Reversible Logic Models: Billiard Ball and Optical

Lecture 4

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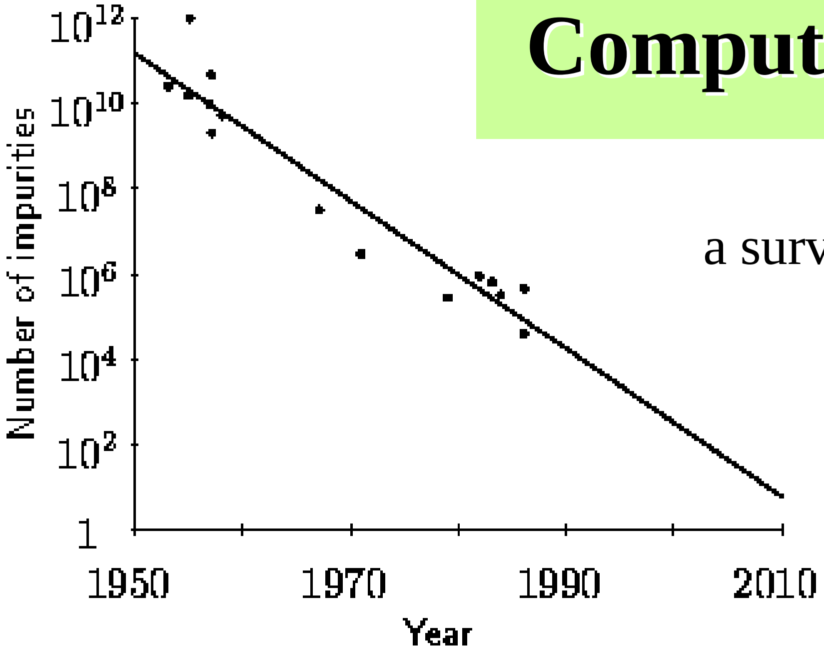
Reversible and Quantum Logic Fundamentals from the Logic Synthesis Point of View

Atom-scale computation:

- What will be the difficulties when we will try to build classical computers (Turing machines) on the **atomic scale**?
- One of the toughest problems to scale down computers is the **dissipated heat that is difficult to remove.**
- Physical limitations placed on computation **by heat dissipation were studied for many years (Landauer, 1961).**

Computing at the atomic scale:



a survey made by Keyes in 1988

R. W. Keyes, IBM J. Res. Develop. **32**, 24 (1988).

- Plot showing the number of dopant impurities involved in logic with bipolar transistors with year.
  - (Copyright 1988 by International Business Machines Corporation)

Reversible Logic

- Bennett showed that for power not be dissipated in the circuit it is necessary that arbitrary circuit should be build from ***reversible gates***.

Information is Physical

- **Is some minimum amount of energy required <u>per one computation step</u>?**

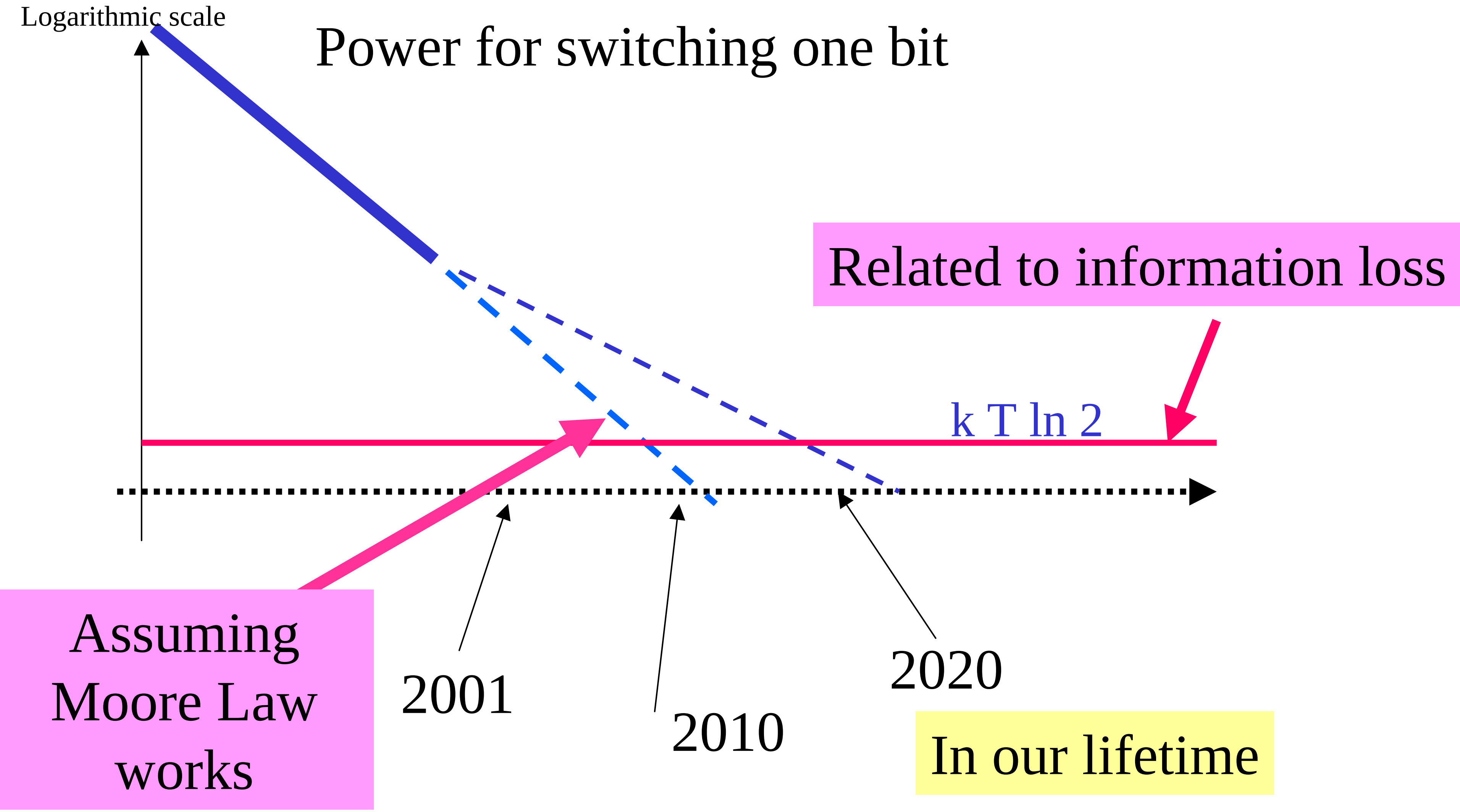
- Rolf Landauer, 1970. Whenever we use a logically irreversible gate we dissipate energy into the environment.

Information loss = energy loss

- The loss of information is associated with laws of physics requiring that *one bit of information lost* dissipates k T ln 2 of energy,
  - where k is ***Boltzmann' constant***
  - and T is the temperature of the system.
- Interest in **reversible computation** arises from the desire to *reduce heat dissipation*, thereby allowing:
  - higher densities
  - speed

R. Landauer, "Fundamental Physical Limitations of the Computational Process", Ann. N.Y. Acad.Sci, 426, 162(1985).

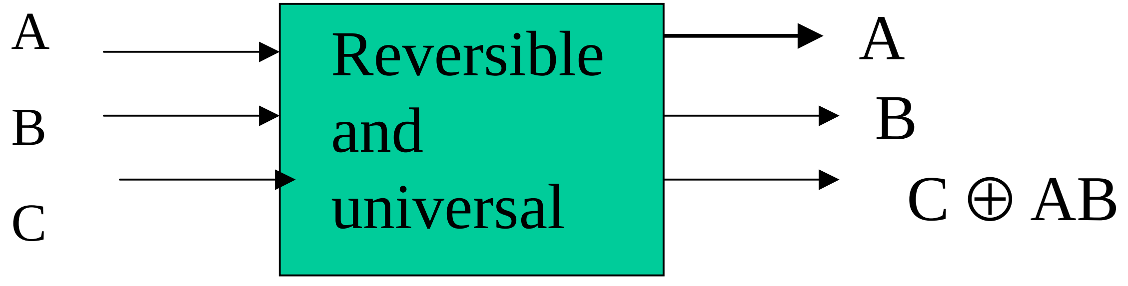
When will IT happen?



Information is Physical

- **Charles Bennett, 1973:** There are no unavoidable energy consumption requirements per step in a computer.
- Power dissipation of reversible circuit, under ideal physical circumstances, **is zero**.

- **Tomasso Toffoli, 1980:** There exists a reversible gate which could play a role of a universal gate for reversible circuits.



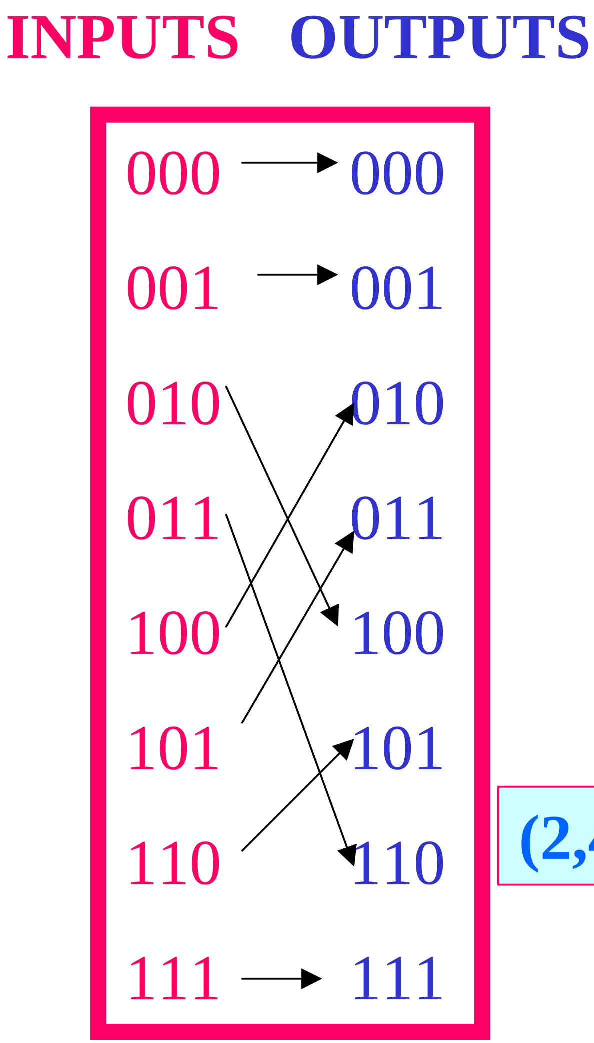
Reversible computation:

- Landauer/Bennett: **almost all operations required in computation** could be performed in a reversible manner, thus dissipating no heat!
- **The first condition** for any deterministic device to be reversible is that its input and output be uniquely retrievable from each other.
  - This is called logical reversibility.
- **The second condition:** a device can actually run backwards - then it is called physically reversible
  - and the second law of thermodynamics guarantees that **it dissipates no heat.**

→ Billiard Ball Model

Reversible logic

Reversible are circuits (gates) that have <u>*one-to-one*</u> ***mapping*** *between vectors of inputs and outputs; thus the vector of input states can be always reconstructed from the vector of output states.*



