Optimization Techniques

Slides of Gang Quan used

Problems

- Decision problems
  - Yes/No
- Optimization problems
  - Some value need to be minimized or maximized

System level design is essentially an optimization problem!

Optimization Techniques

- Mathematical Programming
- Network Analysis
- Branch & Bond
- Genetic Algorithm
- Simulated Annealing Algorithm
- etc.

Mathematical Programming

- Linear Programming
  - What
    - A *linear* multivariable function is to be optimized (maximized or minimized) subject to a number of *linear* constraints.

Linear Programming

$Maximize: f(\mathbf{x}) = f(x_1, x_2, \cdots, x_n)$ } Objective function

$subject \quad to:$

$$\left.\begin{aligned} & h_1(\mathbf{X}) \le 0 \\ & h_2(\mathbf{X}) \le 0 \\ & \ldots \\ & h_m(\mathbf{X}) \le 0 \end{aligned}\right\}$$ Constraints

$f(\mathbf{x}), h_1(\mathbf{X}), h_2(\mathbf{X}) \le 0 \cdots h_m(\mathbf{X}) \le 0,$ are linear functions.

$x_1, x_2, \cdots, x_n,$ are real numbers.

Example

$$Maximize: f(\mathbf{x}) = 4x_1 + 3x_2 - x_4$$

$$subject \quad to:$$

$$3x_1 + 4x_2 + x_3 \leq 12$$
$$3x_1 - 2x_2 + x_4 \leq 10$$
$$4x_1 + x_2 - x_3 \leq 8$$

