

# **Additional Slides to De Micheli Book**

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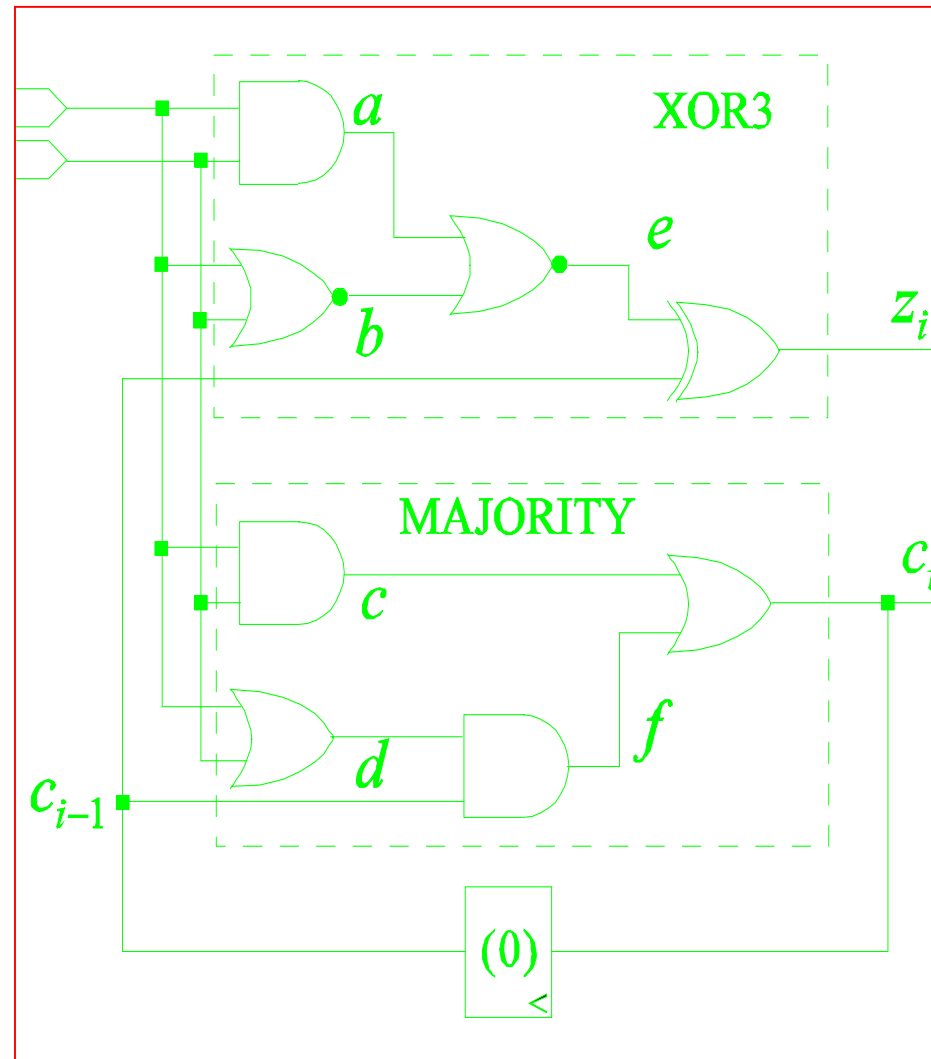
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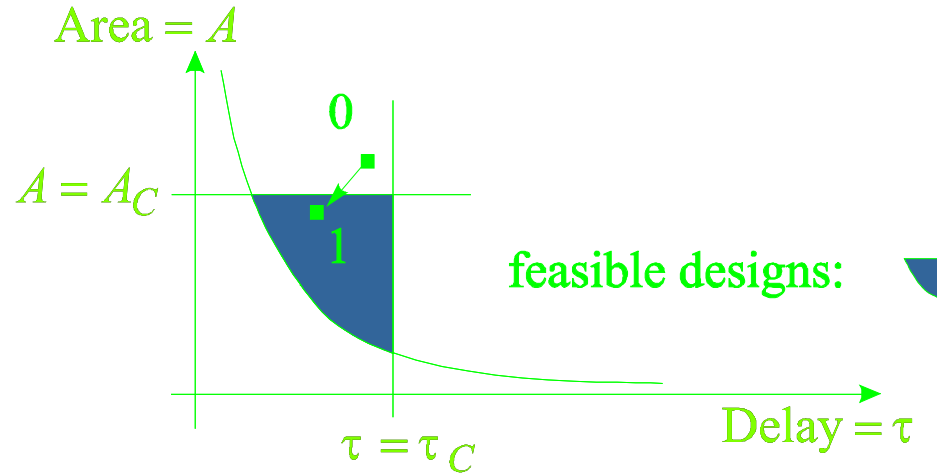
- **Behavioral Synthesis**
  - Resource allocation; Pipelining; Control flow parallelization; Communicating Sequential Processes; Partitioning
- **Sequential Synthesis**
  - Register Movement and Retiming; State Minimization; State Assignment; Synthesis for Testable FSM's; State Machine Verification
- **Logic Synthesis**
  - Extraction of combinational logic to HDL; Two-level minimization; Algebraic Decomposition; Multilevel Logic Minimization; Synthesis for Multifault Testability; Test Generation via Minimization; Technology mapping; Timing optimization
- **Technology Mapping**
  - Mapping to Library of Logic Gates; Timing Optimization
- **Physical Design Synthesis**
  - Cell Placement; Routing; Fabrication; Engineering Changes