2 → 4

3 → 6

4 → 2

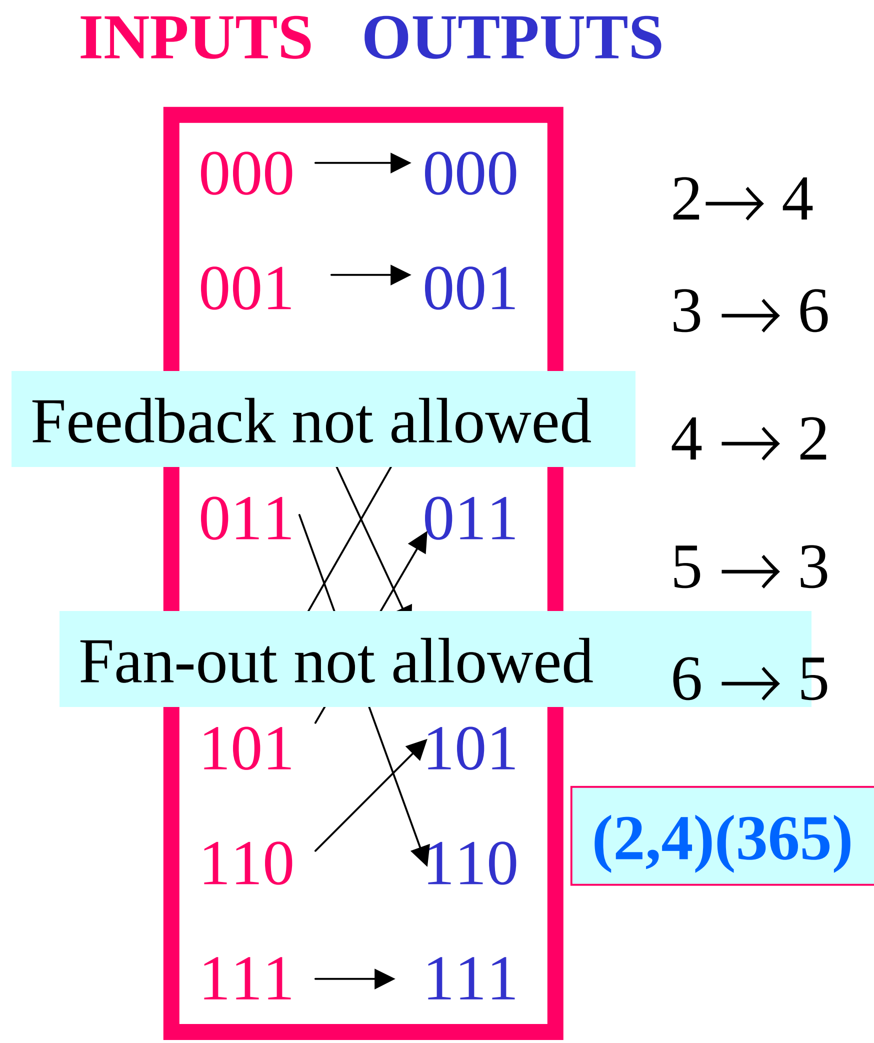
5 → 3

6 → 5

(2,4)(365)

Reversible logic

Reversible are circuits (gates) that ***have the same number of inputs*** ***<u>and</u>*** *outputs and have* *<u>one-to-one</u>* ***mapping*** *between vectors of inputs and outputs; thus the vector of input states can be always reconstructed from the vector of output states.*



Reversible logic constraints

Feedback not allowed in combinational part

In some papers allowed under certain conditions

Fan-out not allowed

In some papers allowed in a limited way in a "near reversible" circuit

Observations

- Every reversible gate can be described by a permutation.
- Every reversible circuit can be described by a **permutation**.
- Synthesis of a reversible circuit can be considered as **decomposition of a permutation** to a sequence of elementary permutations.
- **Group theory** has been successfully used for designing cascades of reversible gates.

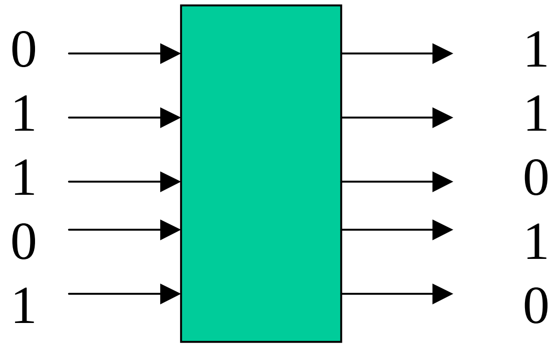
- **To understand reversible logic, it is useful to have intuitive feeling of various models of its realization.**

Our examples will illustrate reversible gates, conservative gates and synthesis principles

Conservative Reversible Gates

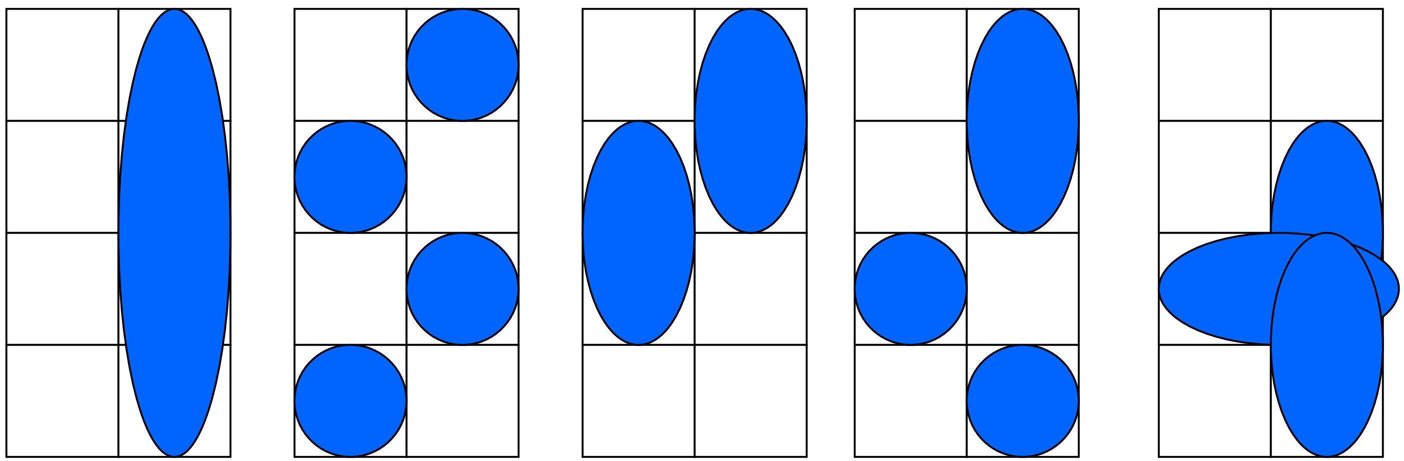
Definitions

- A gate with ***k*** inputs and ***k*** outputs is called a ***k*k*** gate.
- A ***conservative*** circuit preserves the number of logic values in all combinations.
- In ***balanced binary logic*** the circuit has half of minterms with value 1.



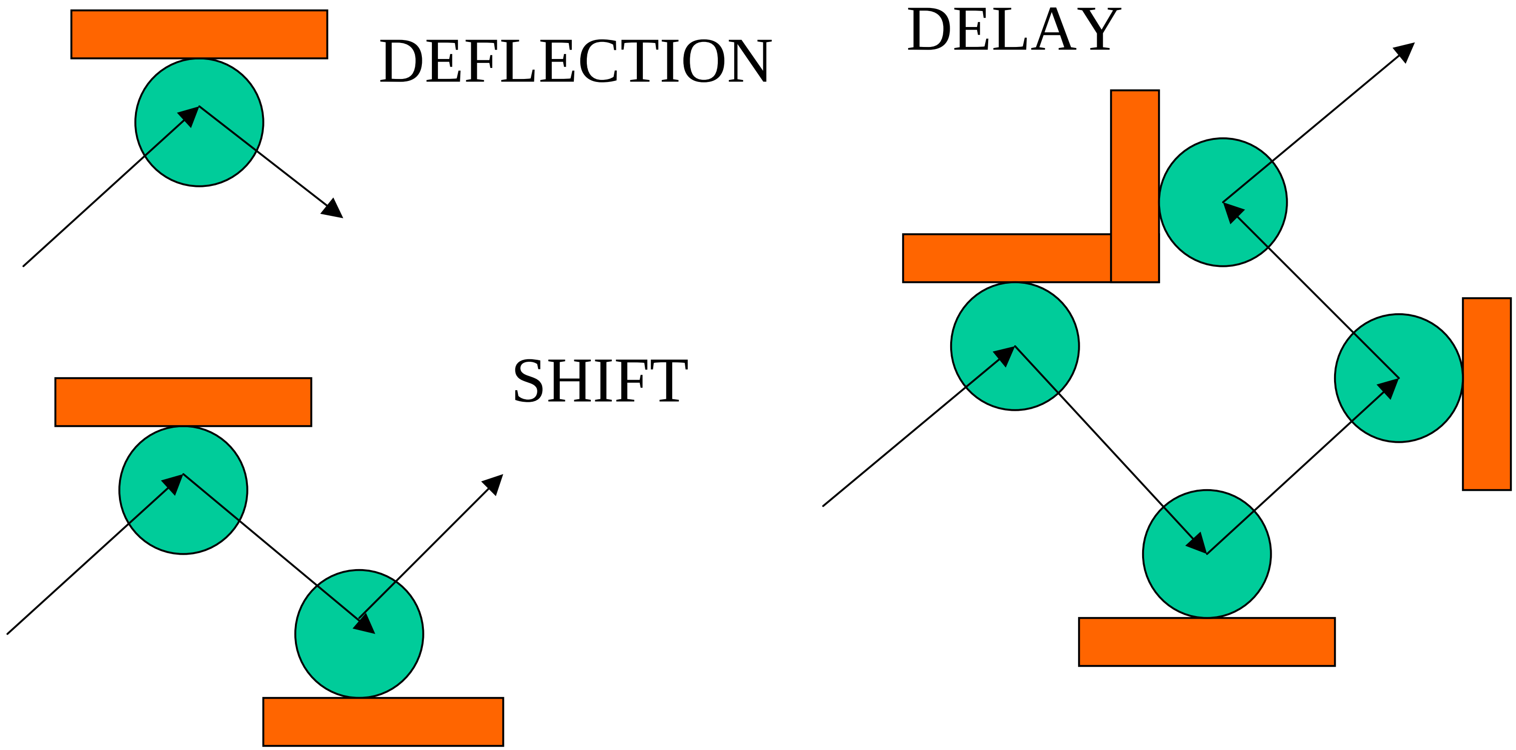
Conservative circuit = the same number of ones in inputs and outputs

variable xor Shannon Davio majority



Examples of balanced functions = half of Kmap are ones

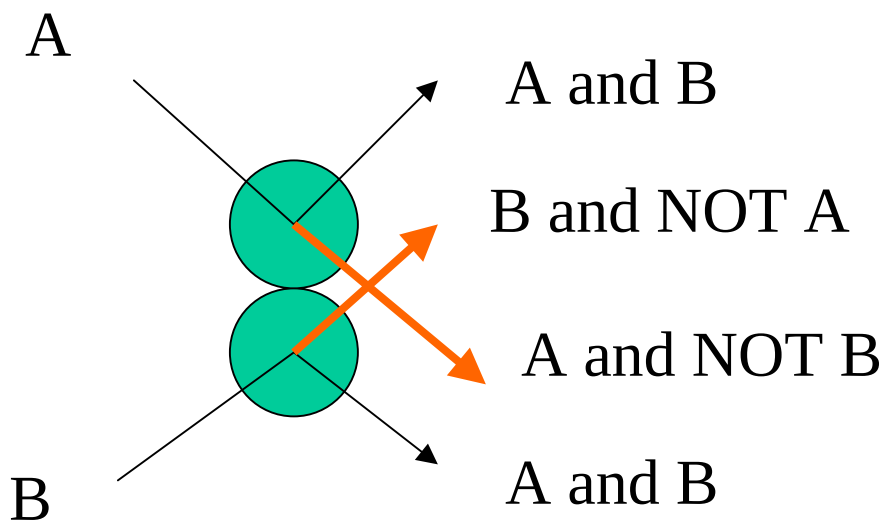
Billiard Ball Model



- This is described in E. Fredkin and T. Toffoli, “Conservative Logic”, Int. J.Theor. Phys. 21,219 (1982).

