

# *Cryptology*

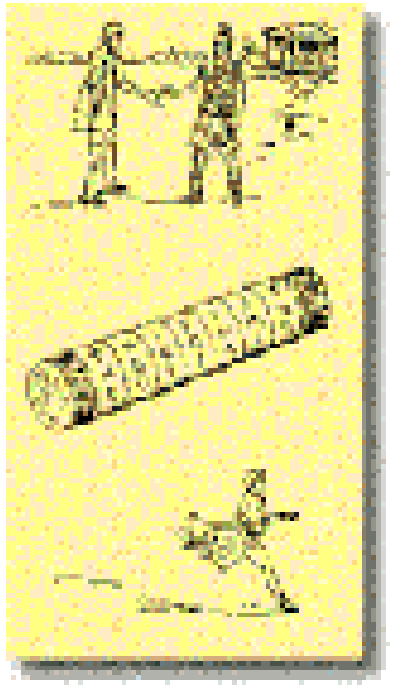


# *Enigma*

*Marian Rejewski,  
Jerzy Różycki,  
Henryk Zygalski*



# From Scytale to Enigma



**SCYTALE 400 BC**

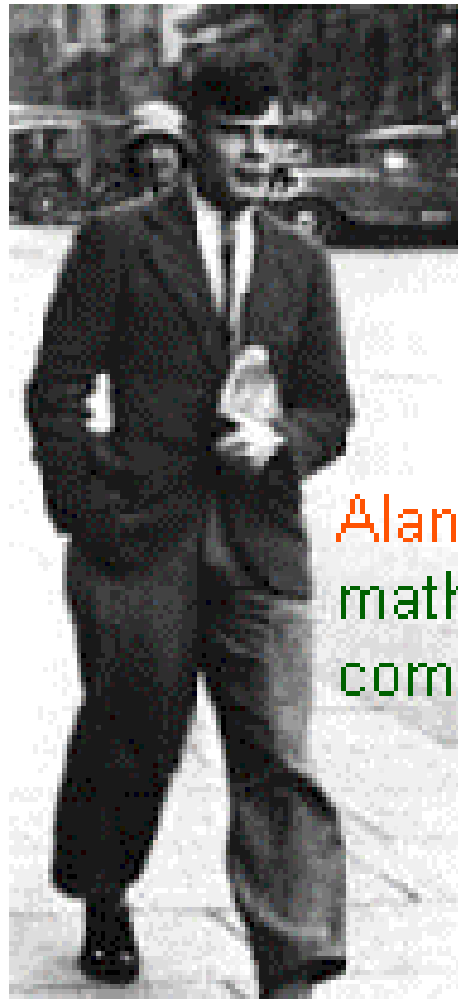


**POLYGRAPHIAE  
1518**

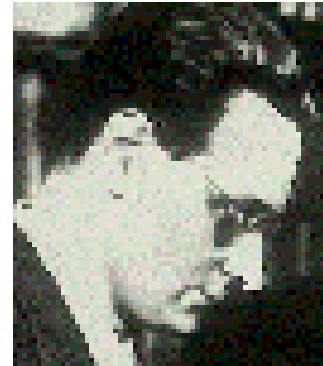


**ENIGMA**

# Computers & Ciphers



Alan Turing provides a mathematical model of computation



Konrad Zuse builds first computing machines

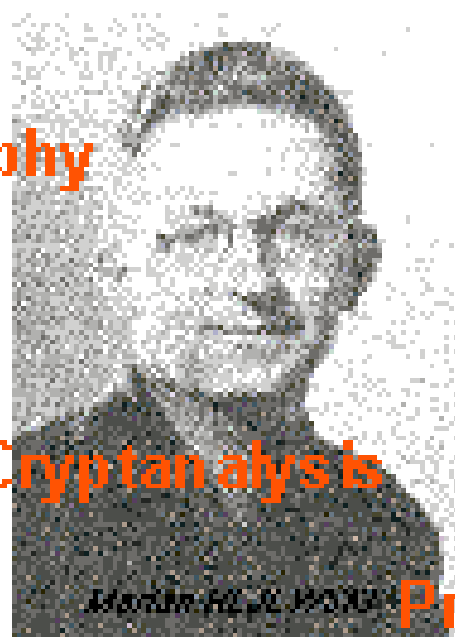


Marian Rejewski breaks Enigma

# From Enigma to Colossus



Cryptography



Cryptanalysis

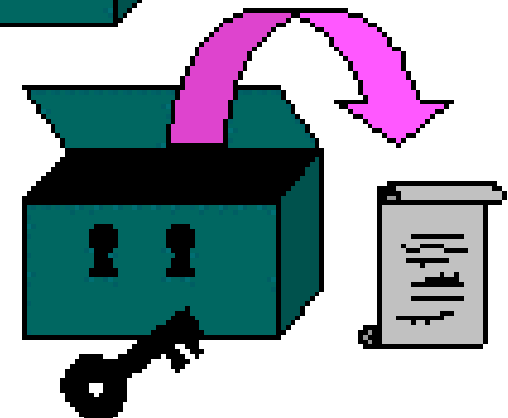
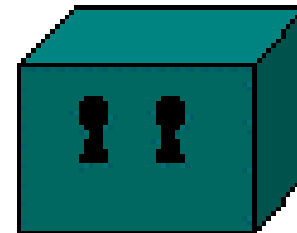
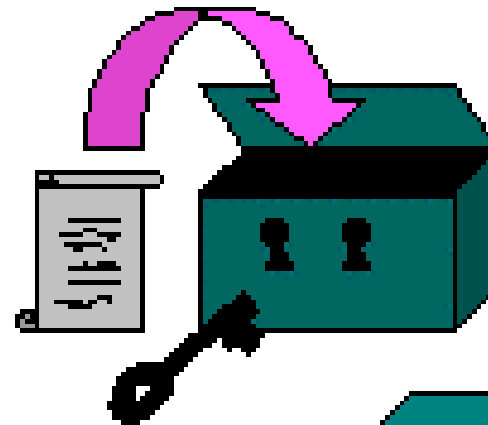
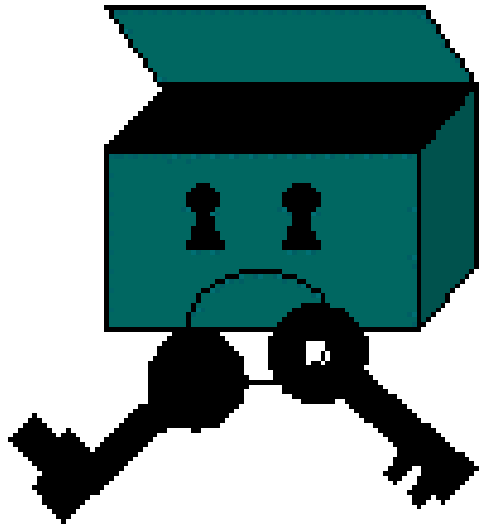
Progress in  
computer  
technology



Bletchley Park



# Public Key Cryptosystems



Public key  
locks the box

Private key  
unlocks the box

**FACTORING**

# The Killer App': Factorization



The *security* of the RSA encryption method uses the *difficulty* of factorizing large numbers.

February 2, 1999:  
140 digits required  
±- 2000 MIPS years.

(MIPS = Millions of  
Instructions Per Second)

```
RSA-140 =  
23298124631825175154149188281621153149188618396321123\  
6218233383215383949984856495913366513853821918336183\  
381381995331238889569238813441936413
```

can be written as the *product* of two 70-digit primes:

```
3398131423828438554538323621633815835633986495969591\  
423498929382113419 * 6264288381483285896353654948264\  
442219382851118623815819133668653946849
```

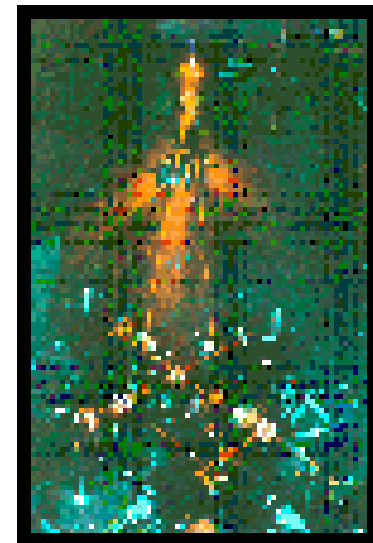
# Quantum Cryptology

Quantum factoring means...



