Scheduling and Assignment

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Scheduling

- Complexity
- Problem Solving Techniques
- Directions

Scheduling using Simulated Annealing

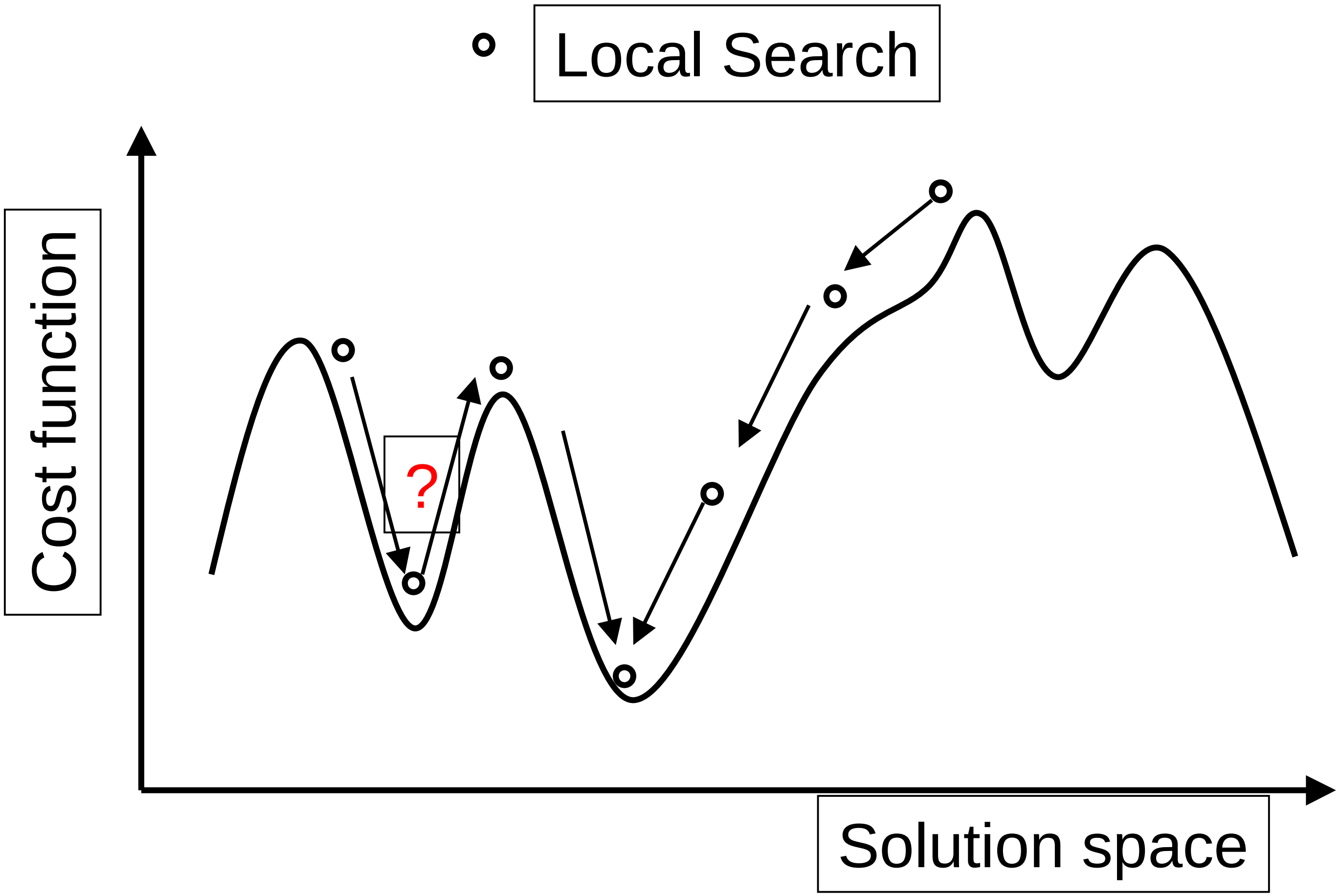
Reference:

Devadas, S.; Newton, A.R.

Algorithms for hardware allocation in data path synthesis.

IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, July 1989, Vol.8, (no.7):768-81.

Simulated Annealing



Statistical Mechanics

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Combinatorial Optimization

State {r:} (configuration -- a set of atomic position)

weight $e^{-E(\{r:])/K_B T}$ -- Boltzmann distribution

E({r:]): energy of configuration

K_B: Boltzmann constant

T: temperature

Low temperature limit ??

Analogy

Physical System		Optimization Problem
State (configuration)	→	Solution
Energy	→	Cost function
Ground State	→	Optimal solution
Rapid Quenching	→	Iteration improvement
Careful Annealing	→	Simulated annealing

Generic Simulated Annealing Algorithm

1. Get an initial solution S

2. Get an initial temperature T > 0

3. While not yet “frozen” do the following:

3.1 For $1 \leq i \leq L$, do the following:

3.1.1 Pick a random neighbor S’ of S

3.1.2 Let Δ=cost(S’) - cost(S)

3.1.3 If $\Delta \leq 0$ (downhill move) set S = S’

3.1.4 If $\Delta > 0$ (uphill move)

set S=S’ with probability $e^{-\Delta/T}$

3.2 Set T = rT (reduce temperature)

4. Return S

Basic Ingredients for S.A.