Billiard Ball Model (BBM)

Input		output			
A	B	1	2	3	4
0	0	0	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	0	0	1

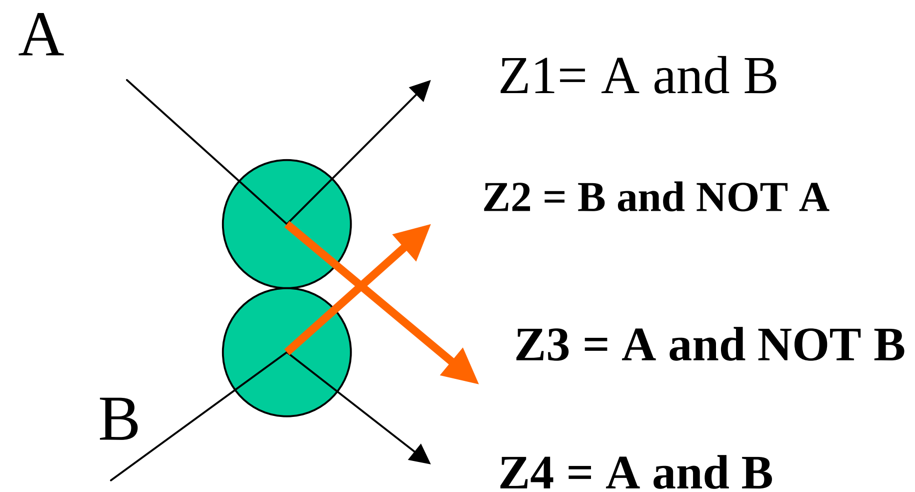


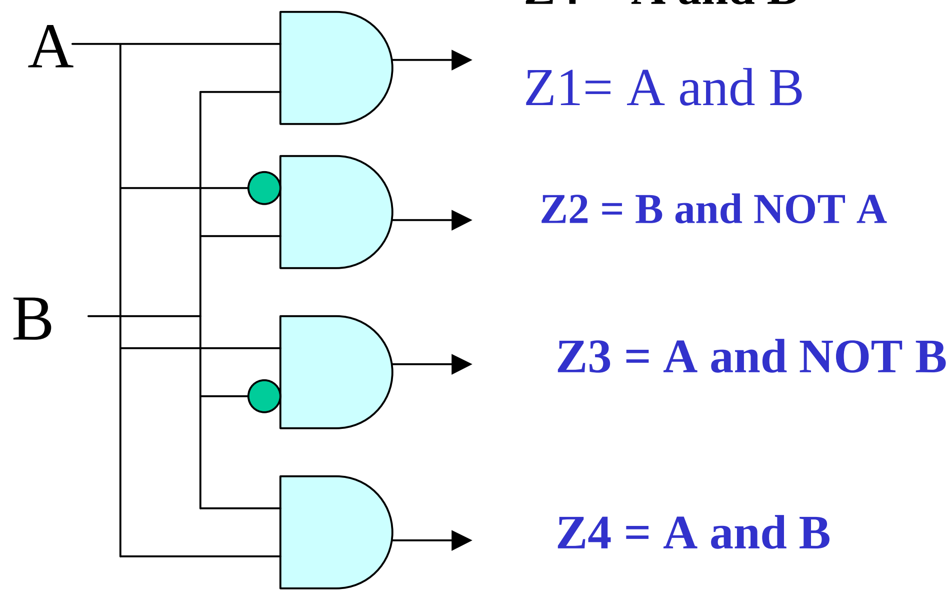
This is called Interaction Gate

This illustrates principle of conservation (of the number of balls, or energy) in **conservative** logic.

Interaction gate

Input		output			
A	B	z1	z2	z3	z4
0	0	0	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	0	0	1

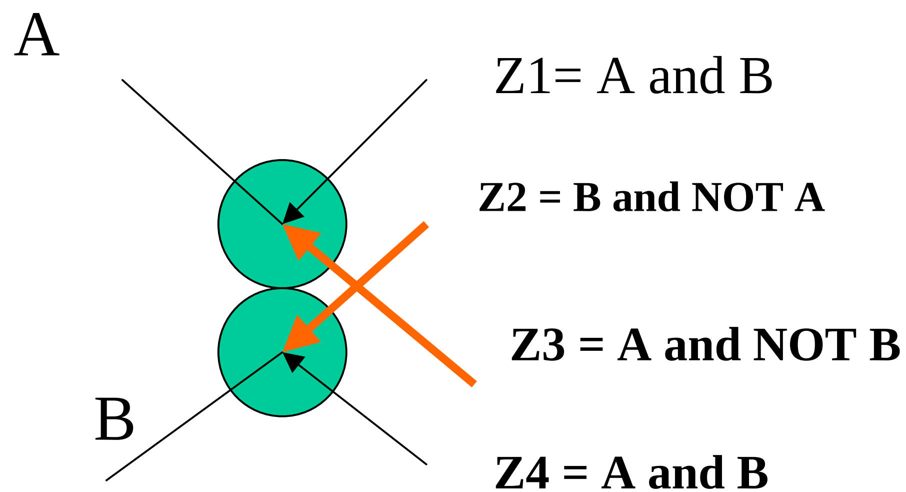


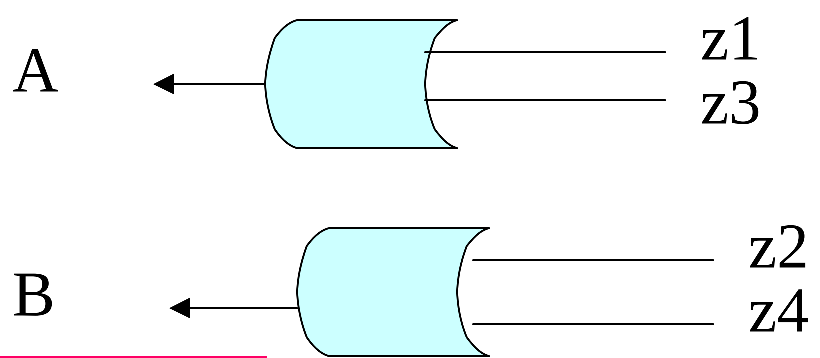


Inverse Interaction gate

input				output	
z1	z2	z3	z4	A	B
0	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
1	0	0	1	1	1

Other input combinations not allowed





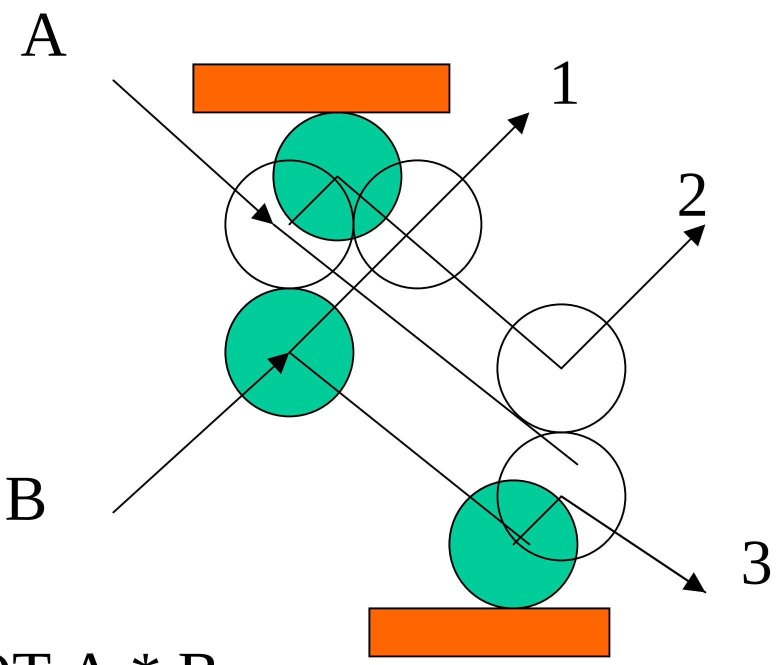
Only disjoint inputs are OR-ed

Designing with this types of gates is difficult

Billiard Ball Model (BBM)

switch

Input		output		
A	B	**z1**	**z2**	**z3**
0	0	0	0	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	1

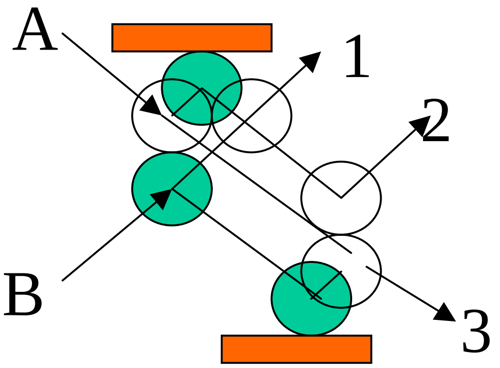


Z1 = NOT A * B

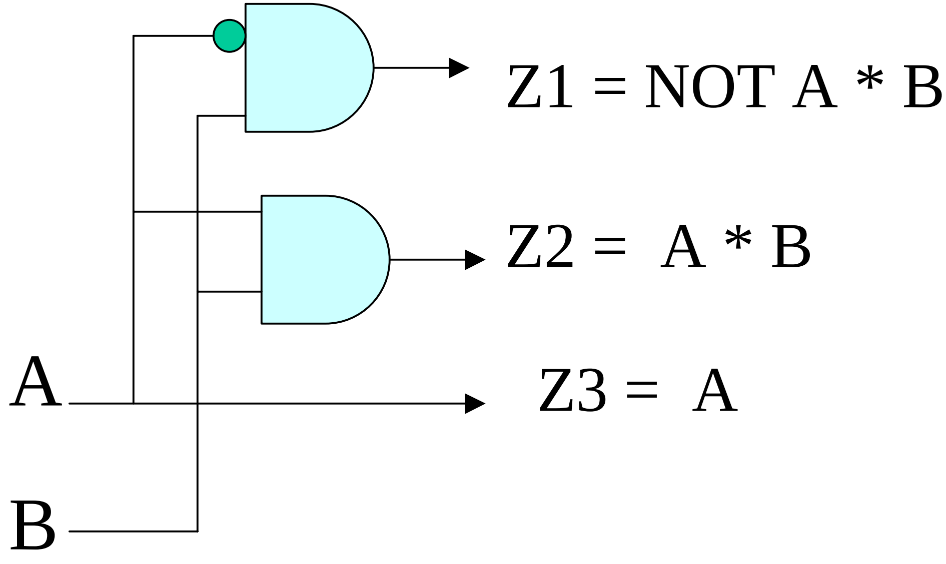
Z2 = A * B

Z3 = A

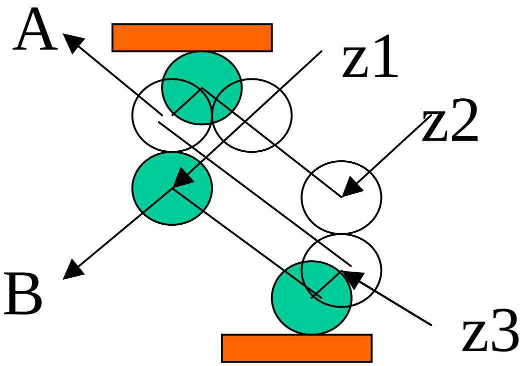
Switch Gate



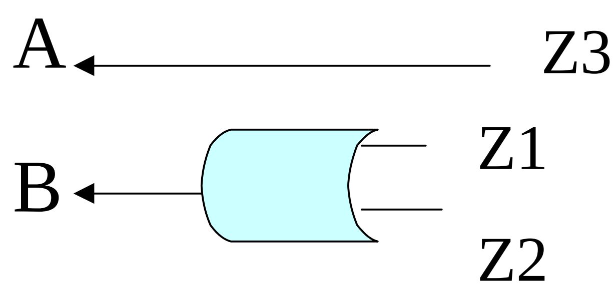
Input		output		
A	B	**z1**	**z2**	**z3**
0	0	0	0	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	1