Alternatives...

**QUANTUM CRYPTOGRAPHY** →





# ***What is the problem with classical cryptography?***

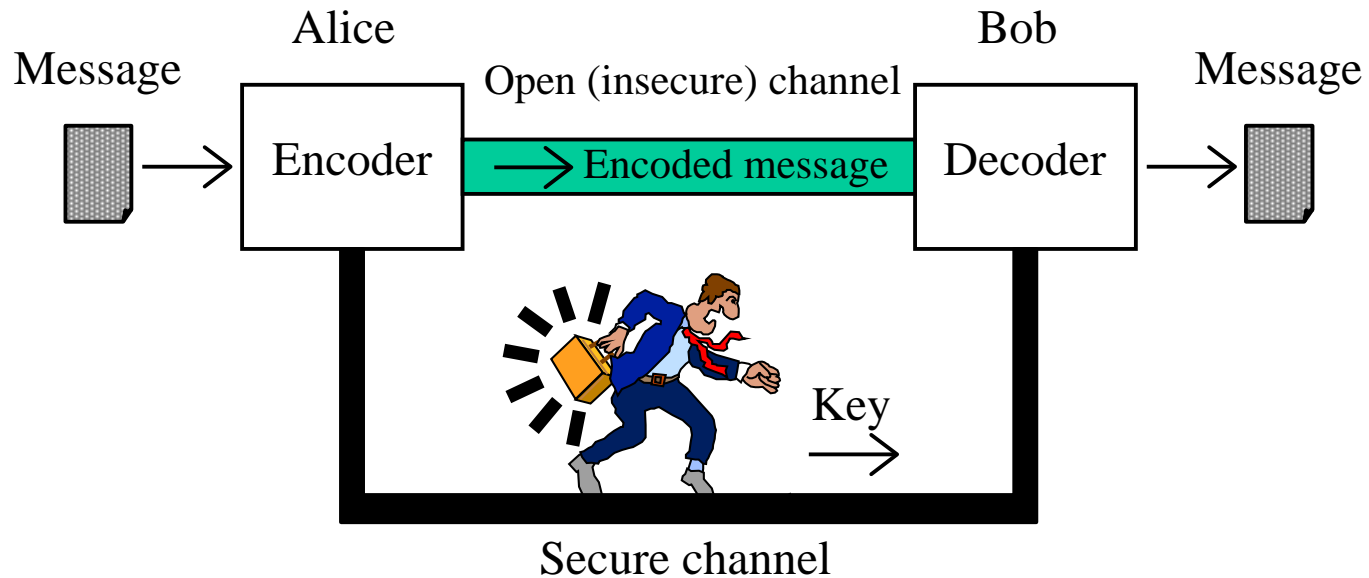
## ❖ **Secret key cryptography**

- Requires secure channel for key distribution
- *In principle* every classical channel can be monitored passively
- Security is mostly based on complicated non-proven algorithms

## ❖ **Public key cryptography**

- Security is based on non-proven mathematical assumptions (e.g. difficulty of factoring large numbers)
- We DO know how to factorize in polynomial time! Shor's algorithm for quantum computers. Just wait until one is built.
- Breakthrough renders messages insecure *retroactively*

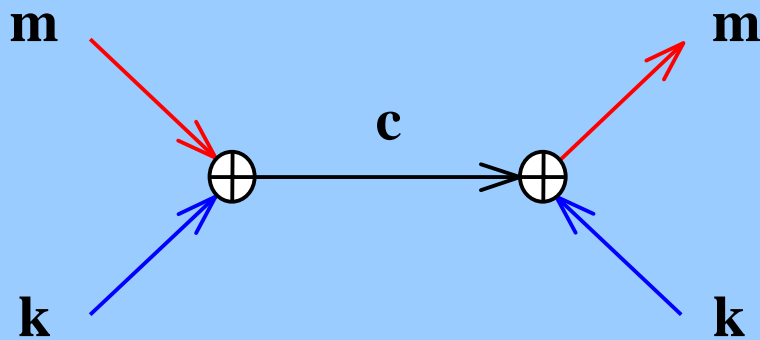
# *Key distribution*



- ❖ **Secret key cryptography requires secure channel for key distribution.**
- ❖ **Quantum cryptography distributes the key by transmitting quantum states in *open channel*.**

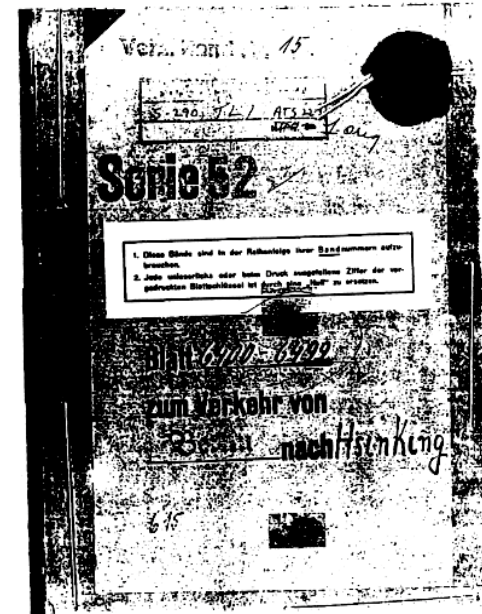
# *The holy grail: One-time pad*

- ❖ The only cipher mathematically proven
- ❖ Requires massive amounts of key material



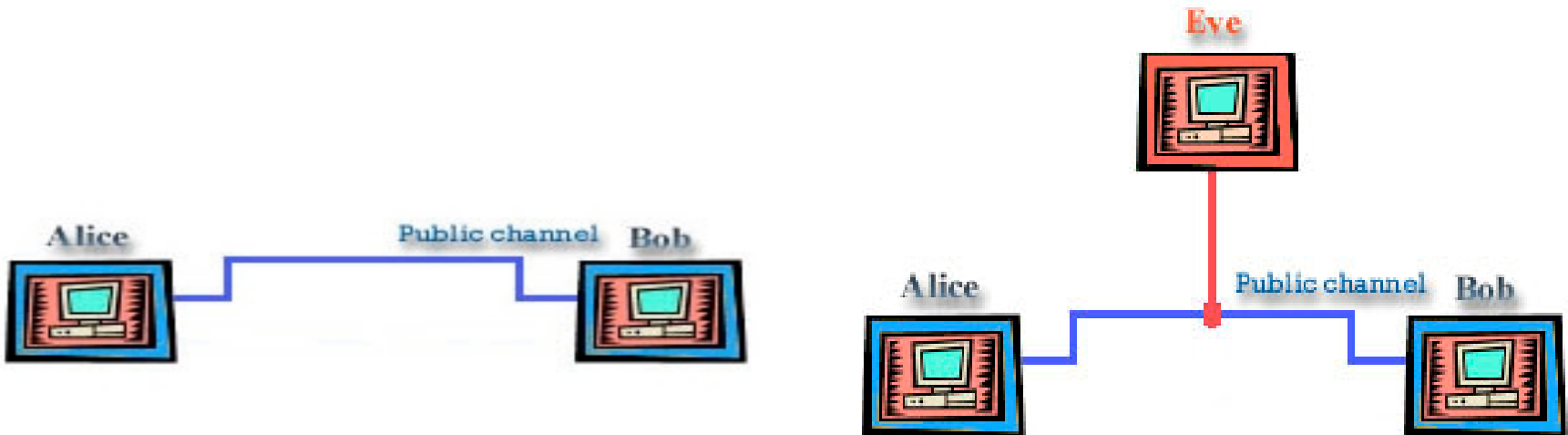
6451

7416R	53047	44636	47649	83461	03137
2966U	52537	72742	00121	80078	27567
66724	35079	44598	76371	29837	70579
43632	72103	80867	17661	27430	71118
72957	55188	45432	49696	26698	31812
75370	76236	91254	50685	76351	40993
90799	41393	21453	96296	89065	4246
81072	5R205	11264	99980	36343	24309



