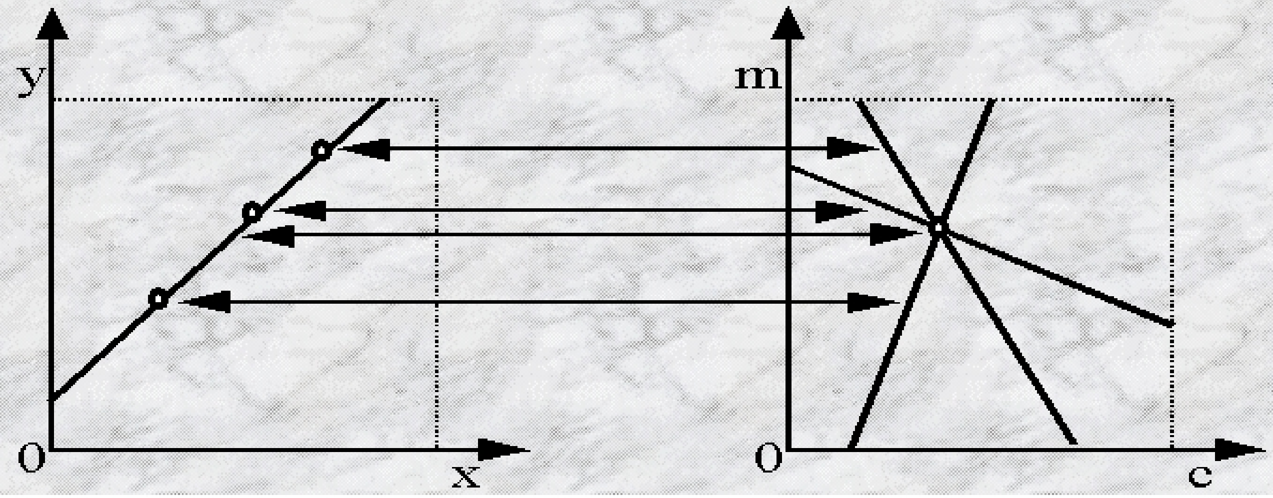
- An Edge Pixel in Real Space would vote into Hough Space all possible lines that contain that point

$$y = mx + b$$

- Continue to Add Votes for different Edge Pixels
- Intersection gives Equation for line
  - Edge Detected Image (real space)
  - Hough Space

Hough Transform for Lines

- Line equation: $y = mx + c \Longleftrightarrow c = -mx + y$
- Each point in image space corresponds to a line in parameter space.
- An intersection point in parameter space corresponds to a line in image space.



The standard Hough Transform for lines (continued)

- Example: Parametric equation of a line

$$x \cos @ + y \sin @ = r$$

- **<u>Generalization:</u>**
  - Technique to isolate curves of a given shape in an image
  - Curve specified by parametric equation