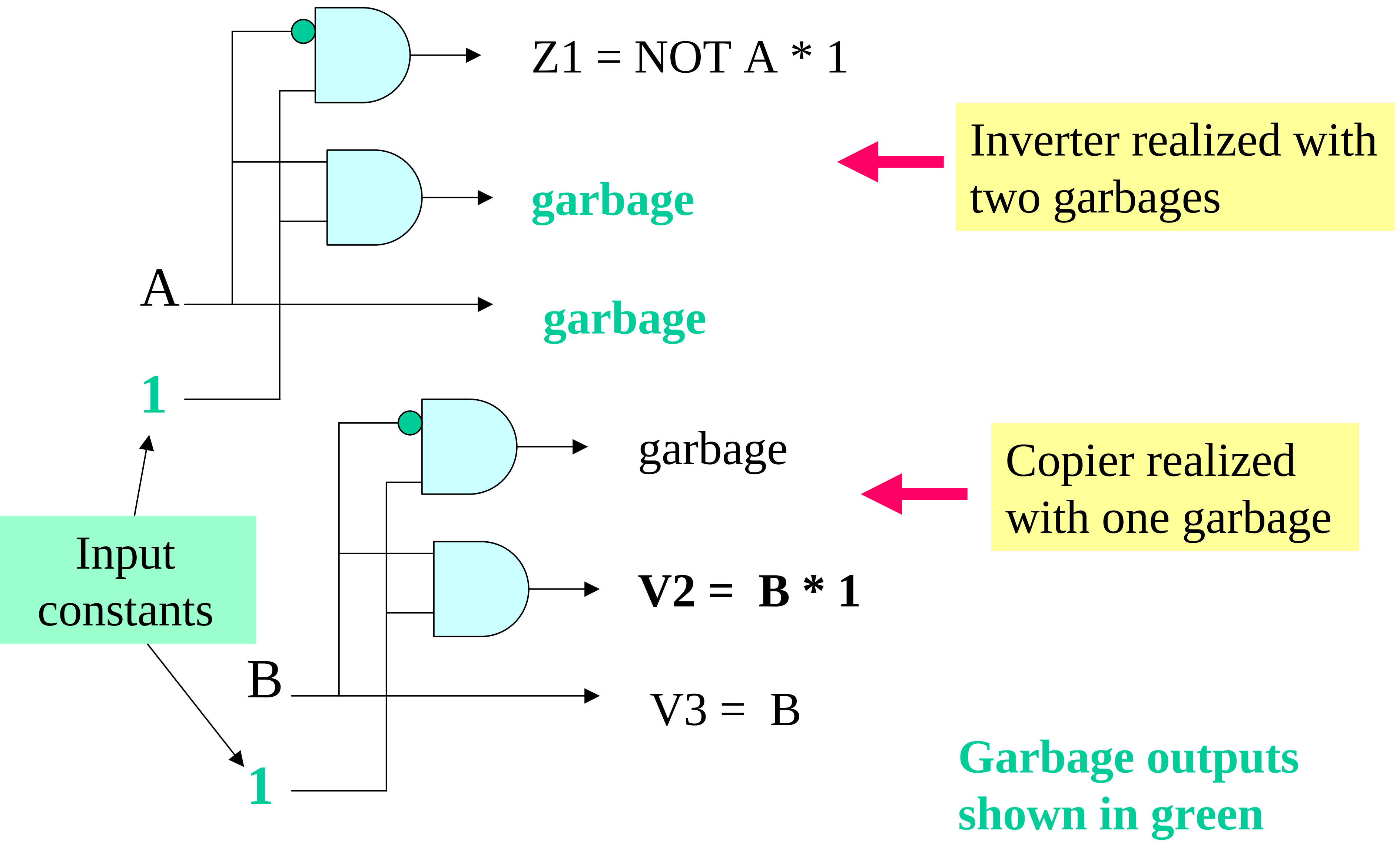
Inverse Switch Gate



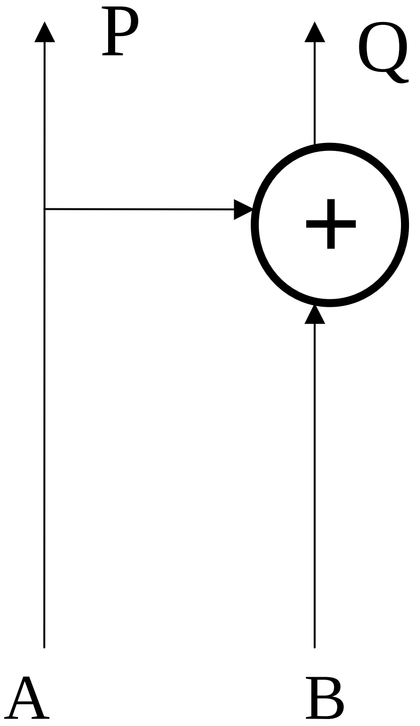
input			output	
z1	**z2**	**z3**	A	B
0	0	0	0	0
1	0	0	0	1
0	0	1	1	0
0	1	1	1	1



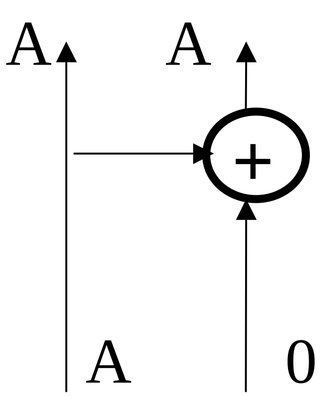
Inverter and Copier Gates from Switch Gate

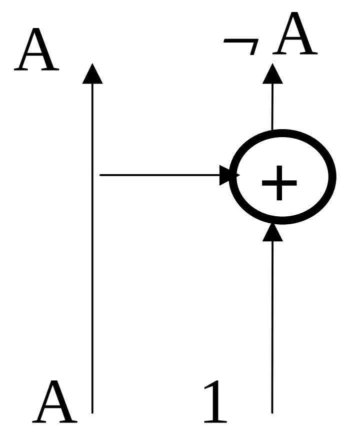


Feynman Gate

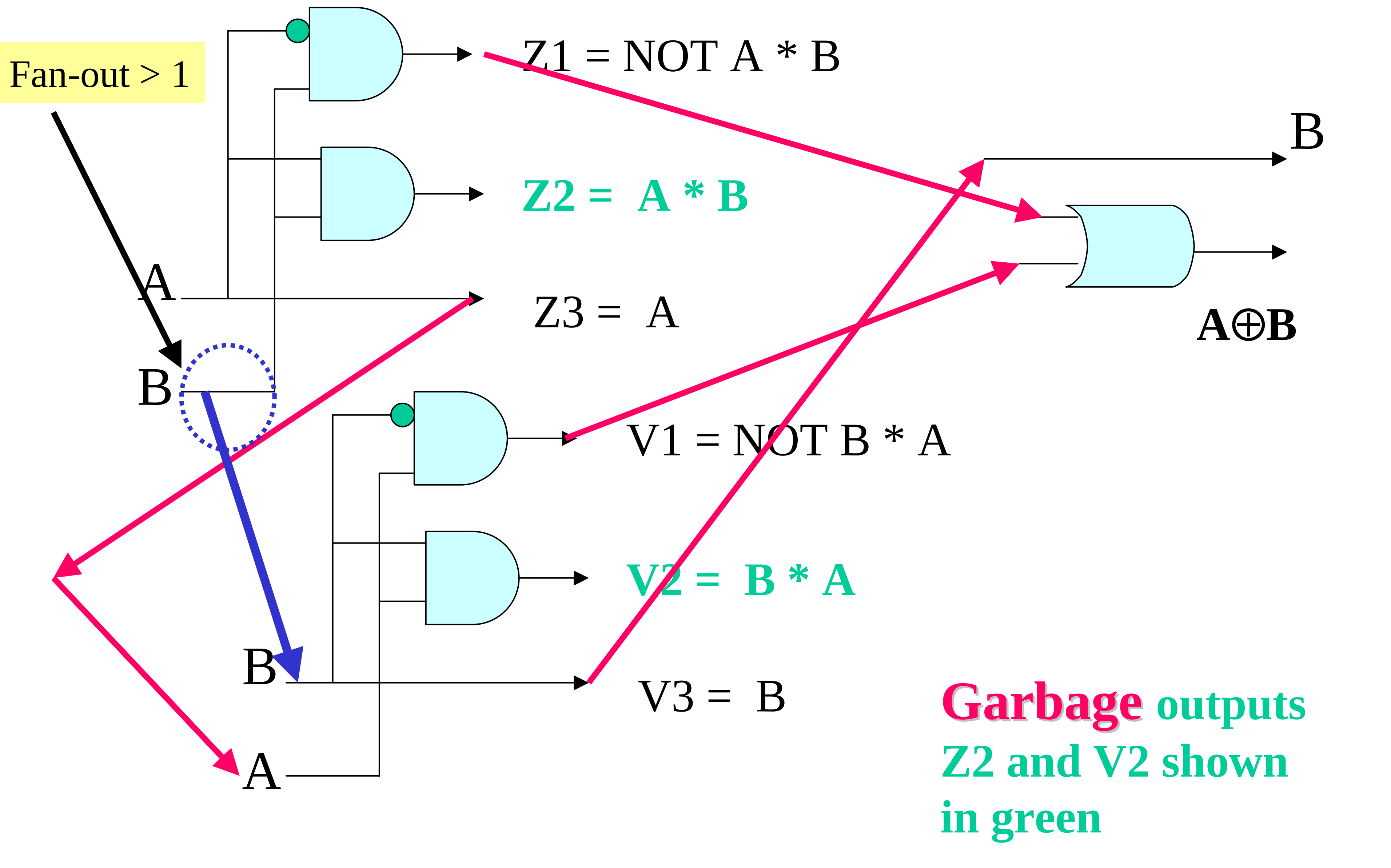


- When ***A = 0*** then ***Q = B,*** when ***A = 1*** then ***Q =*** $\neg$ ***B***.
- Every linear reversible function can be built by composing only 2*2 Feynman gates and inverters
- With ***B=0*** Feynman gate is used as a **fan-out gate.** (**Copying gate**)





Feynman Gate from Switch Gates



Fredkin Gate

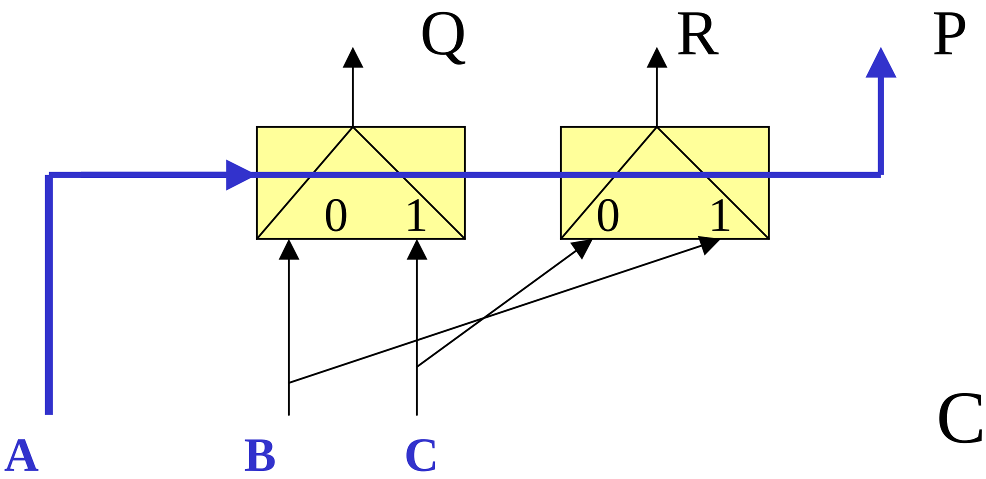
- Fredkin Gate is a fundamental concept in *reversible and quantum computing*.
- Every Boolean function can be build from 3 * 3 Fredkin gates:

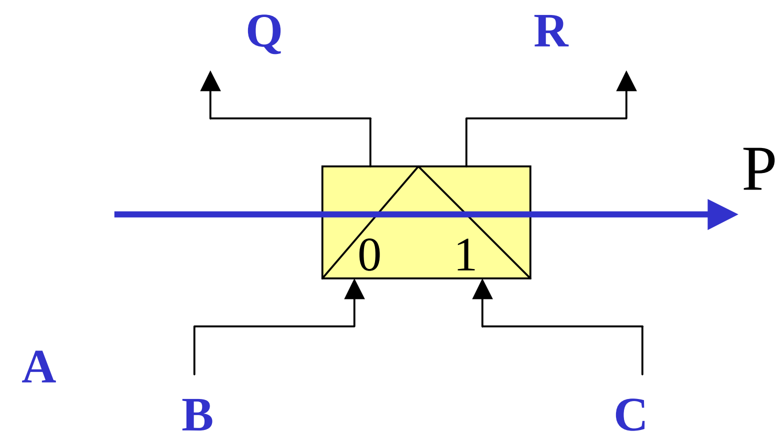
P = A,

Q = if A then C else B,

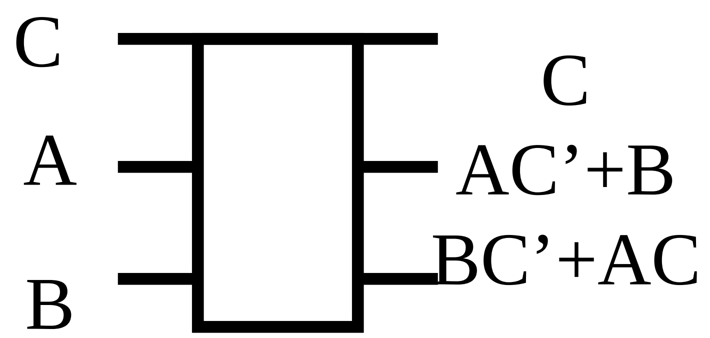
R = if A then B else C.