# *Crypto Definitions: Alice, Bob and Eve*

- ❖ It is a standard in cryptography to define the sender, receiver, and interceptor as:
  - **Alice** is the one who **sends the ciphertext**
  - **Bob** is the one who **receives the ciphertext**
  - **Eve** is the (evil) one who **tries to steal the plaintext or key**



# Quantum Cryptography

# *Quantum Cryptography*

- ❖ Application of Quantum Information rather than Quantum Computing
- ❖ Not really Cryptography – Quantum Key Distribution would be better name
- ❖ Here we describe one possible realization using photon polarizations as qubits
  - Nicholas Gisin and his group in Geneva are one of the leading players in this field

# *Quantum Cryptography - key*

## One-Time Pads

- ❖ Most secure cryptosystem – encode each bit of message using different secret random number

Encode:  $M = N + K \text{ modulo } 2$

Decode:  $M + K \text{ modulo } 2 = N$

- ❖ Problem: both sender (Alice) and receiver (Bob) need to have copy of same set of keys that eavesdropper (Eve) does not have

# *Secure Cipher*

- ❖ A secure cryptosystem can be produced from a random key which is as long as a message.
- ❖ **Process:** The ciphertext is the XOR of the random bits with the plaintext bits

```
Plaintext:  0 1 1 0 1 0 1 0 1 1 0 1 0 0 0
KEY:        1 1 0 0 1 1 1 0 0 1 0 0 1 0 1
Ciphertext: 1 0 1 0 0 1 0 0 1 0 0 1 1 0 1
KEY:        1 1 0 0 1 1 1 0 0 1 0 0 1 0 1
Plaintext:  0 1 1 0 1 0 1 0 1 1 0 1 0 0 0
```



# *Key Distribution*

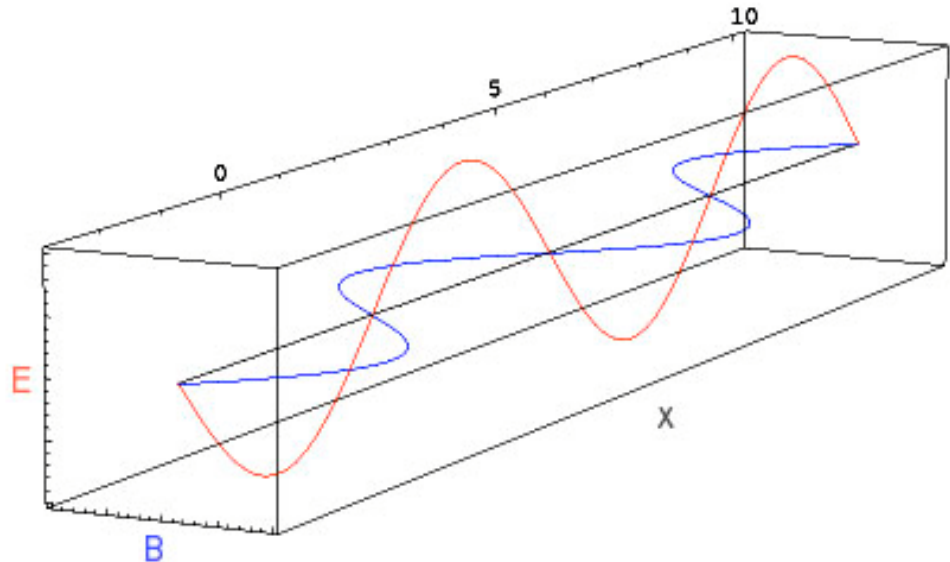
- ❖ **The problem** with a stream cipher of this form is the distribution of a key
- ❖ If the key **is short** so everyone can easily remember it, then it is also easy to break
- ❖ If the key **is long**, so it has to be sent **between the users**, it **could be intercepted** and compromised

# *Quantum Key Distribution*

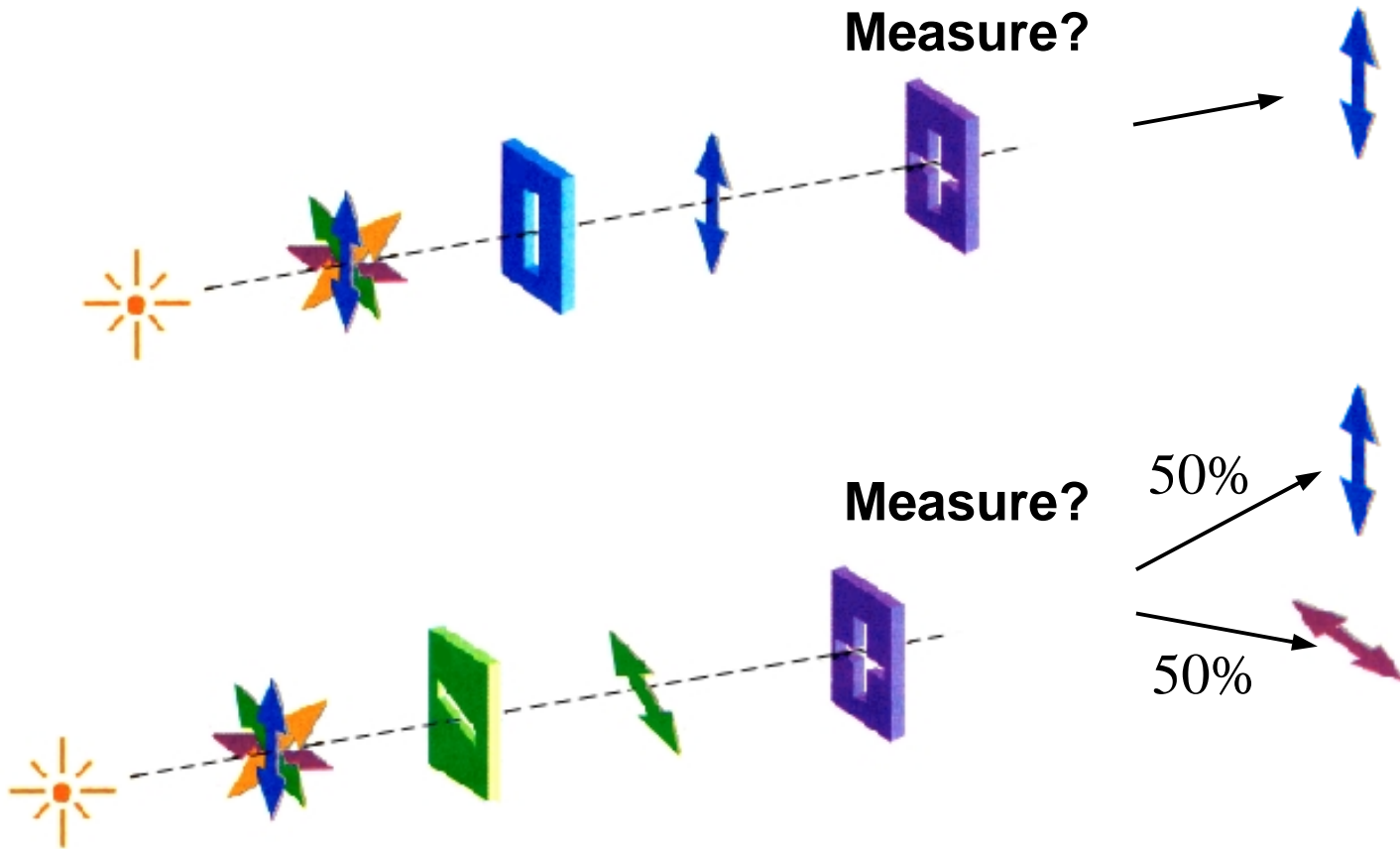
- ❖ **Quantum effects** can be used to distribute a long key with assurances that the key has not be intercepted
- ❖ To understand this process:
  - 1. Consider **another form of a qubit**
  - 2. The standard key distribution format
  - 3. Quantum effects