Line Normal Parameterization

- $\rho = x_i \cos\theta + y_i \sin\theta$

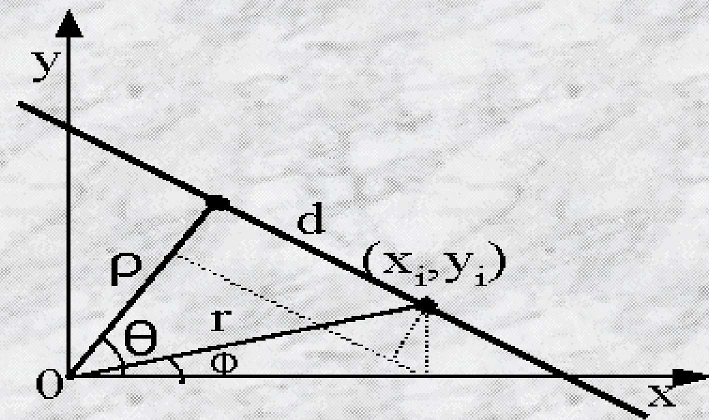
  0 " θ " 2Π , $-N$ " ρ " N

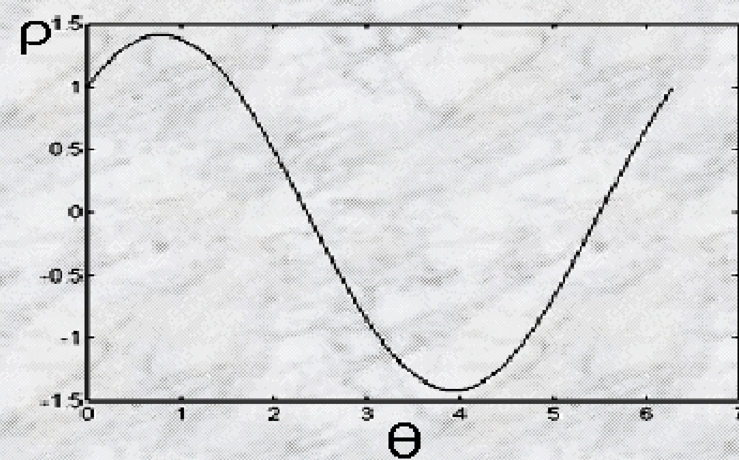
- $\rho = r\,(x_i / r \cos\theta + y_i / r \sin\theta)$

  $= r \cos(\theta - \Phi)$

- Error term:

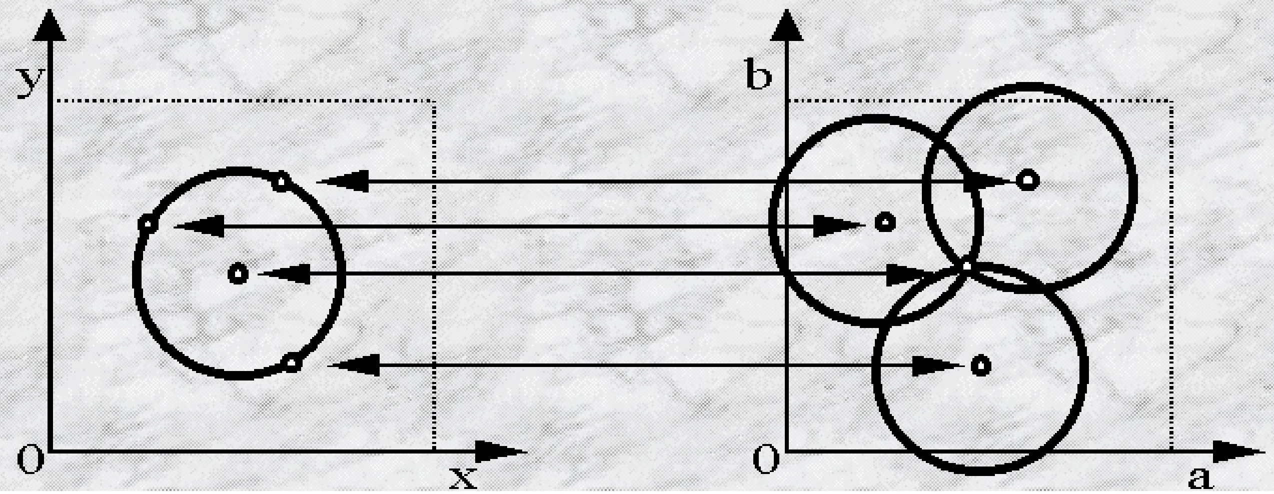
  $\Delta\rho$ " $d\ \Delta\theta$





Circle Fitting

- Implicit circle equation:
  $(x - a)^2 + (y - b)^2 = r^2$
- Parametric equations:
  $a = x + r \cos \theta \quad , \quad b = y + r \sin \theta$
- To find the center of a circle with fixed radius r:



Circle Fitting (cont'd)

For unknown radius r use one of the following:

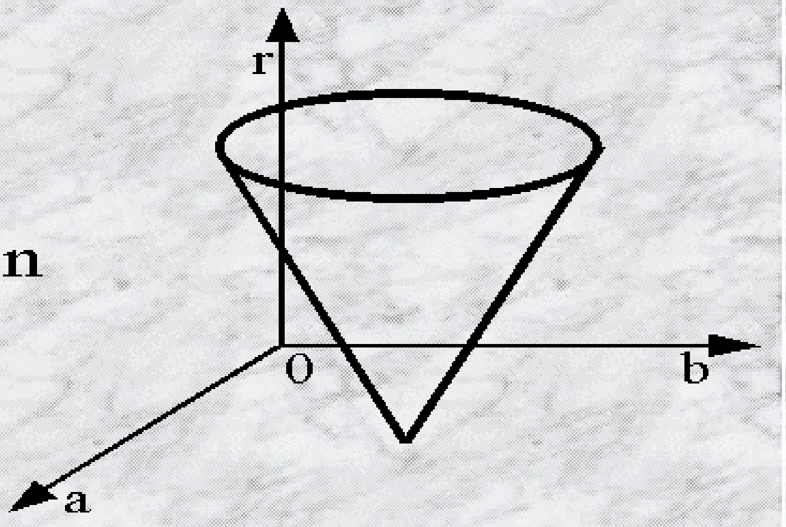
1. Use a 2 dimensional space (a,b), but loose radius size:

   $b = a \tan \theta - x \tan \theta + y$

2. Use a 3 dimensional space (a,b,r) and parametric equations:

   $a = x + r \cos \theta$, $b = y + r \sin \theta$.

   Points in image space correspond to cones.



SHT: Problem

- **Standard Hough Transform requires parametric representation for desired curve**

- **This idea is generalized in the Generalized Hough Transform**

Example: Human Face recognition

- Is there some attribute of the structure of the head that we can exploit to help estimate pose estimation?
- Is this attribute invariant under change in pose?

Or

- "Can we model how this attribute varies with pose?"

Hough Transform in General

- Technique to isolate curves of a given shape in an image
- Standard Hough Transform (HT) uses parametric formulation of curves
- Generalized Hough Transform (GHT) extends for arbitrary curves

Hough Transform Algorithm

1. Form a parameter space to represent all of the unknown parameters in an equation of a family of curves f(x,a)=0.
2. Quantize the parameter space appropriately to form an accumulator array A.
3. Initialize each cell in accumulator A to 0.
4. For each edge point in the image increment all the cells in the accumulator array A that satisfy the curve equation.
5. Local maxima in the accumulator array A correspond to a curve with those parameter in image space.

The Generalized Hough Transform

- **Technique to find arbitrary curves in a given image**
- **Parametric equation no longer required**
- **Look-up table used as transform mechanism**
- **Two phases:**
- **R-Table Generation phase**
- **Object Detection phase**

Conclusions on GHT

- Standard Techniques allow for **invariance** to scale and rotation in the plane
- In general, objects in the real world are 3-dimensional
- Hence a single silhouette provides no invariance to pose (i.e. rotation out of the plane).
- No pose estimation.
- This is generalized to Surface Normal Hough Transform

The Surface Normal Hough Transform

- A technique for computing the *3-D position of a surface* having a specific pose

- A technique for computing the **3-D position and orientation (pose)** of a surface with respect to the pose of a prototypical exemplar of that surface

- Hence a technique for directly registering two **similar surfaces**

Conclusions and Research Directions

- Technique for registration of surfaces
- Invariant to size and orientation in 3-D
- Can be extended to localization in 2-D
- Implementation
- Validation of simple surfaces
- Face Database for testing of human head pose estimation

Performance Issues

Advantages:

- Works for broken curves
- Uses gradient for speed and further noise removal
- Robust to noise
- Can be extended to a General Hough Transform

Disadvantages

- Expensive when number of parameters is large
- Gradient information can have errors

Gradient Information

- Edge gradient in image space can be used in Hough Transform to reduce one dimension in incrementing the accumulator array
- For line detection the gradient is @, and so need only to vote for one cell (p,@) where p is
  - p = x_i cos @ + y_i sin @
- For circle detection the gradient is @, and so need only to vote along a line given by the equations
  - a=x + r cos @, b = y + r sin @