Notation for Fredkin Gates





A circuit from two multiplexers

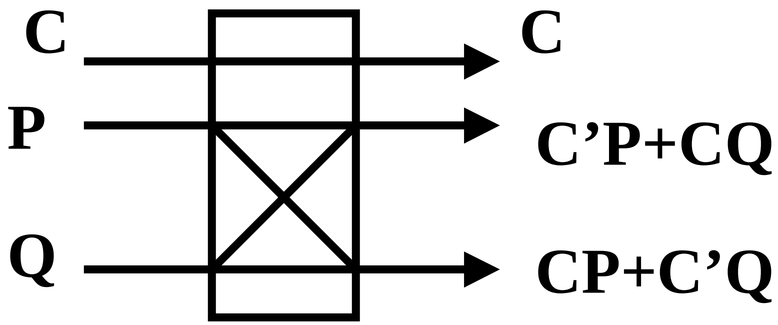


Its schemata

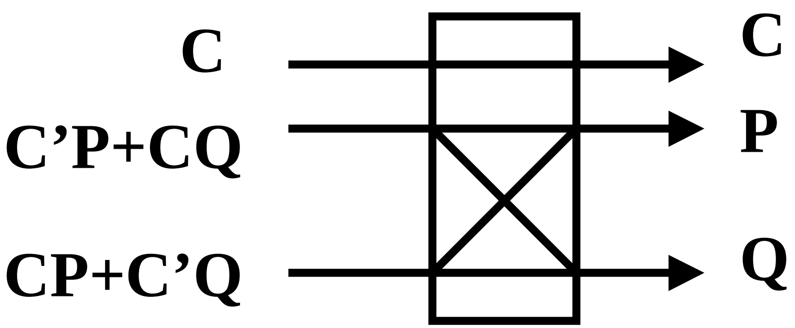
This is a reversible gate, one of many

Yet Another Useful Notation for Fredkin Gate

Fredkin Gate



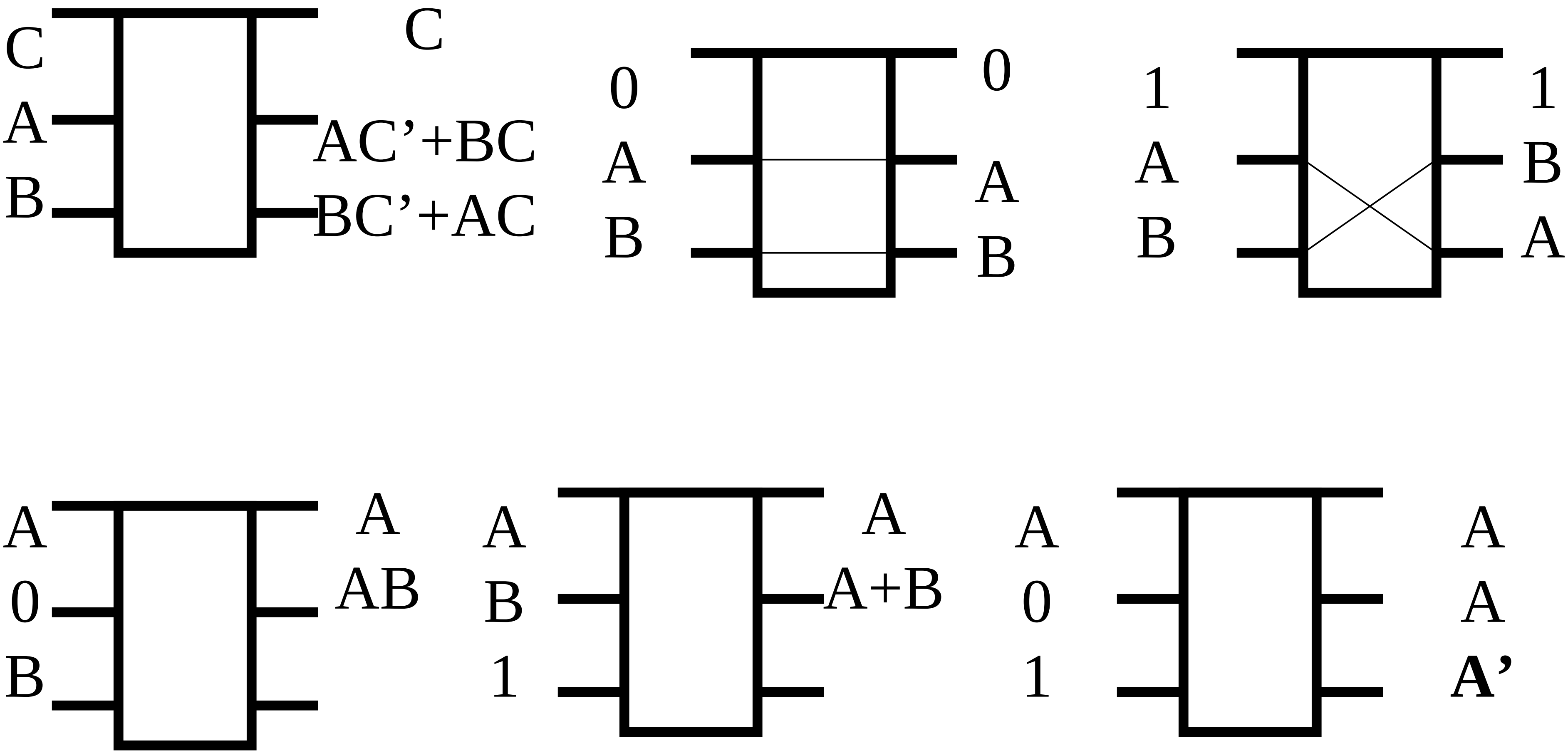
Inverse Fredkin Gate



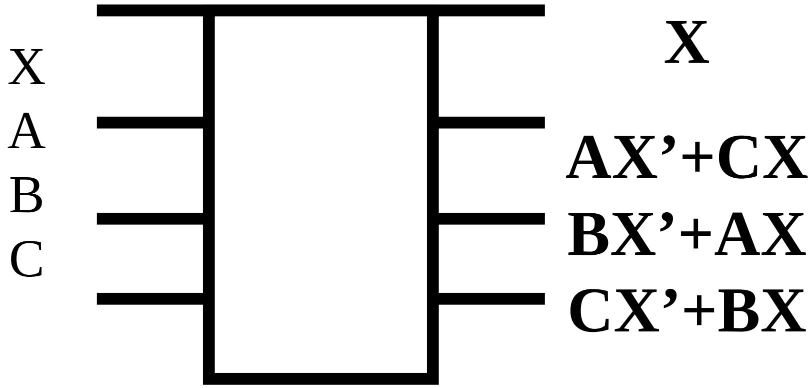
In this gate the input signals P and Q are routed to the same or exchanged output ports depending on the value of control signal C

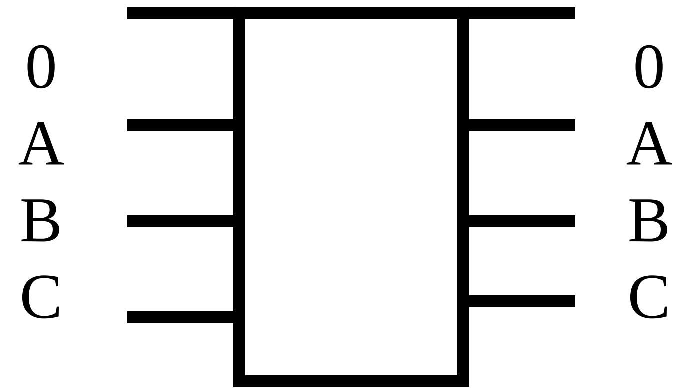
Fredkin gate is conservative and it is its own inverse