# Photons

- ❖ Another form of a qubit could be a **photon**
- ❖ A photon is an **electromagnetic wave** (we know it as light)
  - A photon consists of an **oscillating electric field** and an **oscillating magnetic field** which lie in perpendicular planes

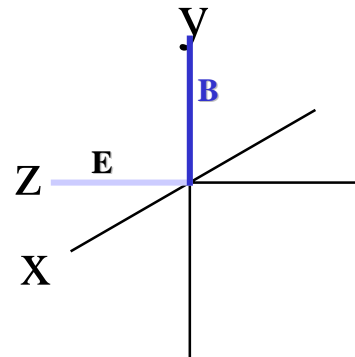
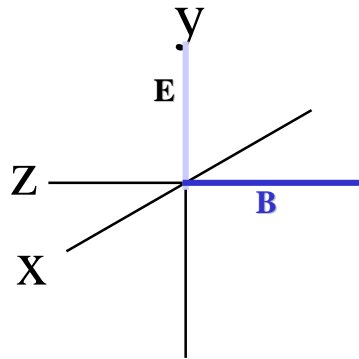


# *Qubit: polarization state of a single photon*



# ***Polarization***

- ❖ One property of a photon *that is of interest* is its polarization
- ❖ Polarization is a relative term that describes the plane of the electrical field
  - If the photon is traveling along the x-axis **then the electric field can be in any plane**



# *Photon Qubits*

- ❖ Define a reference line
- ❖ A photon which is polarized **at either 0 or 45 degrees of the line** is a 0
- ❖ A photon which is polarized **at either 90 or 135 degrees of the line** is a 1

**0 and 90 degrees are called *rectilinear polarization***  
**45 and 135 degrees are called *diagonal polarization***

# *Quantum Key Distribution I*

- ❖ Alice and Bob want to **exchange a key** on a **quantum channel** and **ciphertext** on a **normal channel**



If **photons** are exchanged,  
then the quantum channel  
could be a **fiber optics link**

# *Quantum Cryptography - polarization bases*

- ❖ Giles Brassard and Charles Bennett proposed using qubits to exchange secret keys in 1984
- ❖ **BB84 Scheme** uses polarization states of photon as qubit
- ❖ Alice can send photons :
  - Either - in Horizontal-Vertical Basis with polarizers set at 0 and 90 degrees
  - Or - in Diagonal Basis with polarizers set at 45 and 135 degrees



# *Quantum Cryptography - bases*

- ❖ Bob can also choose to receive photons either in H-V basis or in Diagonal basis
  - he does not know Alice's settings in advance
- ❖ If Alice sends a '1' using H-V setting but Bob measures photon in Diagonal setting, Bob will measure a '1' 50% of the time and a '0' for the remaining 50%

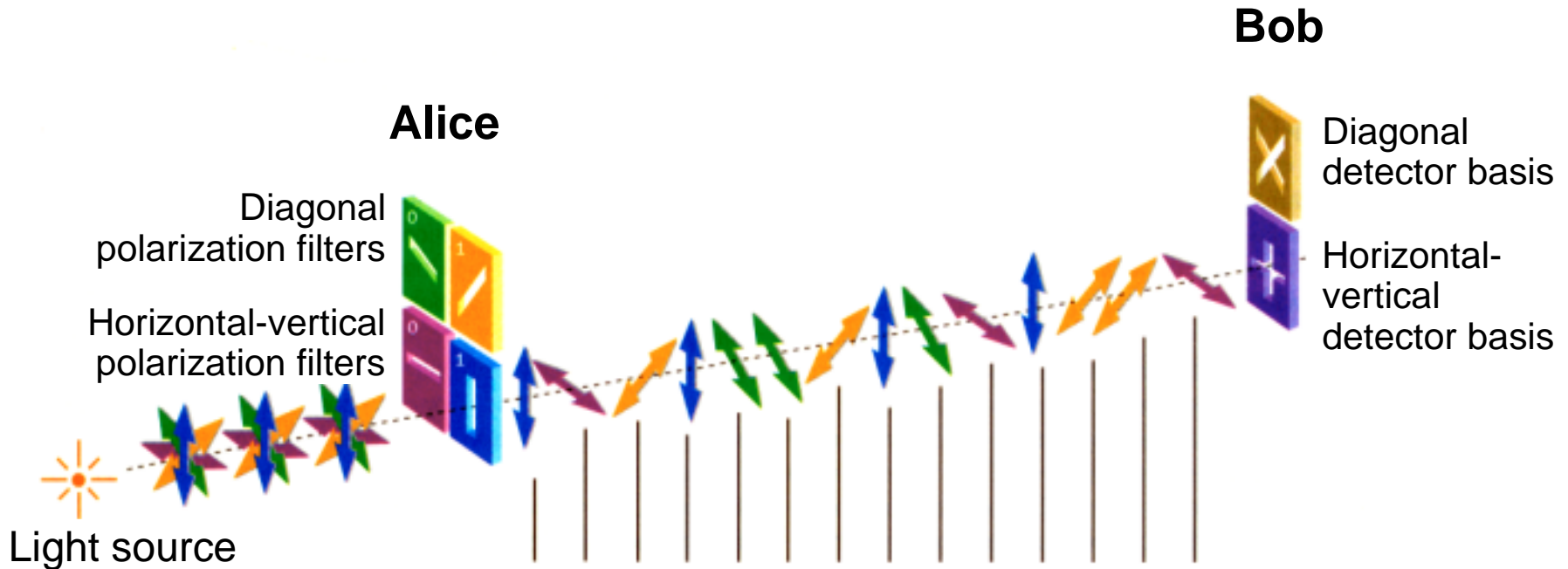
How does this help?

# *Quantum Cryptography*

## *State of the Art*

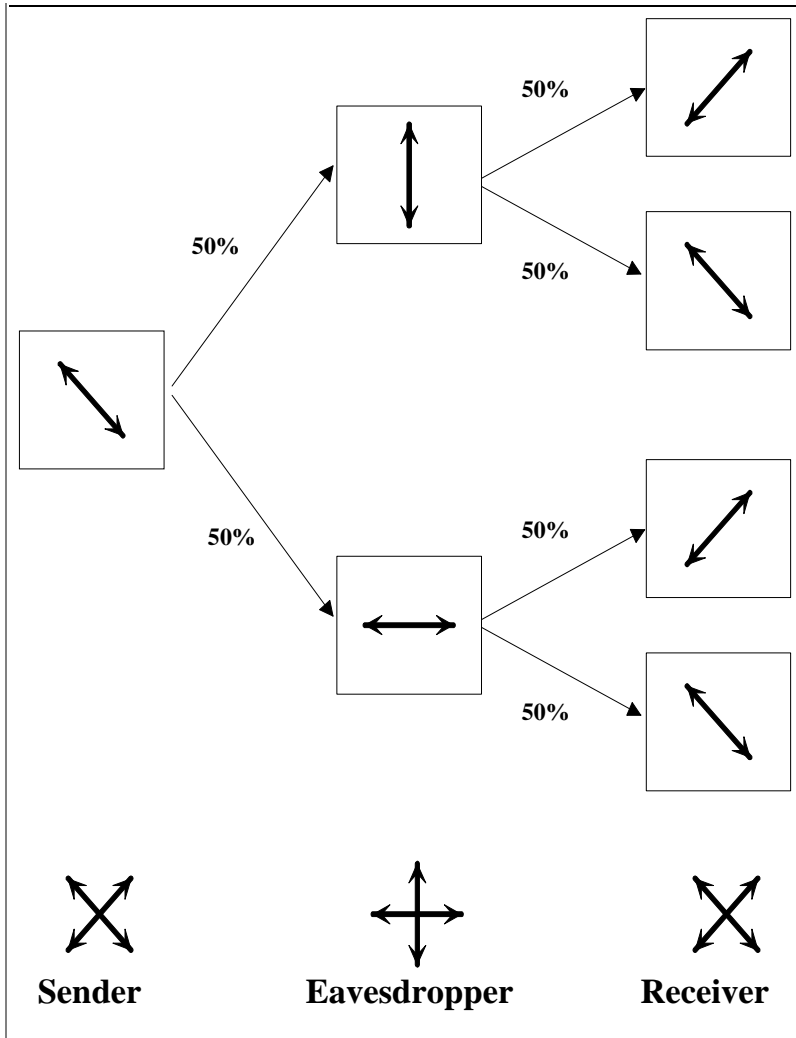
- ❖ First demonstration system built by Charles Bennett at IBM in 1989
- ❖ Many groups now demonstrated real systems transmitting keys down commercial optical fibres over many kilometres e.g. Richard Hughes at LANL, Nicholas Gisin under Lake Geneva, Paul Townsend at BT
- ❖ Hughes group also demonstrated free space transmission possible