$$x_i \geq 0, \quad i = 1,2,3,4$$

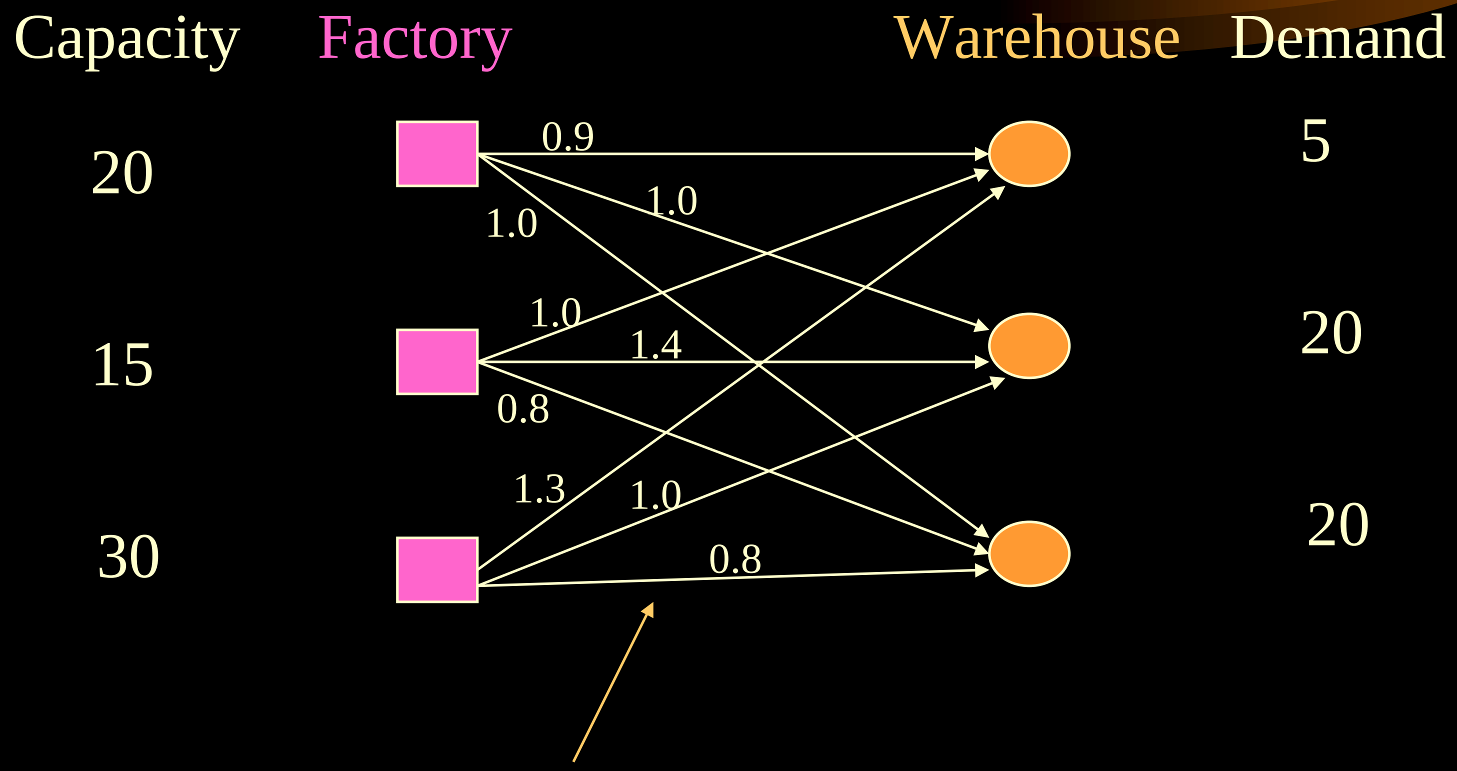
A Practical Example

A supply system comprising *three* factories which must supply the needs for a single commodity of *three* warehouses. Given:

1. The unit cost from one factory to any warehouse
2. The maximum product capacity by each factory
3. The minimum product demand by each warehouse

How much of the product from each factory should be shipped to each warehouse to make the cost minimum?

A Practical Example (cont'd)



The unit cost from one factory to any warehouse

A Practical Example (cont'd)

Let x_{ij} be the units sent from factory i to warehouse j.

Objectives ?

Minimize: $0.9x_{11}+1.0x_{12}+1.0x_{13}+1.0x_{21}+1.4x_{22}+0.8x_{23}$
$+1.3x_{31}+1.0x_{32}+0.8x_{33}$

Subject to ?

for each factory

$x_{11} + x_{12} + x_{13} <= 20$
$x_{21} + x_{22} + x_{23} <= 15$
$x_{31} + x_{32} + x_{33} <= 30$

for each warehouse

$x_{11} + x_{21} + x_{31} >= 5$
$x_{12} + x_{22} + x_{32} >= 20$
$x_{13} + x_{23} + x_{33} >= 20$

And ?

$x_{ij} >= 0$, i = 1,2,3   j=1,2,3

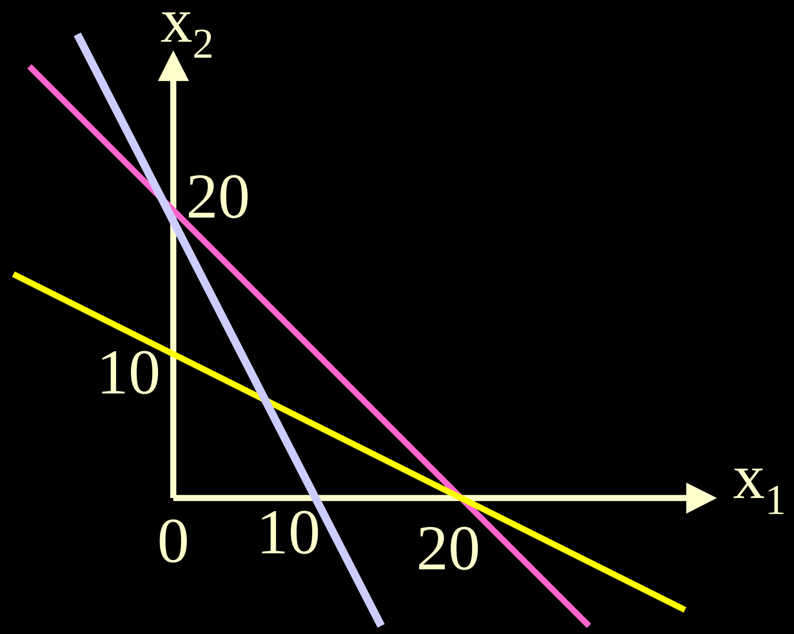
Linear Programming Solvers

- Matlab, Maple, Execl
- http://elib.zib.de/pub/Packages/mathprog/linprog/lp-solve/

Infeasible Solution

- There is no such vector **X** that satisfies all the constraints simultaneously.