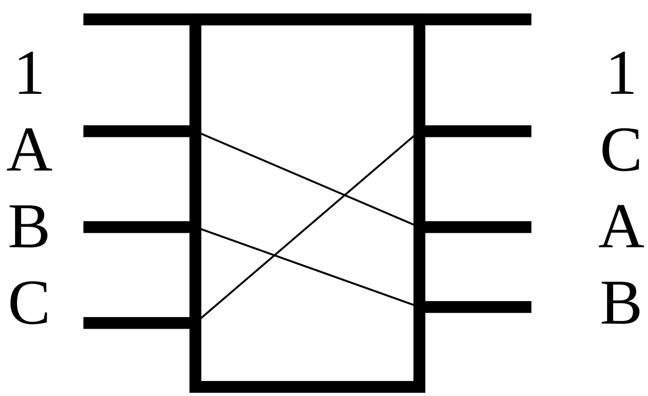
Operation of the Fredkin gate

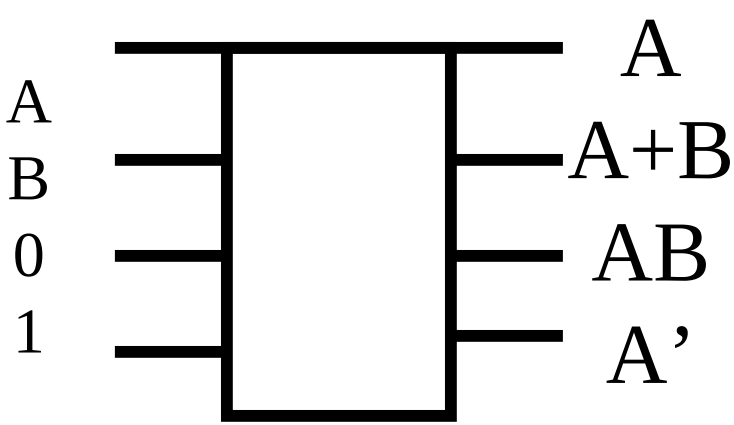


A 4-input Fredkin gate

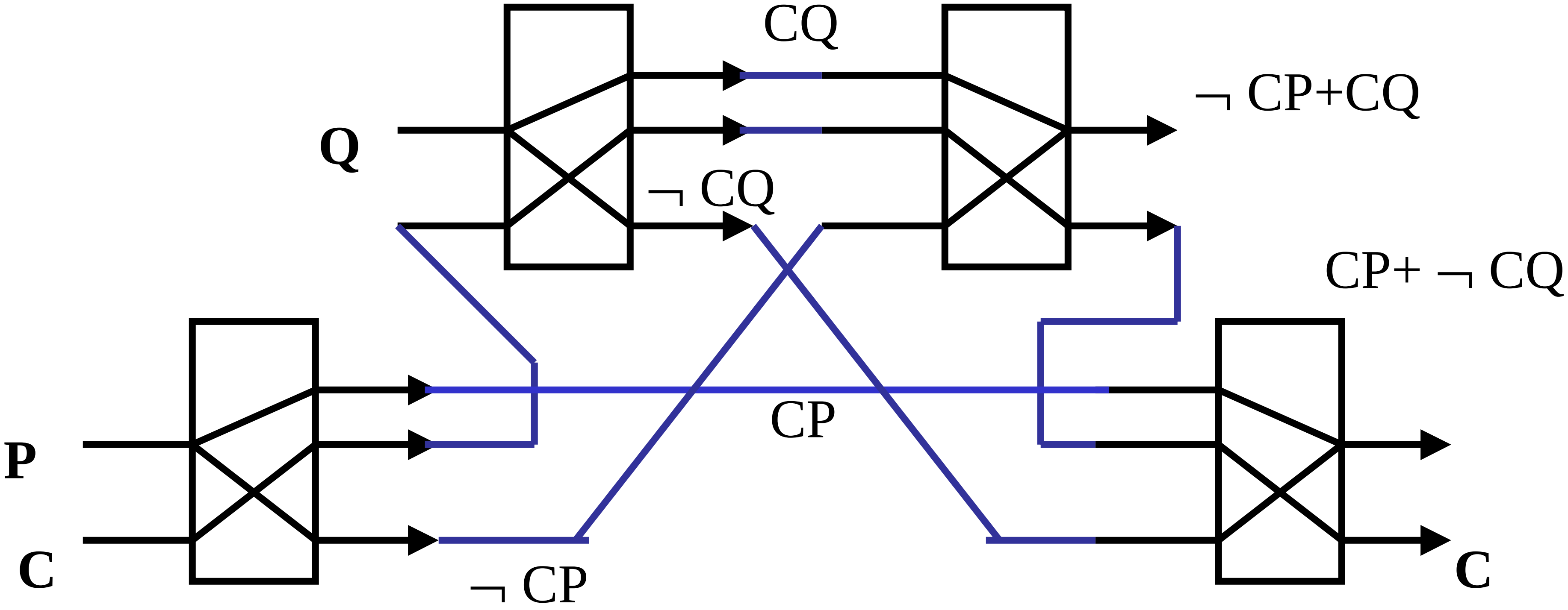








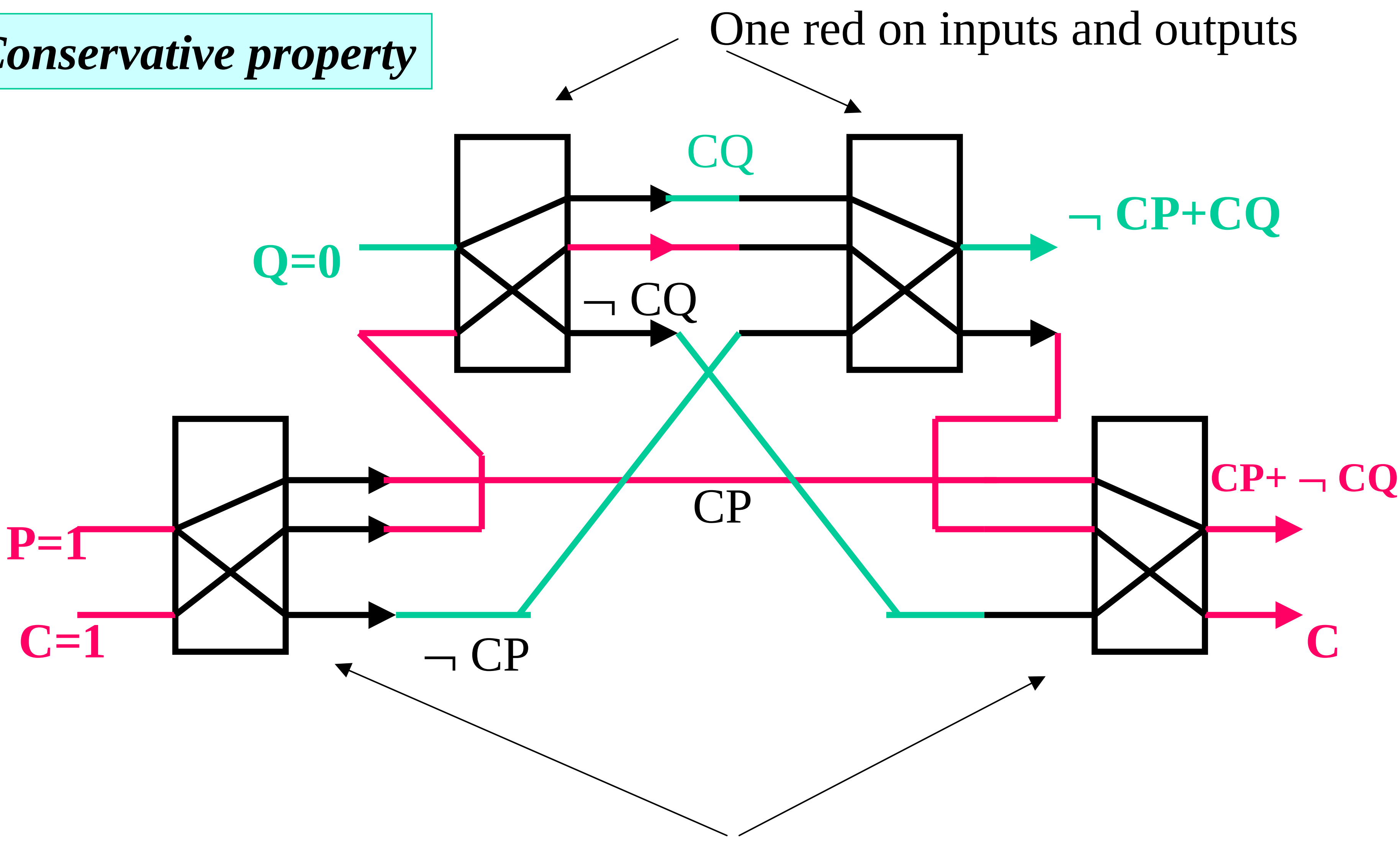
Fredkin Gate from Switch Gates



Another Illustration of Conservative Property of a Circuit

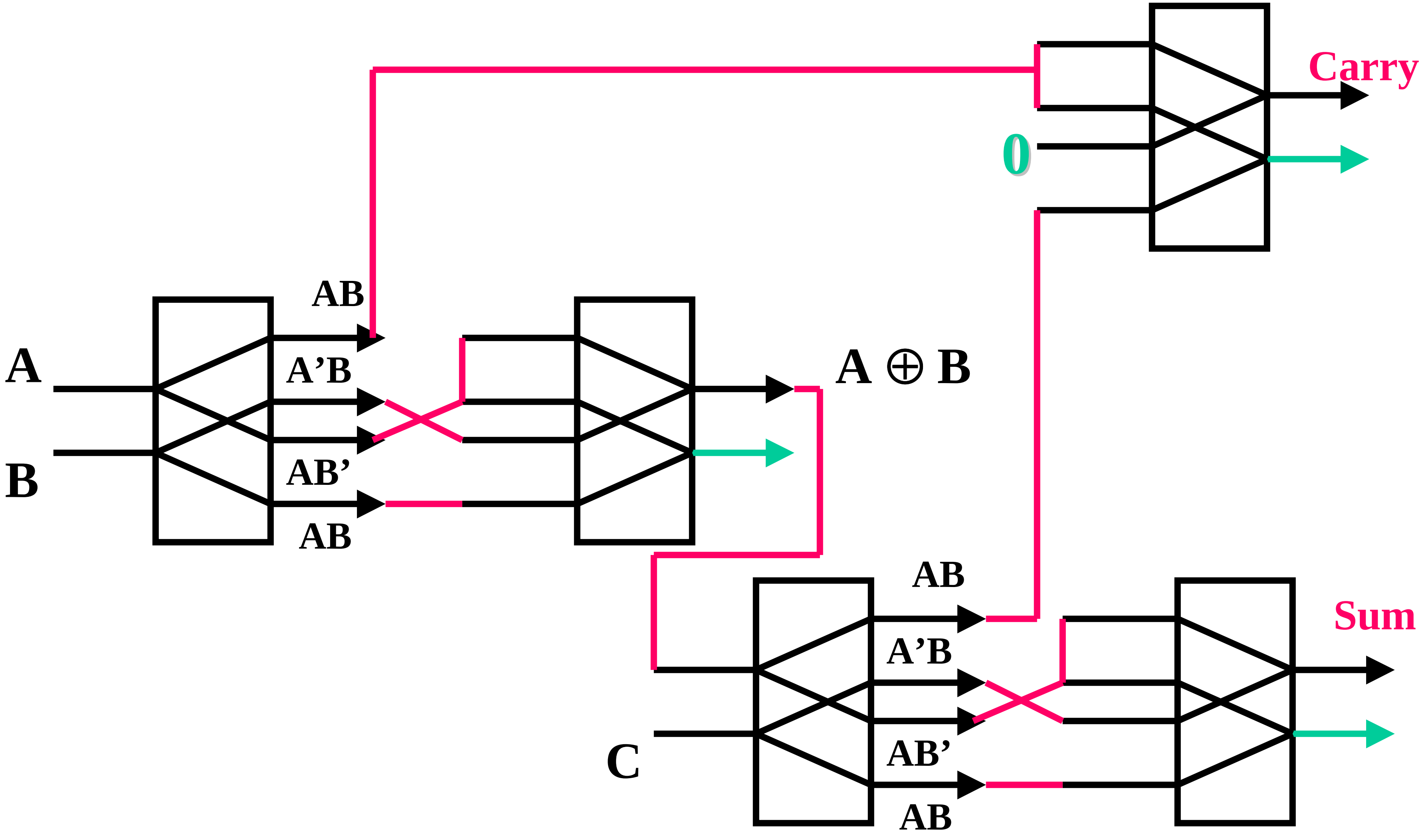
Operation of a circuit from Switch Gates

Conservative property



Red signals are value 1

Minimal Full Adder using Interaction Gates



Garbage signals shown in green

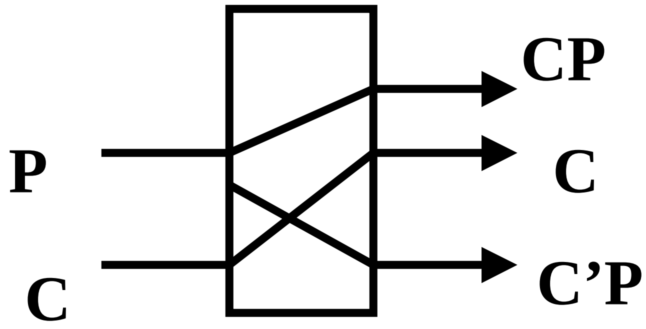
3 garbage bits

Reversible logic: Garbage

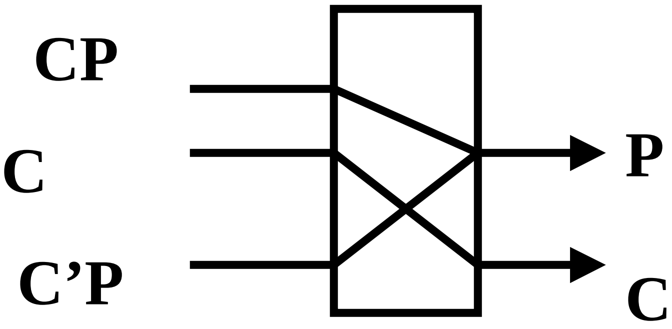
- A k*k circuit without constants on inputs which includes only reversible gates realizes on all outputs only balanced functions.
- Therefore, k1 * k1 circuit can realize non-balanced functions only with ***garbage*** outputs.

Switch Gate

Switch Gate

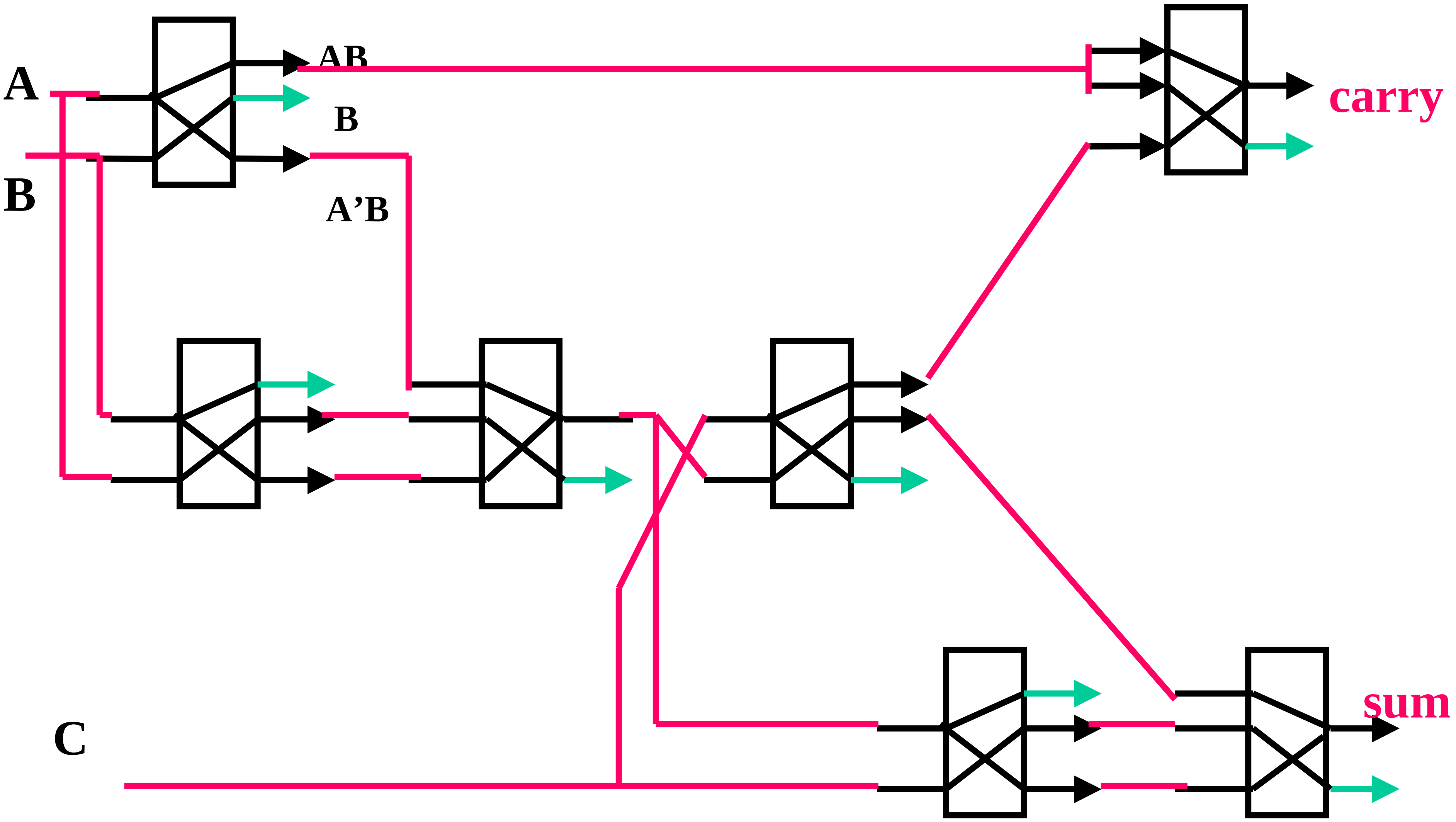


Inverse Switch Gate



In this gate the input signal P is routed to one of two output ports depending on the value of control signal C