# Quantum key distribution



Alice's bit sequence	1	0	1	1	0	0	1	1	0	0	1	1	1	0
Bob's detection basis	+	x	+	+	x	x	+	+	x	+	x	x	+	+
Bob's measurement	1	0	0	1	0	0	1	1	0	0	0	1	0	0
Retained bit sequence	1	-	-	1	0	0	-	1	0	0	-	1	-	0

# Eavesdropping with wrong reference system



Sender	Tyvlytter		Mottaker
	Referanse	Resultat av måling	
"0"	Rett	"0" Rett	Rett
	Galt	"0" Rett "1" Galt	<del>Rett </del> Galt
"1"	Rett	"1" Rett	Rett
	Galt	"0" Galt "1" Rett	<del>Rett </del> Galt
"0"	Rett	"0" Rett	Rett
	Galt	"1" Galt "0" Rett	<del>Rett </del> Galt
			Rett
"1"	Rett	"1" Rett	Rett
	Galt	"1" Rett "0" Galt	<del>Rett </del> Galt
			Rett

# *Complete Example - Set Up*

- ❖ Alice selects a random sequence of bits
  - Out of this sequence, Alice and Bob will ultimately construct a common key

❖ Example:

1 1 1 1 1 0 0 1 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 1 1

**Alice must encode these into polarized photons and send them to Bob along the quantum channel**

# *Encoding Process*

❖ Alice chooses to encode each bit in either the **rectilinear polarization (+)** form or the **diagonal polarization (x)** form

❖ Summary of polarization forms: **1**:  $\mathbf{x} /$   
+ -

**0**:  $\mathbf{x} \backslash$   
+ |

1 in  
rectilinear



# *Alice's Choice*

❖ Say Alice sends the random bits with the following choice of polarization:

1	1	1	1	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1
x	+	x	x	x	x	x	+	x	x	+	+	+	+	x	x	+	+	x	+	+	x	x	+	+	x	x	+	x	x
\	-	\	\	\	/	/	-	/	\					\	/			/	-		/	/			/	\		\	\

polarization

received

Alice sends this sequence of polarized photons to Bob

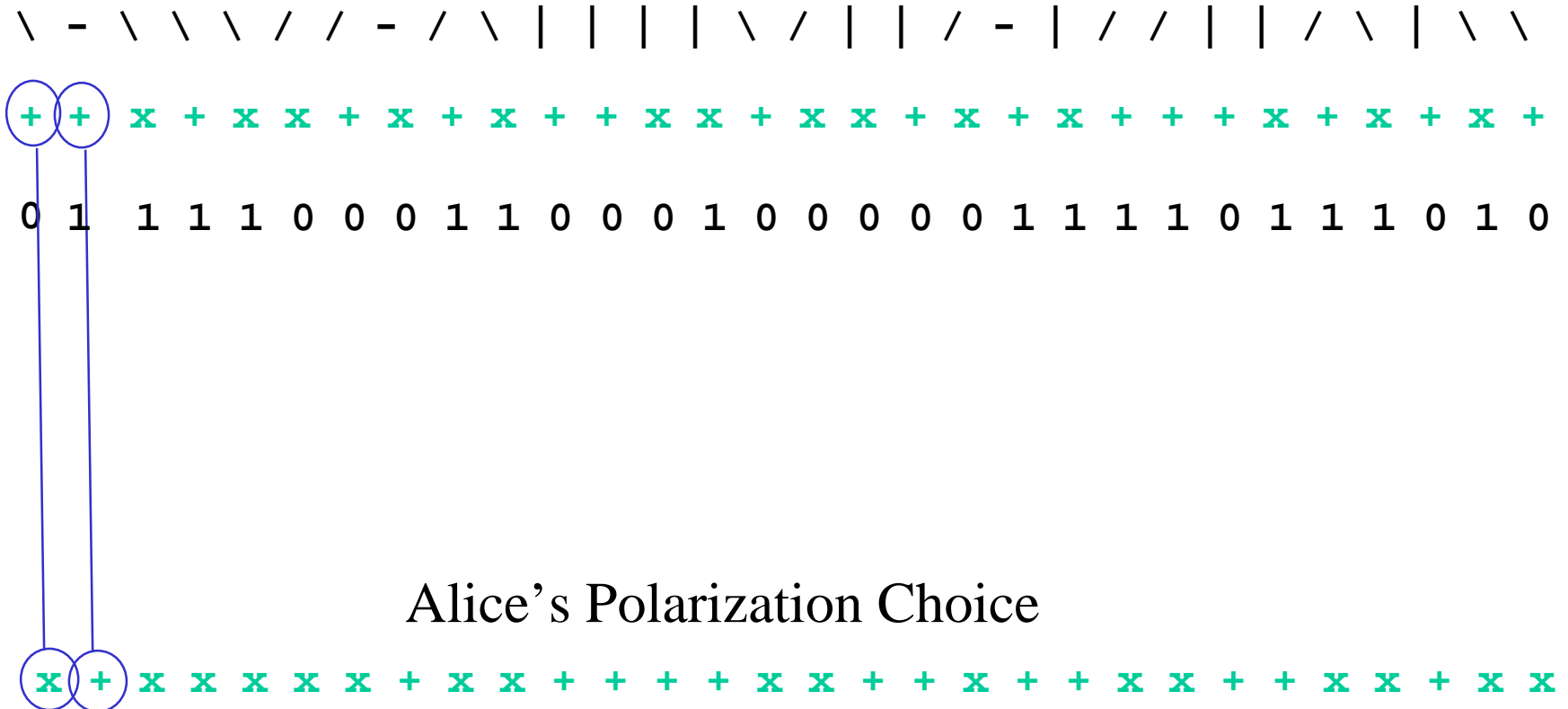
# ***Bob's Task***

- ❖ Bob must **measure** the **direction** of the **polarization** of Alice's photons to reconstruct the set of bits
  - However, Bob does not know when Alice used rectilinear or diagonal polarization **so he has to guess**
  - **If his guess is correct** then he will recover the correct bit
  - **If his guess is wrong**, he has **only a 50-50 chance** of recovering the correct bit



# Decoding

❖ Bob receives Alice's polarized photons:



# *Eavesdropping Test*

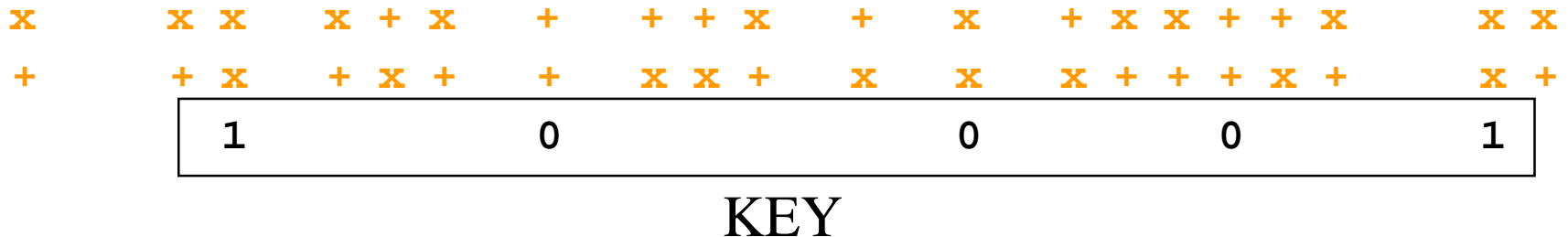
- ❖ Now **Alice tells Bob** the polarizer orientation for a subset of the bits and **Bob tells Alice** the orientations he used on that same subset
  - For those cases where Alice and Bob agreed, Alice tells him what bit values he should have received

x	+	x	x	+	x	+	x	+	x	+	x
+	+	+	+	x	+	x	+	+	+	+	+
1					0			0		0	
1					0			0		0	

**They agree so there was no eavesdropping**

# Common Key

- ❖ Since the channel is secure, Alice sends Bob the polarization orientation for another subset of the bits
  - Bob compares the actual polarization with his guess
  - He only uses the bits for which the two polarizations match (and so does Alice)



# *Quantum Cryptography - bad Eve*

- After sending stream of bits in randomly chosen settings, Alice then telephones Bob and they agree which are the 'good' bits

## What use is this?

- ❖ Suppose Eve is intercepting the bits from Alice and re-sending them on to Bob
- ❖ Since Eve has to guess which setting Alice used (H-V or Diagonal) there is now a probability of  $\frac{1}{4}$  for Alice and Bob to **disagree** on the bit sent even when they use the same settings

# *The Effect of Eve*

- ❖ If Eve intercepts and measures the photons, she has to send her measured values on to Bob

Alice's Message

\ - \ \ \ / / - / \ | | | | \ / | | / - | / / | | / \ | \ \

Eve's Guess

+ + x + x + + x x + + x + + x + x + x x x + x x + x + x +

0 1 1 1 1 0 0 1 0 0 0 0 1 0 0 0 0 0 1 1 1 0 1 1 0 0 1 0 1 0

Eve's Message

| - \ - \ | | \ / | | | | \ | | / | / - \ \ / - \ / | \ | \ |

# *Bob's Test*

❖ Bob, unaware of Eve's presence, decodes the photons:

Eve's Message  
| - \ - \ | | \ / | | | \ | | / | / - \ \ / - \ / | \ | \ |  
Bob's Guess  
+ + x + x x + x + x + + x x + x x + x + x + + + x + x + x +  
0 1 1 1 1 1 0 1 1 1 0 0 1 0 0 0 0 0 1 1 1 1 1 0 0 0 1 0 1 0

Alice selects check bits – sends polarization and bit information

1 1 0 1 0 0 0 1 0 0

**ERROR**

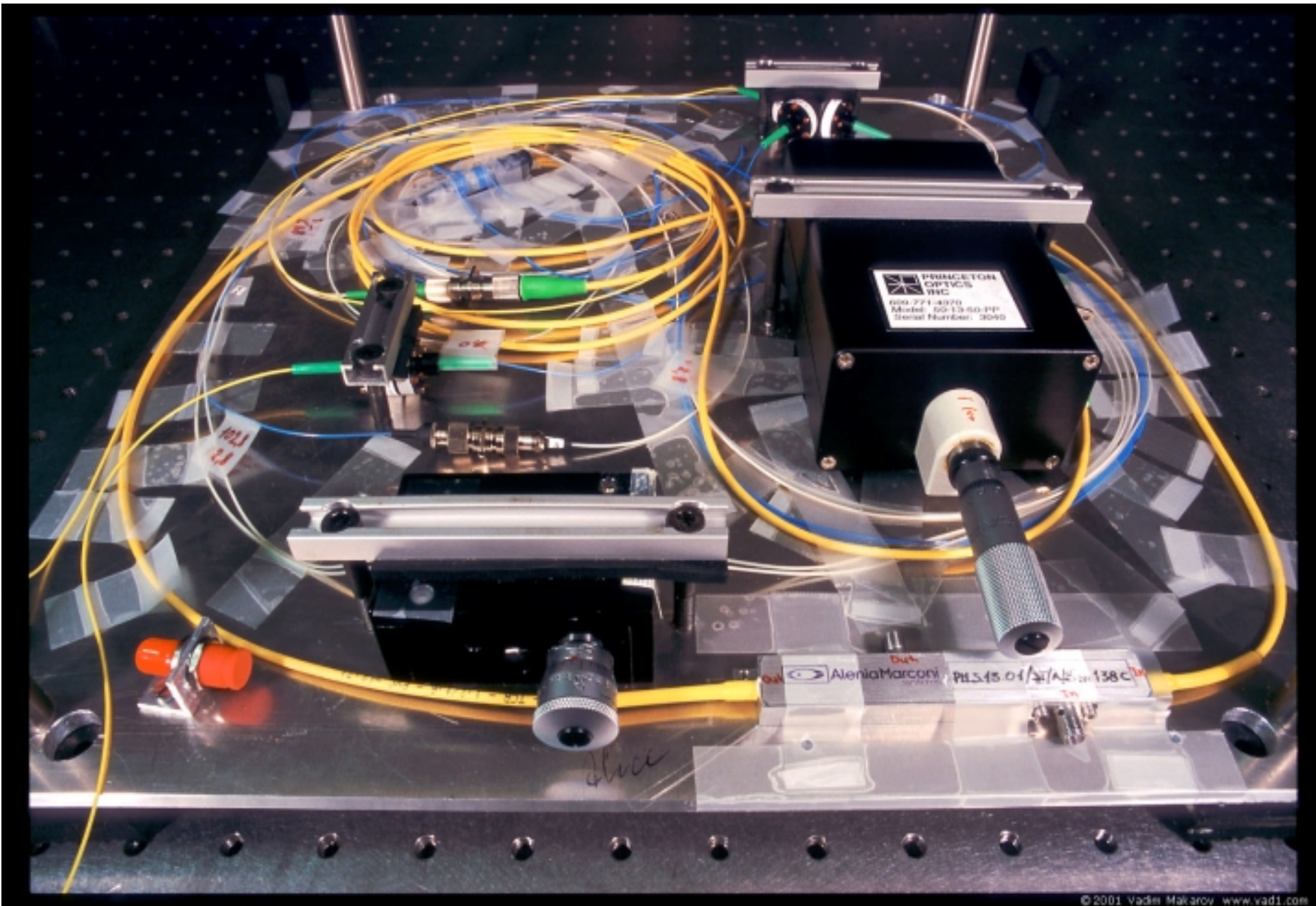
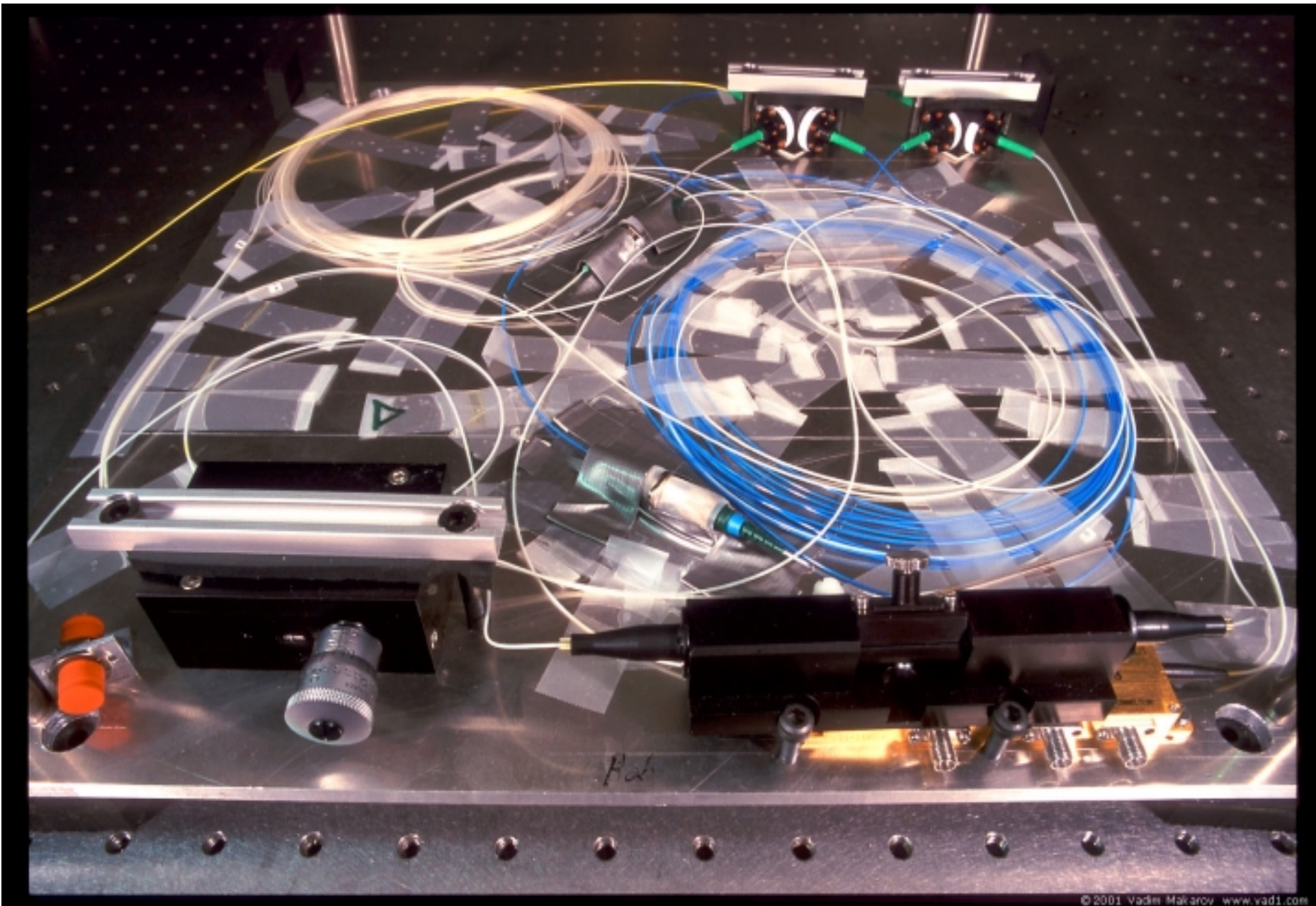


