Motivations for Hough Transform (example)
Hough Transform

• One of the most popular utilizations of a voting mechanism

• A kind of structured Neural Network

• A transformation from an image space to a parameter space (vote space, Hough space).

• Voting is performed in the parameter space

• This transform can be also treated as template matching
Hough Transform

- It locates straight lines (SHT) - standard, simple HT
- It locates straight line intervals
- It locates circles
- It locates algebraic curves
- It locates arbitrary specific shapes in an image

- But you pay progressively for complexity of shapes by time and memory usage
Hough Transform idea

- An Edge Pixel in Real Space would vote into Hough Space all possible lines that contain that point

\[ y = mx + b \]

- Continue to Add Votes for different Edge Pixels

- Intersection gives Equation for line
  - Edge Detected Image (real space)
  - Hough Space
Hough Transform for Lines

- Line equation: $y = mx + c \iff c = -mx + y$
- Each point in image space corresponds to a line in parameter space.
- An intersection point in parameter space corresponds to a line in image space.
The standard Hough Transform for lines (continued)

• Example: Parametric equation of a line

\[ x \cos \@ + y \sin \@ = r \]

• **Generalization:**
  – Technique to isolate curves of a given shape in an image
  – Curve specified by parametric equation
Line Normal Parameterization

- $\rho = x_i \cos \theta + y_i \sin \theta$
  - $0 \leq \theta \leq 2\pi$, $-N \leq \rho \leq N$

- $\rho = r \left( \frac{x_i}{r \cos \theta} + \frac{y_i}{r \sin \theta} \right)$
  - $= r \cos (\theta - \Phi)$

- Error term:
  - $\Delta \rho = d \Delta \theta$
Circle Fitting

- Implicit circle equation:
  \[(x - a)^2 + (y - b)^2 = r^2\]
- Parametric equations:
  \[a = x + r \cos \theta, \quad b = y + r \sin \theta\]
- To find the center of a circle with fixed radius \(r\):
Circle Fitting (cont’d)

For unknown radius $r$ use one of the following:

1. Use a 2 dimensional space $(a,b)$, but loose radius size:
   
   \[ b = a \tan \theta - x \tan \theta + y \]

2. Use a 3 dimensional space $(a,b,r)$ and parametric equations:
   
   \[ a = x + r \cos \theta, \quad b = y + r \sin \theta. \]

Points in image space correspond to cones.
SHT: Problem

• Standard Hough Transform requires parametric representation for desired curve

• This idea is generalized in the Generalized Hough Transform
Example: Human Face recognition

• Is there some attribute of the structure of the head that we can exploit to help estimate pose estimation?
• Is this attribute invariant under change in pose?
  Or
• “Can we model how this attribute varies with pose?”
Hough Transform in General

• Technique to isolate curves of a given shape in an image
• Standard Hough Transform (HT) uses parametric formulation of curves
• Generalized Hough Transform (GHT) extends for arbitrary curves
Hough Transform Algorithm

1. Form a parameter space to represent all of the unknown parameters in an equation of a family of curves \( f(x,a)=0 \).
2. Quantize the parameter space appropriately to form an accumulator array \( A \).
3. Initialize each cell in accumulator \( A \) to 0.
4. For each edge point in the image increment all the cells in the accumulator array \( A \) that satisfy the curve equation.
5. Local maxima in the accumulator array \( A \) correspond to a curve with those parameter in image space.
The Generalized Hough Transform

• Technique to find arbitrary curves in a given image

• Parametric equation no longer required

• Look-up table used as transform mechanism

• Two phases:

• R-Table Generation phase

• Object Detection phase
Conclusions on GHT

• Standard Techniques allow for invariance to scale and rotation in the plane.

• In general, objects in the real world are 3-dimensional.

• Hence a single silhouette provides no invariance to pose (i.e. rotation out of the plane).

• No pose estimation.

• This is generalized to Surface Normal Hough Transform.
The Surface Normal Hough Transform

- A technique for computing the 3-D position of a surface having a specific pose

- A technique for computing the 3-D position and orientation (pose) of a surface with respect to the pose of a prototypical exemplar of that surface

- Hence a technique for directly registering two similar surfaces
Conclusions and Research Directions

• Technique for registration of surfaces
• Invariant to size and orientation in 3-D
• Can be extended to localization in 2-D
• Implementation
• Validation of simple surfaces
• Face Database for testing of human head pose estimation
Advantages:

- Works for broken curves
- Uses gradient for speed and further noise removal
- Robust to noise
- Can be extended to a General Hough Transform

Disadvantages:

- Expensive when number of parameters is large
- Gradient information can have errors
**Gradient Information**

- Edge gradient in image space can be used in Hough Transform to reduce one dimension in incrementing the accumulator array.

- For line detection the gradient is $\theta$ and so need only to vote for one cell $(p, \theta)$ where $p$ is

  \[ p = x_i \cos \theta + y_i \sin \theta \]

- For circle detection the gradient is $\theta$ and so need only to vote along a line given by the equations

  \[ a = x + r \cos \theta \quad b = y + r \sin \theta \]
Figure 6.3 A Hough accumulator array
Drift Chamber Tracking Method: Hough Transform

- Optimum choice of particle trajectory
- Calculate $\phi/\alpha$ for each $x_1-x_2$ pair at the reference radius ($R$) after most of the bending has occurred.
  - 6000 bins in $\sin(\phi)$
  - 300 bins in $\alpha$
Sample Drift Chamber Hough Transform

- Peaks are clearly distinguishable from the background in phase space
Sources

• Zinovi Tauber