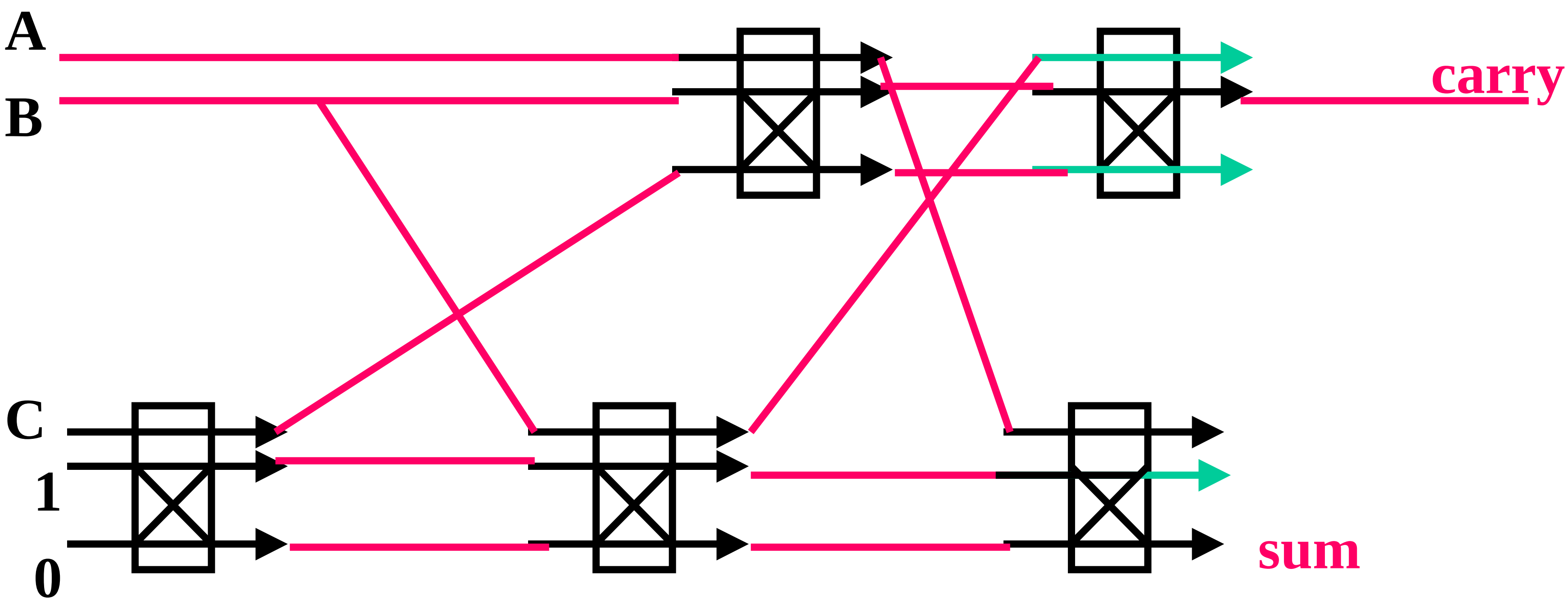
Minimal Full Adder Using Switch Gates



Garbage signals shown in green

7 garbage bits

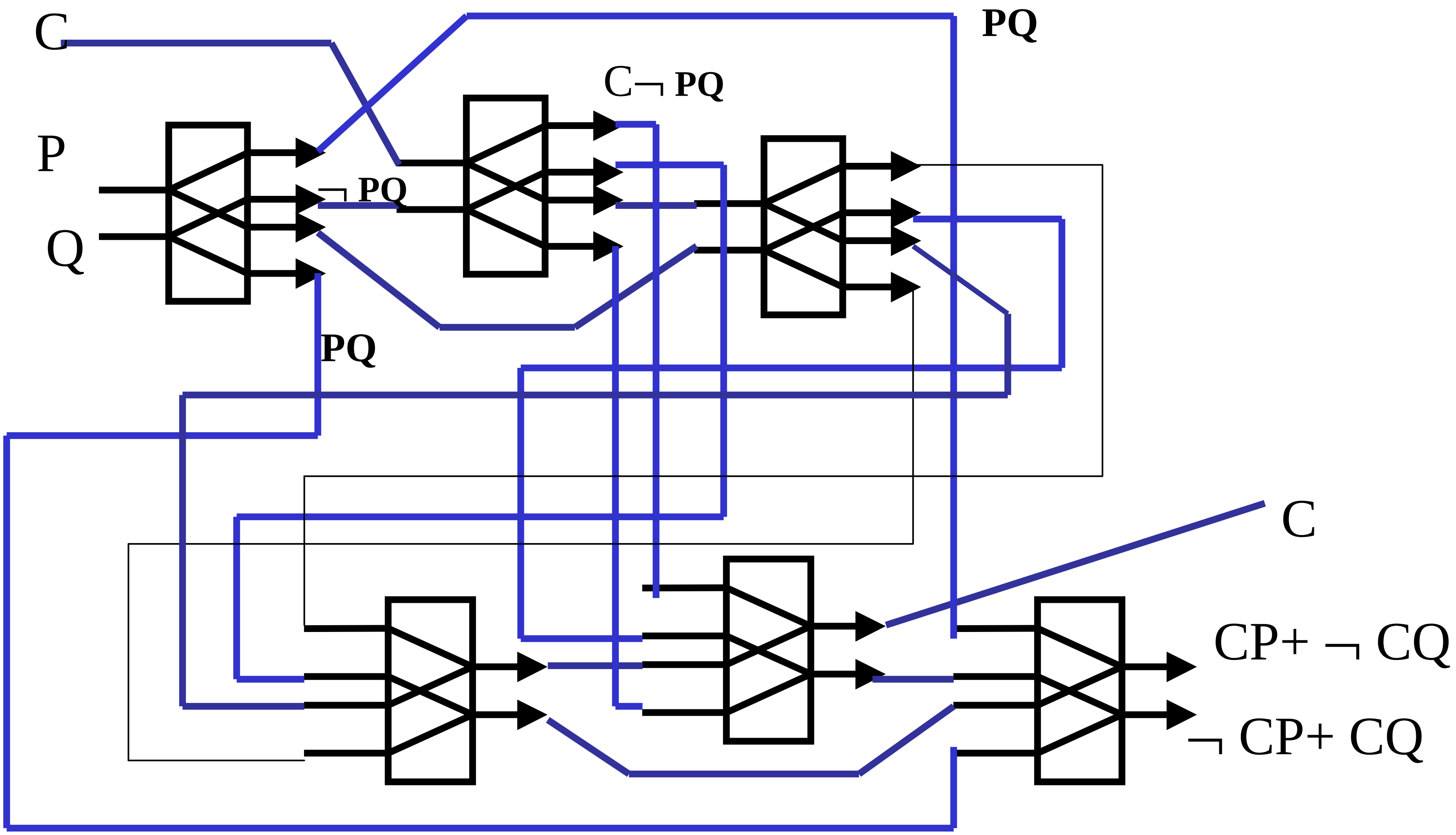
Minimal Full Adder Using Fredkin Gates



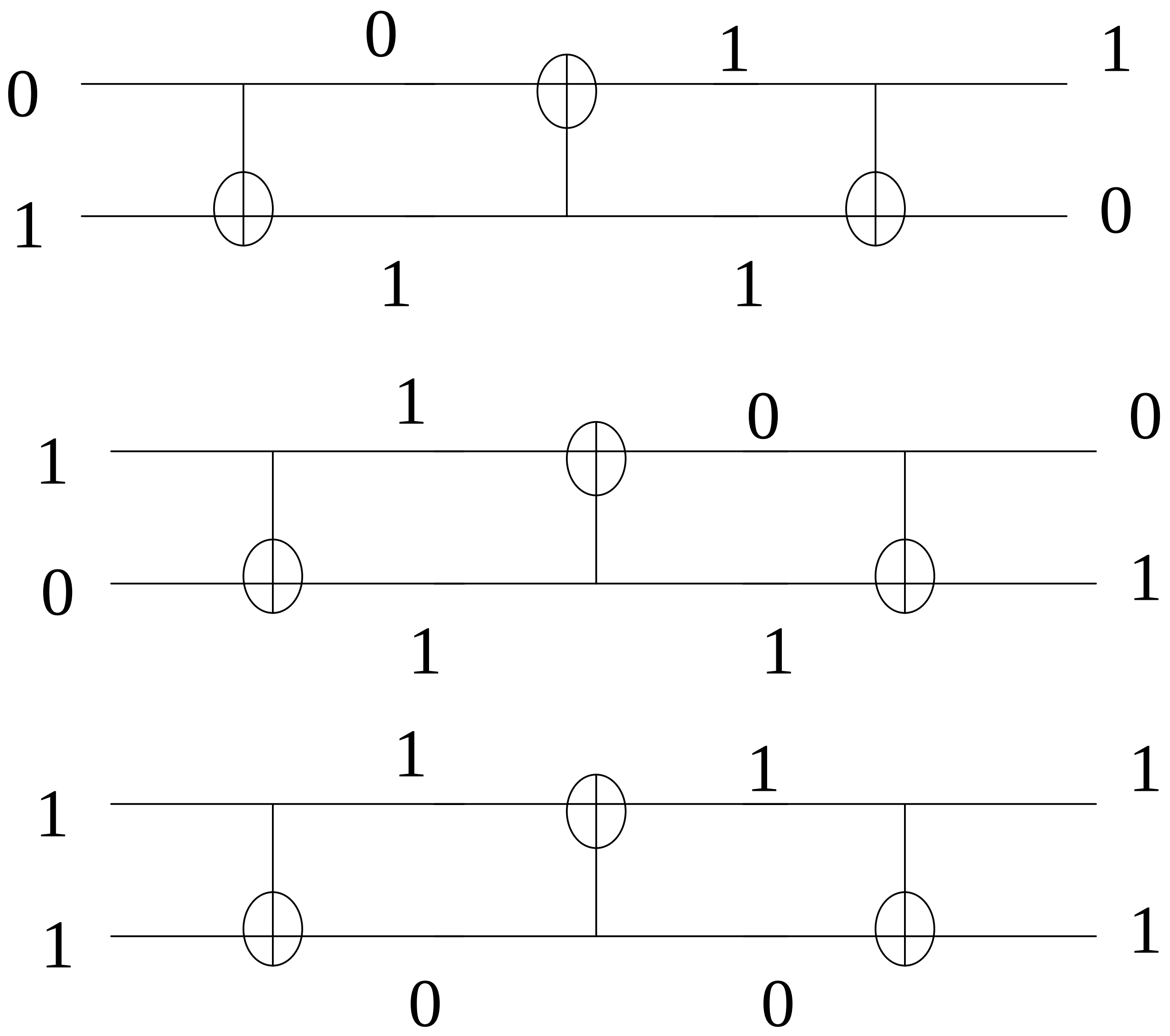
In this gate the input signals P and Q are routed to the same or exchanged output ports depending on the value of control signal C