Hough Transform

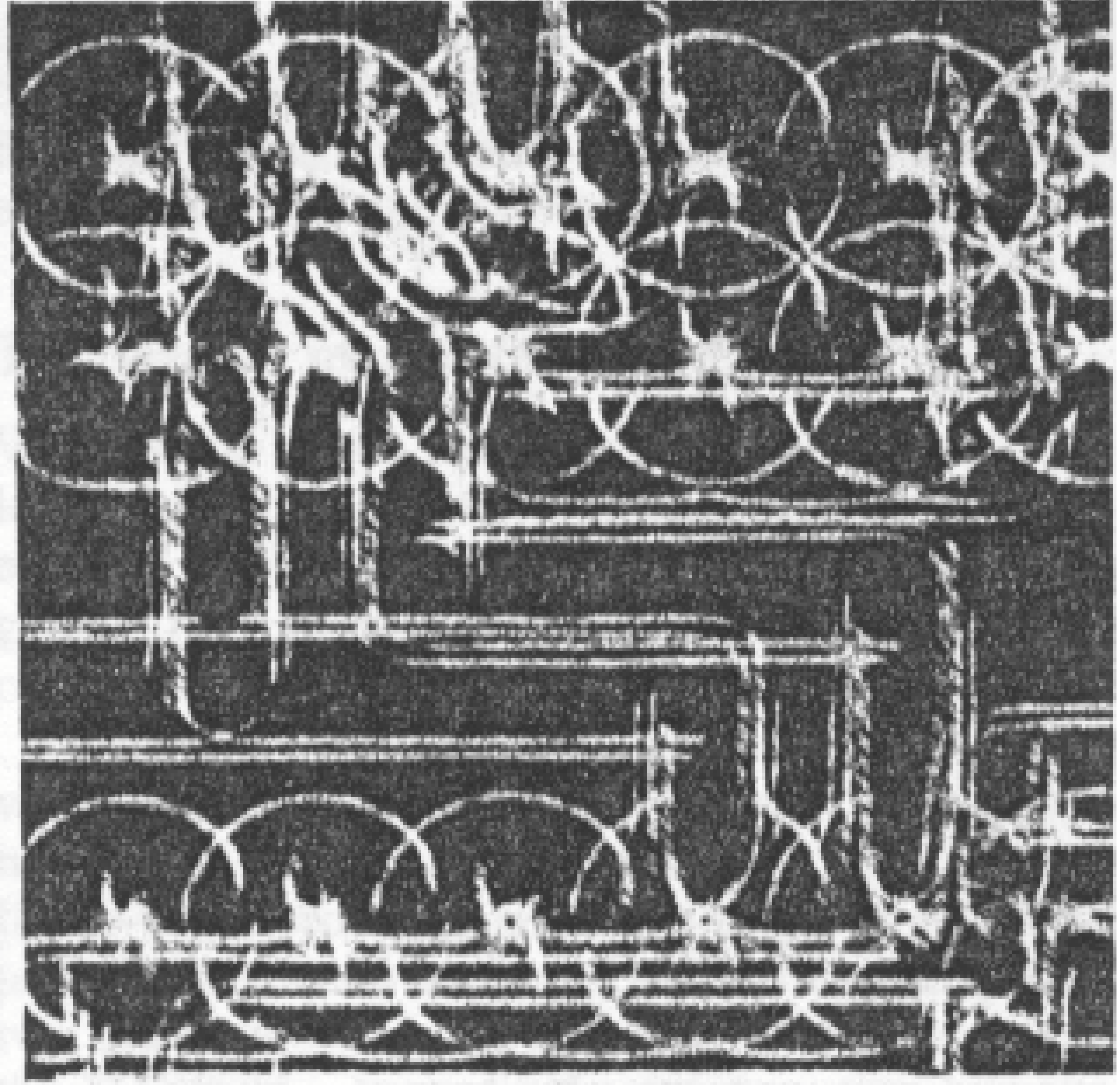