- Solution space
- neighborhood Structure
- Cost function
- Annealing Schedule

Integer Linear Programming

- **Given**: integer-valued matrix A_{mxn},
  vectors $B = (b_1, b_2, \ldots, b_m)$, $C = (c_1, c_2, \ldots, c_n)$
- **Minimize**: $C^T X$
- **Subject to:**

  $AX \leq B$

  $X = (x_1, x_2, \ldots, x_n)$ is an integer-valued vector

Integer Linear Programming

- Problem: For a set of (dependent) computations $\{t_1, t_2, \ldots, t_n\}$, find the minimum number of units needed to complete the execution by k control steps.
- Integer linear programming:

Let y_0 be an integer variable.

For each control step i ($1 \leq i \leq k$):

define variable x_{ij} as

$x_{ij} = 1$, if computation t_j is executed in the ith control step.

$x_{ij} = 0$, otherwise.

define variable $y_i = x_{i1} + x_{I2} + \ldots + x_{in}$.

Integer Linear Programming

- Integer linear programming:

  For each computation dependency: t_i has to be done before t_j, introduce a constraint:

$$k \bullet x_{1i} + (k-1) \bullet x_{2i} + \dots + x_{ki} < k \bullet x_{1j} + (k-1) \bullet x_{2j} + \dots + x_{kj} + 1 \quad (*)$$

Minimize: y_0

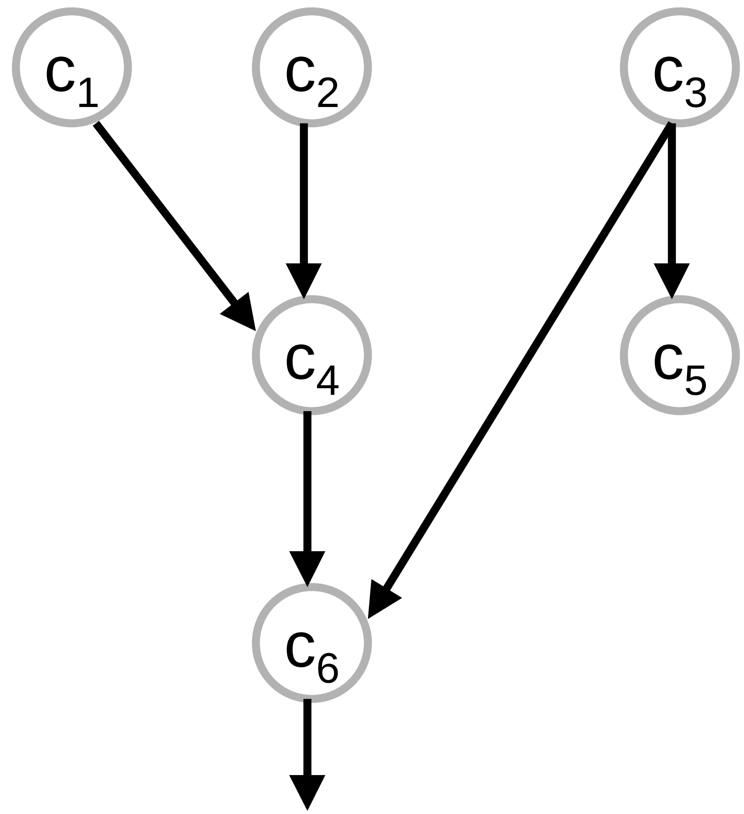
Subject to: $x_{1i} + x_{2i} + \dots + x_{ki} = 1$ for all $1 \leq i \leq n$

$y_j \leq y_0$ for all $1 \leq i \leq k$

all computation dependency of type (*)

An Example

6 computations
3 control steps



An Example

- Introduce variables:
  - x_{ij} for $1 \leq i \leq 3$, $1 \leq j \leq 6$
  - $y_i = x_{i1}+x_{i2}+x_{i3}+x_{i4}+x_{i5}+x_{i6}$ for $1 \leq i \leq 3$
  - y_0
- Dependency constraints: e.g. execute c_1 before c_4

  $3x_{11}+2x_{21}+x_{31} < 3x_{14}+2x_{24}+x_{34}+1$
- Execution constraints:

  $x_{1i}+x_{2i}+x_{3i} = 1$ for $1 \leq i \leq 6$

An Example

- Minimize: y_0
- Subject to: $y_i \le y_0$ for all $1 \le i \le 3$
  dependency constraints
  execution constraints
- One solution: $y_0 = 2$
  $x_{11} = 1$, $x_{12} = 1$,
  $x_{23} = 1$, $x_{24} = 1$,
  $x_{35} = 1$, $x_{36} = 1$.
  All other $x_{ij} = 0$