3 garbage bits

Fredkin Gate from Interaction Gates

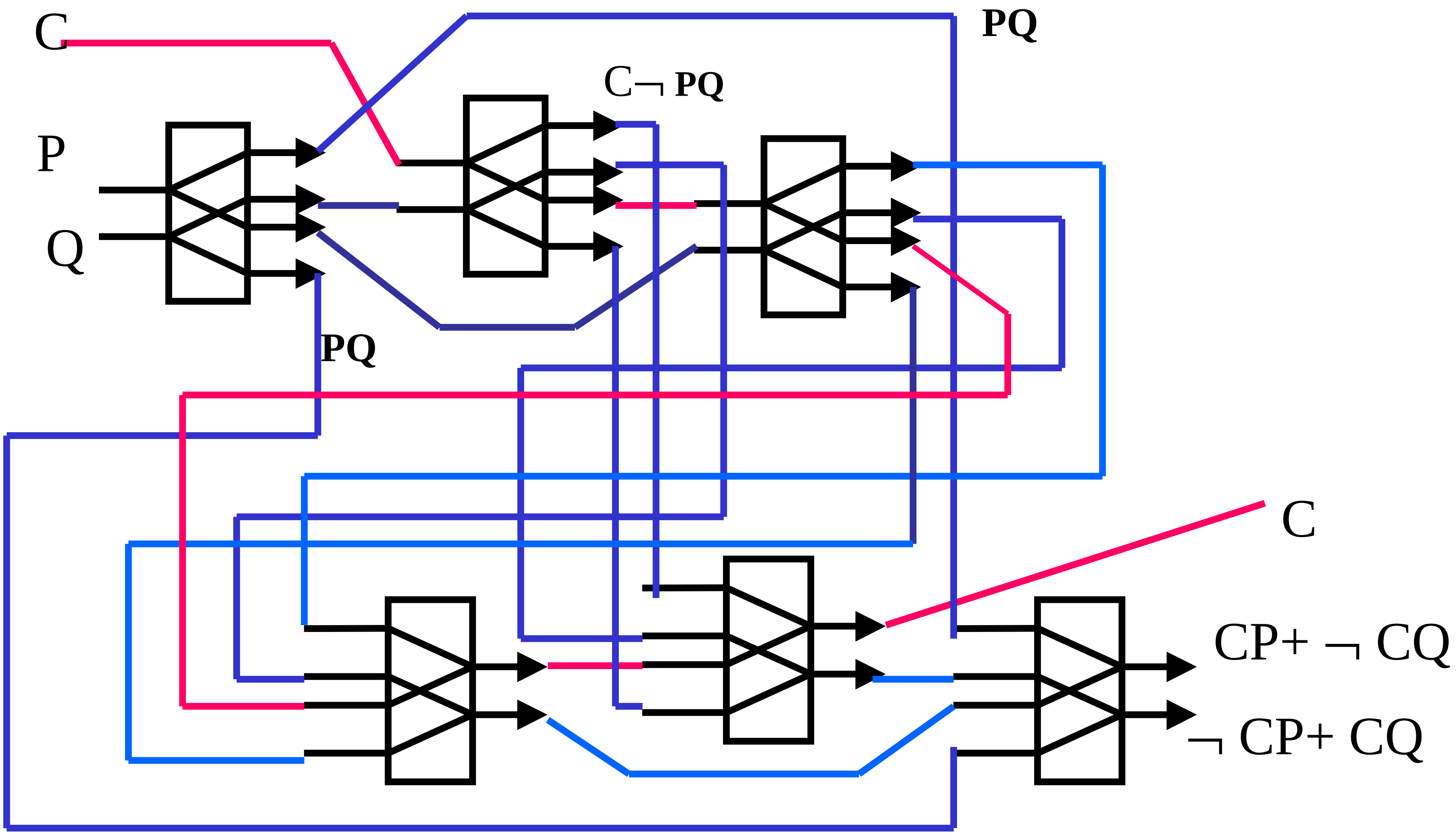


Swap gate from three Feynman Gates



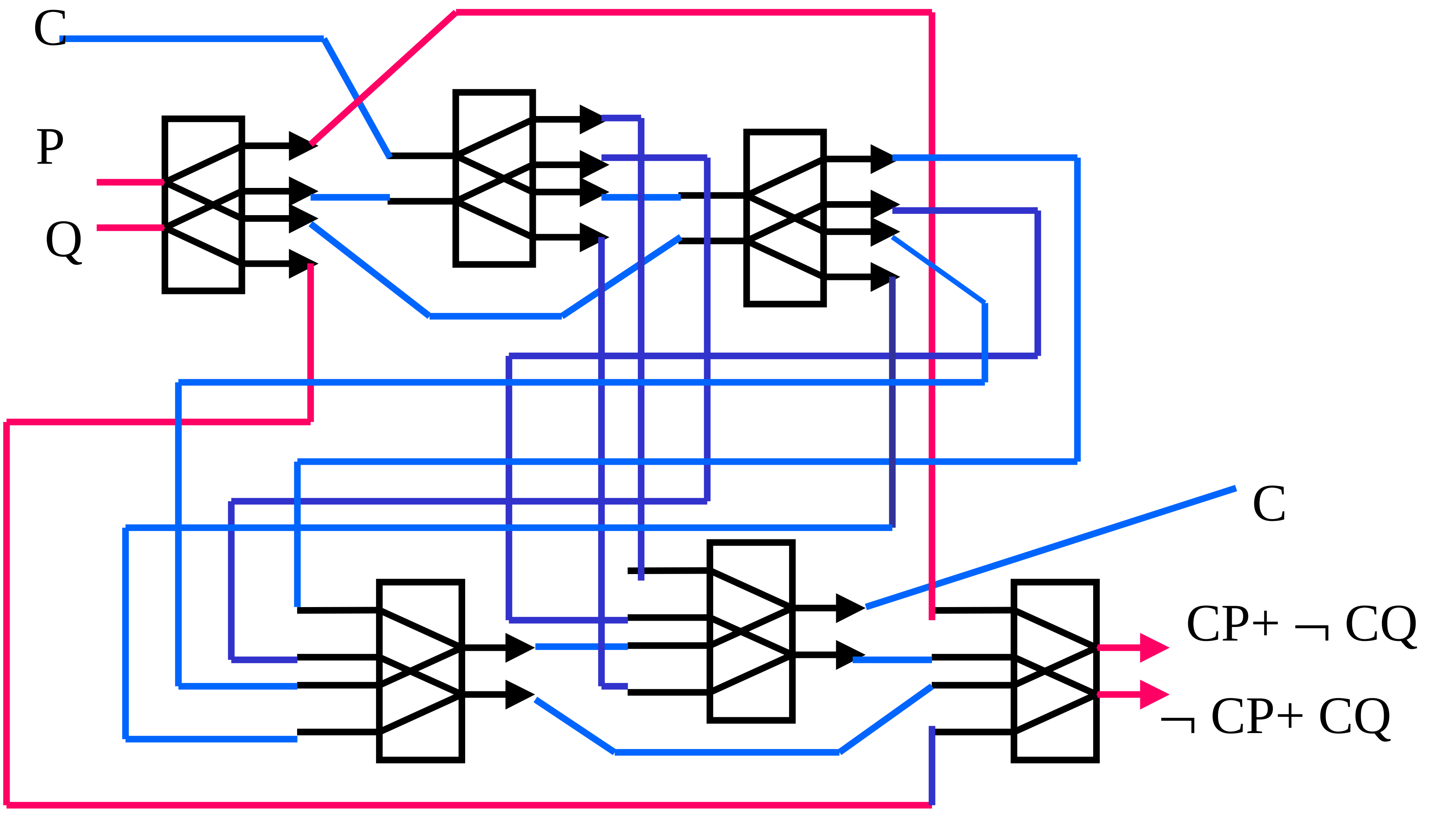
Thus every non-planar function can be converted to planar function

Fredkin Gate from Interaction Gates



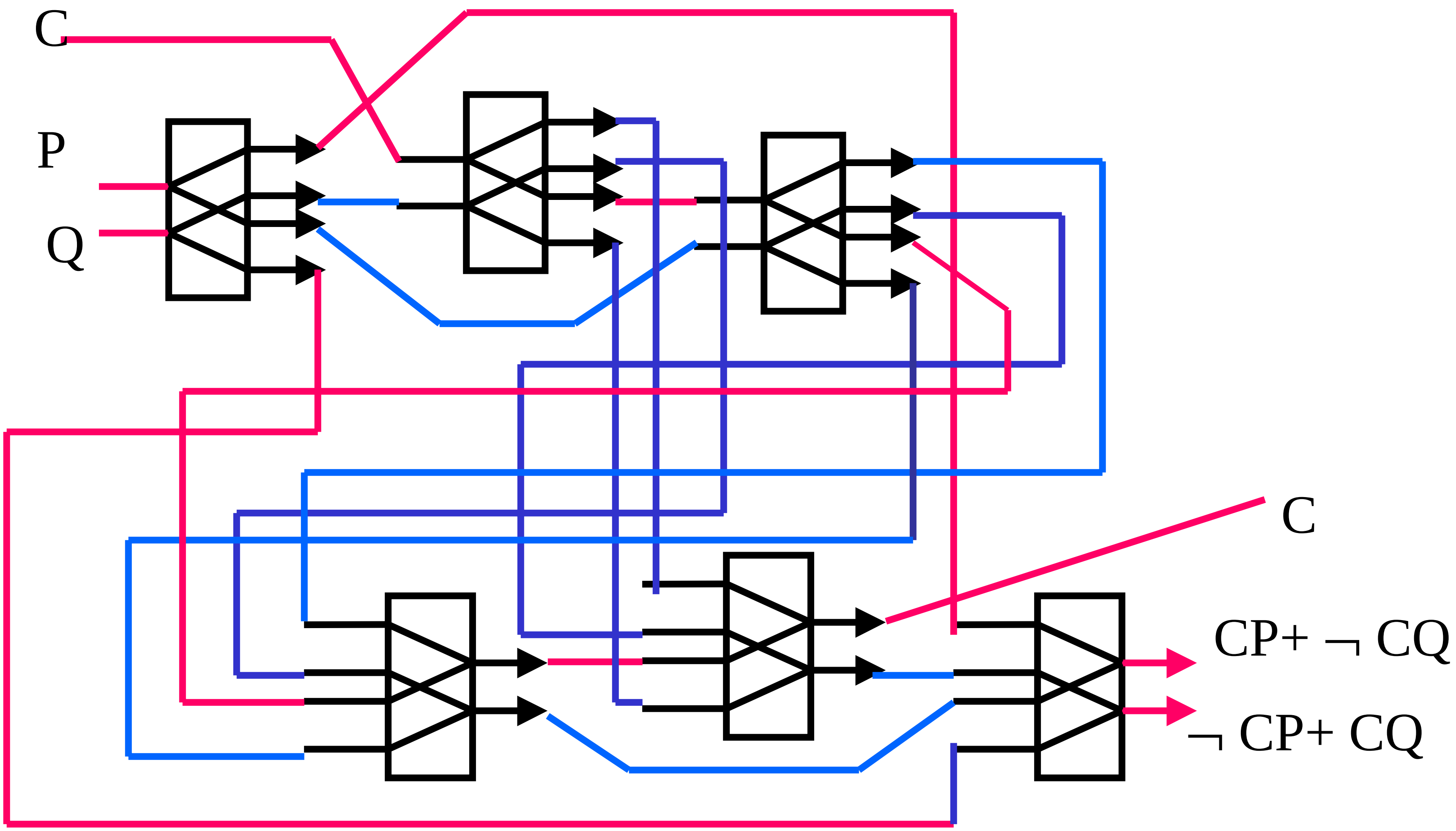
To verify conservative property signal ON is shown in red

Fredkin Gate from Interaction Gates



To verify conservative property signal ON is shown in red

Fredkin Gate from Interaction Gates



To verify conservative property signal ON is shown in red

Concluding on the Billiard Ball Model

- The **Interaction** and **Switch Gates** are reversible and conservative, but have ***various numbers of inputs and outputs***
- Their inverse gates required to be given **only some input combinations**
- **Logic Synthesis methods** can be developed on the level of k*k gates such as Fredkin and Toffoli (quantum)
- Logic synthesis methods can be developed on level of simpler gates such as Interaction gate and Switch gate that have direct counterparts in physical processes.

Concluding on the Billiard Ball Model

- **INVERTER, FREDKIN** and **FEYNMAN** gates can be created from Billiard Ball Model.
- There is a close link of **Billiard Ball Model** and quantum gates and other physical models on micro level
- Many ways to realize universal (for instance – optical) gates, completely or partially reversible but conservative

End of Lecture 4