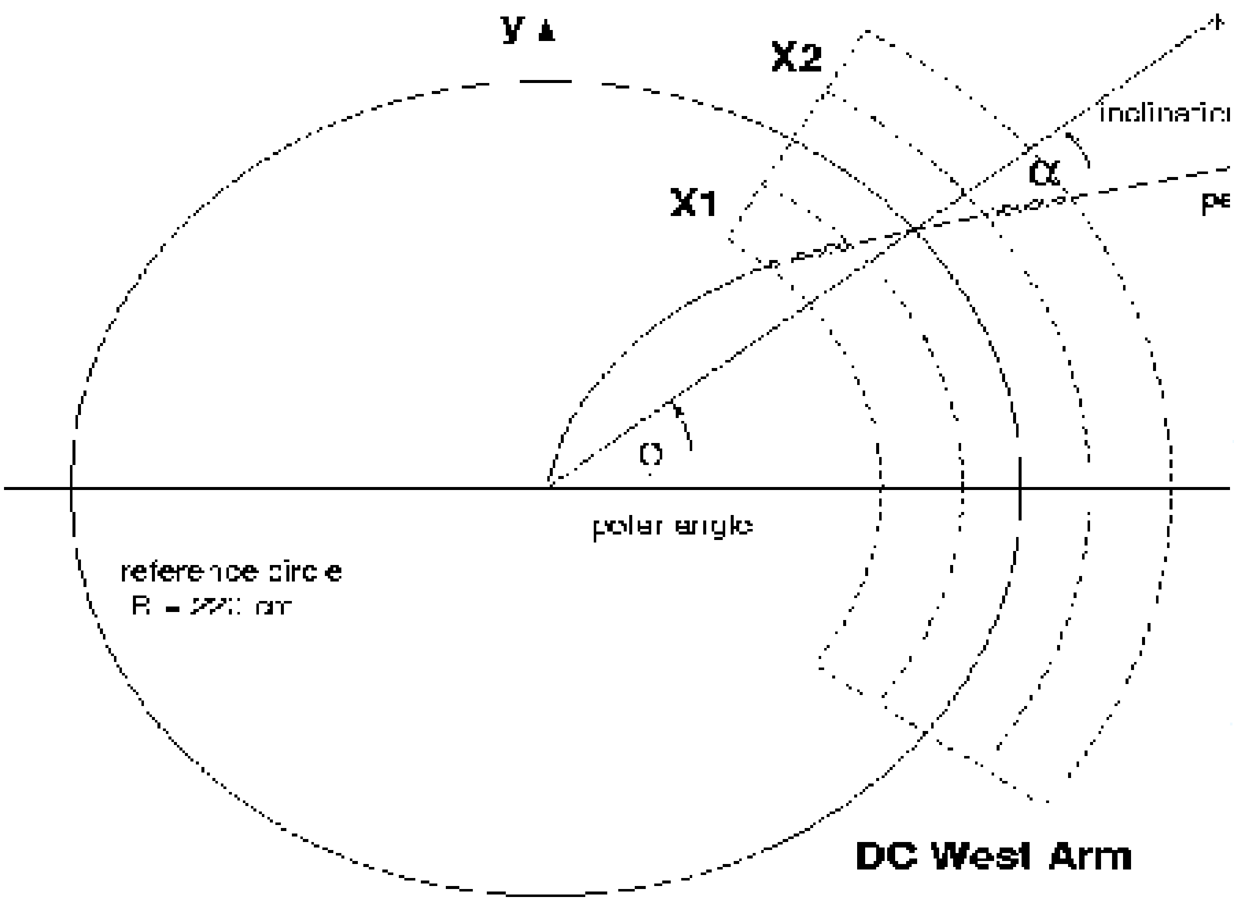