Photo 1. **Alice** (uncovered, no thermoisolation installed)



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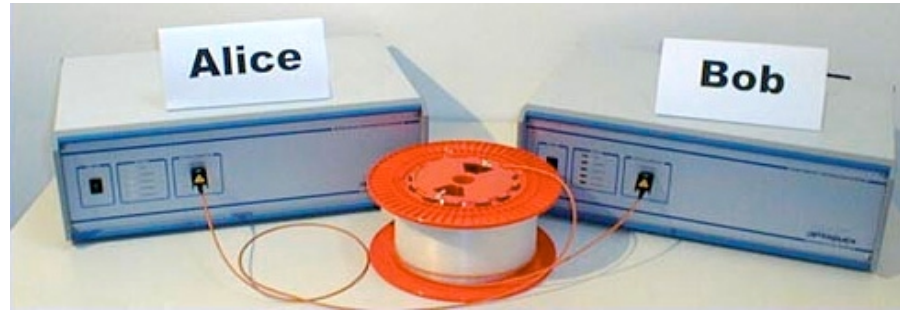
Photo 2. **Bob** (uncovered, no thermoisolation installed)



# *Commercial status*

- ❖ id Quantique (Geneva)

first commercially available quantum key distribution system:



- ❖ MagiQ Technologies (Boston)

- ❖ EQUIS project (Heriot-Watt University and Corning; UK)

compact integration into standard PCs

- ❖ + several research groups, telecom/ electronics companies

## Main features

- ▶ First quantum cryptography system
- ▶ Security guaranteed by quantum physics
- ▶ Point-to point key distribution
- ▶ Standard optical fiber
- ▶ Distances up to 70 km
- ▶ Key rate up to 1000 bits/s
- ▶ Compact and reliable

Key distribution is a central problem in cryptography. Currently, public key cryptography is commonly used to solve it. However, these algorithms are vulnerable to increasing computer power. In addition, their security has never been formally proven.

Quantum cryptography exploits a fundamental principle of quantum physics - observation causes perturbation - to distribute cryptographic keys with absolute security.

id Quantique is introducing the first quantum key distribution system. It consists of an emitter and a receiver, which can be connected to PC's through the USB port.

## *id Quantique*

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email: [info@idquantique.com](mailto:info@idquantique.com)  
web: <http://www.idquantique.com>



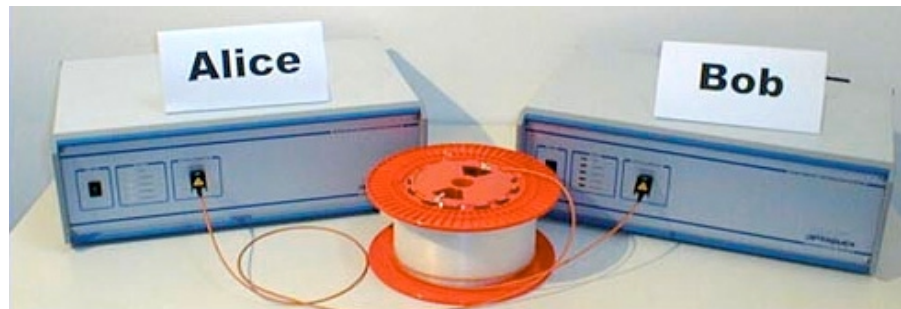
# *Geneva University spin-off makes quantum leap in secure communications*

- ❖ Swiss company **id Quantique** has announced what it claims are the first commercially available quantum cryptography products - a key distribution system **a random number generator**.
- ❖ **id Quantique** is a spin-off of the **University of Geneva**. It was created in October 2001 by four researchers of the Group of Applied Physics.
- ❖ **id Quantique** is in the news for receiving the coveted **Vigier Foundation Prize**.
- ❖ The company has set its sights on becoming a leader in **novel secure communication systems** based on quantum photonics.
- ❖ In this domain, **id Quantique** has launched **two products**:
  - The first one, the "**Quantum Random Number Generator**" (**QRNG**), is a physical random number generator based on a quantum physical law.
  - It exploits a **truly random process** - the reflection or the transmission of a single photon upon incidence on a semi-transparent mirror -
  - to generate high quality random numbers for **cryptographic purposes, numerical simulations, statistical studies or gambling**.

- ❖ The company's main product is a **QKD** system, which enables remote parties to exchange a private cryptographic key in absolute secrecy, **even if other parties are trying to eavesdrop**.
- ❖ The **key** is **exchanged in the form of a sequence of single photons** in an optical fiber.
- ❖ Because of the quantum properties of the photons, **eavesdropping inevitably perturbs the communication and so is immediately detected**.