$x_{11} + x_{12} >= 20$
$x_{11} + 2x_{12} <= 20$
$2x_{11} + x_{12} <= 20$



Unboundedness

- The maximized (minimized) objection function can grow infinitely large (small)

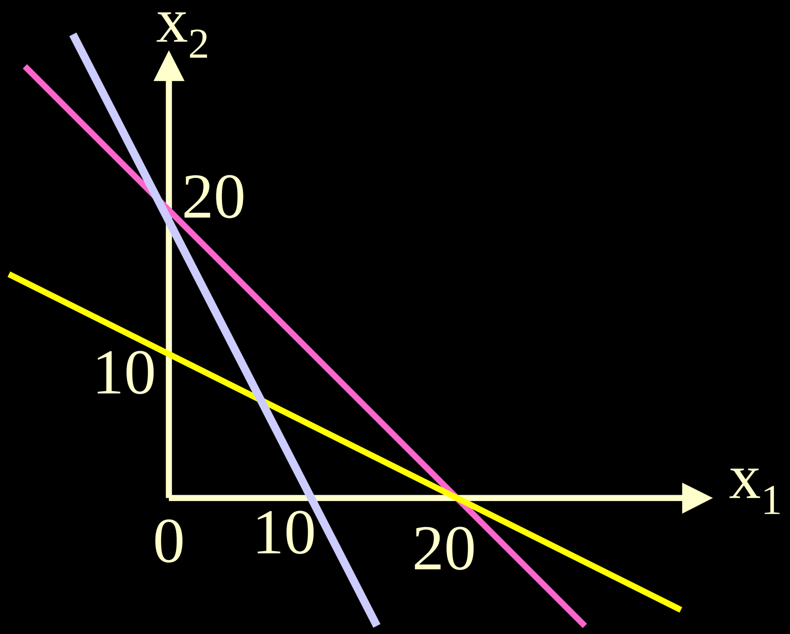
Max: $3x_{11}+4x_{12}$

s.t.:

$x_{11} + x_{12} >= 20$

$x_{11} + 2x_{12} >= 20$

$2x_{11} + x_{12} >= 20$



Integer Linear Programming

- The variables can only be integers

$$\text{Maximize} : f(\mathbf{x}) = f(x_1, x_2, \cdots, x_n)$$

$$\text{subject to} :$$

$$h_1(\mathbf{X}) \le 0$$
$$h_2(\mathbf{X}) \le 0$$
$$\dots$$
$$h_m(\mathbf{X}) \le 0$$

$$x_1, x_2, \dots, x_n \text{ are integers}.$$

- NP-complete

Example

- Knapsack problem
  - *Given a knapsack of capacity* $\mathbf{c}$*, into which we may put* $\mathbf{k}$ *types of objects. Each object of type* i *has a profit ,* $\mathbf{p_i}$*, and a weight* $\mathbf{w_i}$*, (we have an unbounded number of items of each object type), determine the number of each type of the objects to be chosen to put in the knapsack so as to maximize the total profit.*

Example (cont'd)

Let n_i be the number of items should be chosen for type i.

Objectives ?

$$Maximize\ :\ f(\mathbf{x}) = \sum_i n_i p_i$$

profit

Subject to ?

$$\sum_i n_i w_i \le c$$

And ?

$$n_i \ge 0, n_i \text{ is an integer}, \quad i = 1, 2, \ldots, k$$

Other Mathematical Programming

- Mixed integer linear programming
- Quadratic programming
- Geometric Programming

Other Mathematical Programming (Cont'd)

- Quadratic programming

$$Maximize\ :\ f(\mathbf{x}) = \mathbf{C}^T\mathbf{x} + \mathbf{xDx}$$

$$S.t.: \mathbf{Ax} \le \mathbf{B}$$

$$\mathbf{x} \ge \mathbf{0}$$

$$Maximize\ :\ f(\mathbf{x}) = 3x_1 + 2x_2 - x_1^2 - x_1x_2 - x_2^2$$

$$S.t.: 4x_1 + 5x_2 \le 20$$

$$x_1, x_2 \ge 0$$

Other Mathematical Programming (Cont'd)

- Geometric Programming

$$Minimize\ : f(\mathbf{x}) = \sum_{j=1}^{m} (C_j \prod_{i=1}^{n} x_i^{a_{ij}})$$

$$S.t.: x_i > 0,$$

$$c_j > 0$$

$$a_{ij}\ are\quad arbitrary\quad real\quad numbers.$$

$$i = 1,2,\ldots, n$$

$$j = 1,2,\ldots, m$$

Other Mathematical Programming (Cont'd)

- Geometric Programming Example

$$Minimize\ :\ f(\mathbf{x}) = 2x_1x_2^{-1} + 3x_2x_3^{-2} + 2x_1^{-2}x_2x_3 + x_1x_2$$

$$S.t.: x_1, x_2, x_3 > 0$$