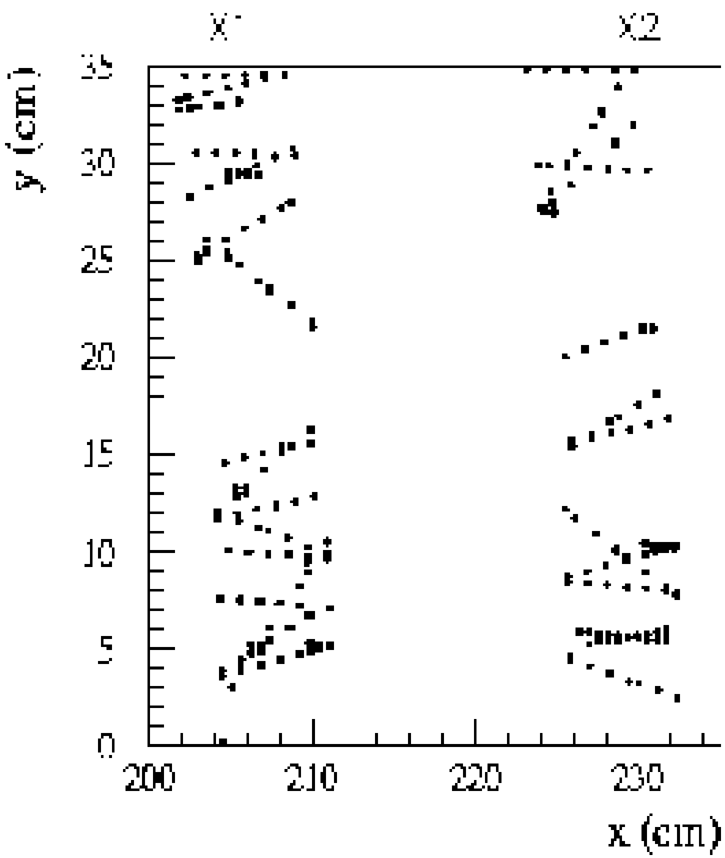
Figure 6.3 A Hough accumulator array

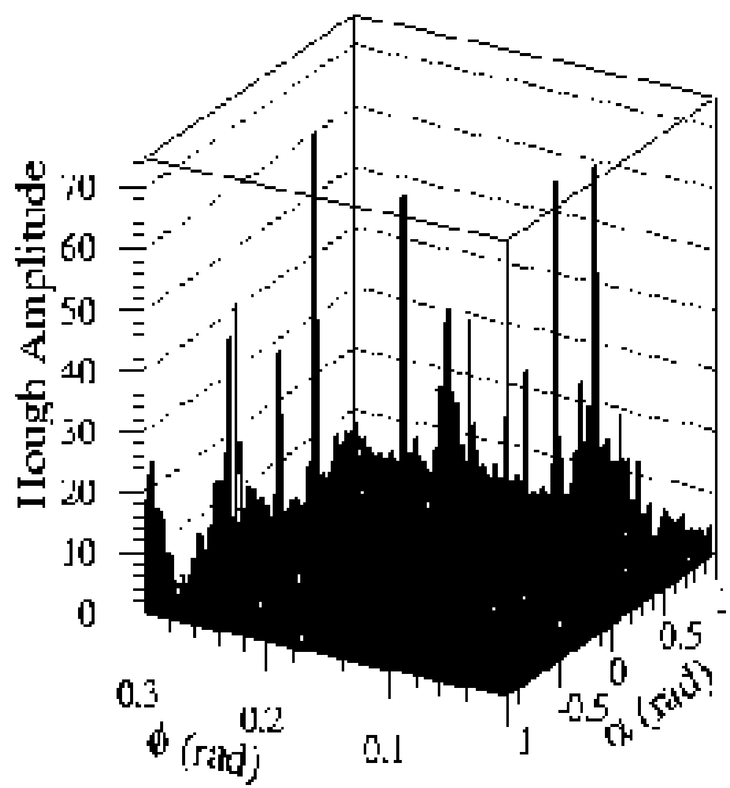
Drift Chamber Tracking Method: Hough Transform

- Optimum choice of particle trajectory
- Calculate ϕ/α for each x1-x2 pair at the reference radius (R) after most of the bending has occurred.
  - 6000 bins in sin(ϕ)
  - 300 bins in α



Sample Drift Chamber Hough Transform





- Peaks are clearly distinguishable from the background in phase space

Sources

- **Zinovi Tauber**