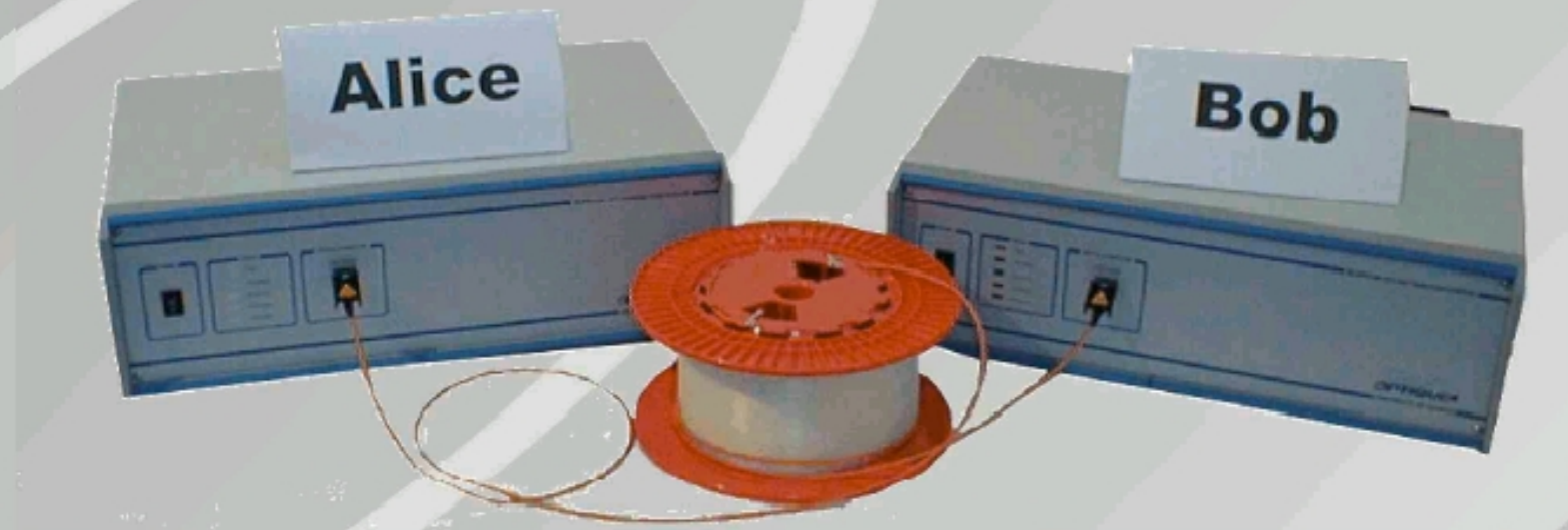
- ❖ The prototype commercial system consists of two PC-sized boxes at either end of a fiber-optic cable, **containing lasers** for generating the photons, detectors and cooling devices.
- ❖ It has been **successfully tested over a 67-kilometer** fiber-optic link between Geneva and Lausanne, Switzerland, with a net key distribution rate over that distance of 60 bps (bits per second).
- ❖ The system can be **deployed over existing fiber-optic cables** and the distribution rate rises to over 1000 bps over shorter distances, id Quantique said.

- ❖ Eavesdropping inevitably perturbs the communication, and is immediately revealed.
- ❖ Hence, unlike all other technologies, this quantum key distribution (QKD) system allows to exchange a cryptographic key with absolute security, guaranteed by the laws of physics.



# Quantum Security... at last

## Quantum Key Distribution System



Key distribution over optical fiber  
with absolute security

## Performance

Key exchange rate <sup>1</sup> over		Units
10 km :	4000	Bits/s
20 km :	1500	Bits/s
50 km :	100	Bits/s

### Notes

<sup>1</sup>: The key exchange rate depends on the actual fiber attenuation.

## Interface

Platform	Windows 98 2 <sup>nd</sup> ed, 2000, ME, XP
Interface	USB version 1.0, 12 Mbit/s Plug & Play connection

## General information

	Emitter	Receiver	
Optical connector <sup>2</sup>		FC/PC	
Operating temperature		+10 to +30	°C
Dimension (L×W×H)		32 x 46 x 16	cm
Weight	13	7	kg
Power supply		110 - 230	VAC

### Notes

<sup>2</sup>: Other connector type available upon request

# True randomness upon request

## Quantum Random Number Generator QRNG



Although random numbers are required in countless applications, their generation is often overlooked. Being deterministic devices, computers are not capable of producing random numbers. A physical source of randomness is necessary. Quantum physics is intrinsically random. Therefore, it is natural to exploit a quantum process for such a source. It offers the advantage over conventional randomness sources like electronic noise of being robust and invulnerable to environmental perturbations.

The QRNG is a physical random number generator exploiting an elementary quantum optics process. Photons - light particles - are sent one by one onto a semi-transparent mirror and detected. The exclusive events (reflection - transmission) are associated to "0" - "1" bit values.

The operation of the QRNG is continuously monitored to ensure immediate detection of a failure.

A program supplied with the QRNG produces files of random numbers and implements standard randomness tests to analyze the data. A software development kit (option) allows simple and fast integration of the QRNG in an existing application.



# Never worry again about the quality of your random numbers!

## Main features

- ▶ First low-cost device based on quantum randomness
- ▶ Output data pass all randomness tests
- ▶ High generation rate
- ▶ Compact and reliable
- ▶ Immediate interruption in case of failure
- ▶ Easy integration in existing applications
- ▶ Acquisition and Randomness Tests program
- ▶ USB plug & play connection
- ▶ Self-powered through USB

## Applications

- ▶ Cryptography
- ▶ Numerical Simulations
- ▶ Statistical research

## *id Quantique*

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email: [info@idquantique.com](mailto:info@idquantique.com)  
web: <http://www.idquantique.com>



## Performance

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	Standard Version	Improved Version	Units
Random bits rate	10	100	kBits/s
Thermal noise contribution	< 2	< 0.5	%
Biasing factor*	< 0.1	< 0.1	-

---

### Notes

\*:  $|p_1 - p_0|$  where  $p_1$  and  $p_0$  are the probabilities to register a "0", respectively a "1".

## Interface

---

Platform	Windows 98 2 <sup>nd</sup> ed, 2000, ME, XP
Interface	USB version 1.0, 12 Mbit/s Plug & Play connection QRNG power supply

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### Notes

Alternate platform (Unix) and interface (ethernet) in development

## Software

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- <i>QRNG Tester</i>	Distributed with the QRNG Intuitive graphical user interface Acquisition of random number files (binary or text) Unbiasing procedure (Von Neuman, and Peres) Standard randomness tests
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## Software

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- *QRNG Tester*

Distributed with the QRNG  
Intuitive graphical user interface  
Acquisition of random number files (binary or text)  
Unbiasing procedure (Von Neuman, and Peres)  
Standard randomness tests

- *Software development kit*

Option  
Simple and fast integration in existing applications

## General information

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Operating temperature

+10 to +30

°C

Dimension (L×W×H)

68×150×188

mm

Weight

850

g

## Sales Contact

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For further information on this or other products, please contact *id Quantique* by phone: (+41) 022 702 69 29 or email: [info@idquantique.com](mailto:info@idquantique.com)

## Disclaimer

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The information and specification set forth above are subject to change at any time by *id Quantique* without prior notice. February 2002.

# *Conclusions?*

Feynman “not sure if there was a real problem with quantum mechanics”

- “Squeeze the difficulty of quantum mechanics into a smaller and smaller place”

Perhaps the foundation of a new multi-billion dollar industry!

# ***Possible Homework Projects***

- ❖ For a possible homework project, work out examples of quantum error correction schemes and compare them to digital error correction

# *Possible Exam Questions*

- ❖ Remember that even though each question is worth only 5 to 10 points, the points do add up to a significant contribution to your overall grade
- ❖ If there is a quiz it *might* cover these issues:
  - What is a quantum dot?
  - Why are errors a problem with quantum systems?
  - What does a controlled NOT gate do?