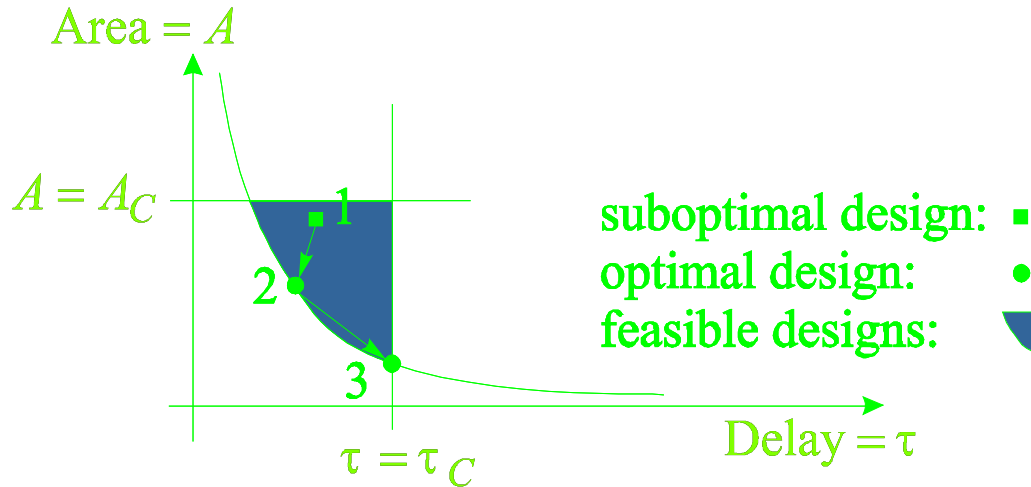
# Area vs Delay: The Bit-Serial Adder

- A typical **tradeoff** is area versus delay
- With just one full adder, this circuit can do 32-bit addition
- But, it is 32x slower than a parallel adder (32 full adders, 1 bit output per clock cycle)

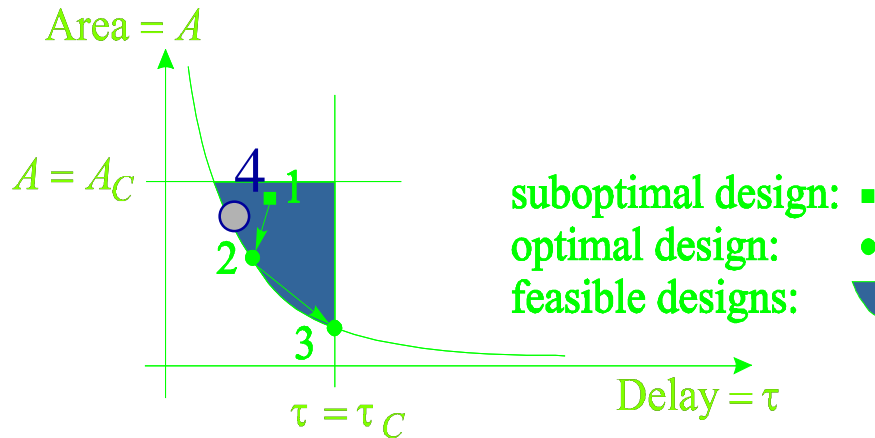




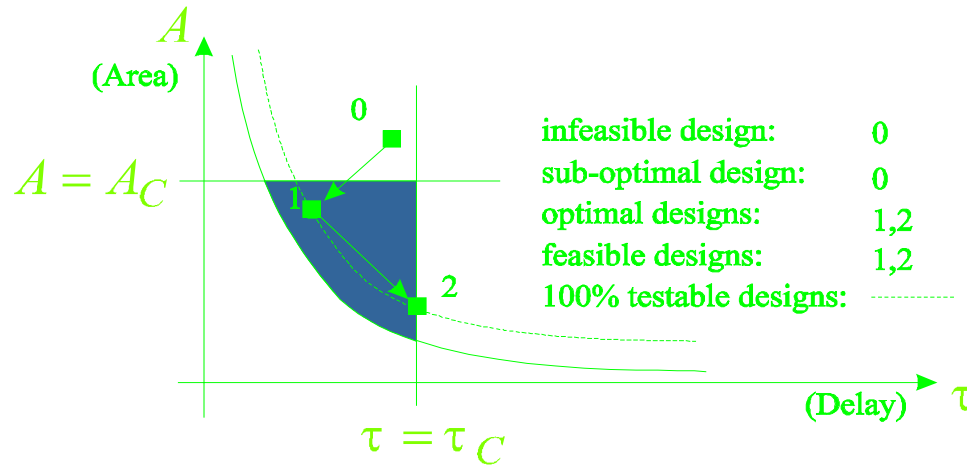
- Holding other factors constant, the Area vs Delay tradeoff curve is typically parabolic
- The first design requirement is meet **Constraints** on Chip Area and Critical Path Delay (0 to 1)



- The next priority is to optimize a feasible design
- Design 2 is optimal, in the sense that area and delay cannot both be decreased from this point
- Tradeoff is now necessary, according to **Policy**



- A typical design **Policy** is to optimize area subject to a delay constraint (2 to 3)
- Often a preferred policy is to optimize delay subject to an area constraint (2 to 4)



- Sometimes other factors, such as **testability** or power, take priority
- Typically this moves the area-delay tradeoff curve up and to the right
- Thus designs 1 and 2 are optimal

- Typically performed in a ***technology independent view*** of the circuit
- In this view gates are regarded as logic functions
- These functions are converted to physical gates by ***Technology Mapping***





$$a = x_i y_i$$

$$b = x_i' y_i'$$

$$e = a' b'$$

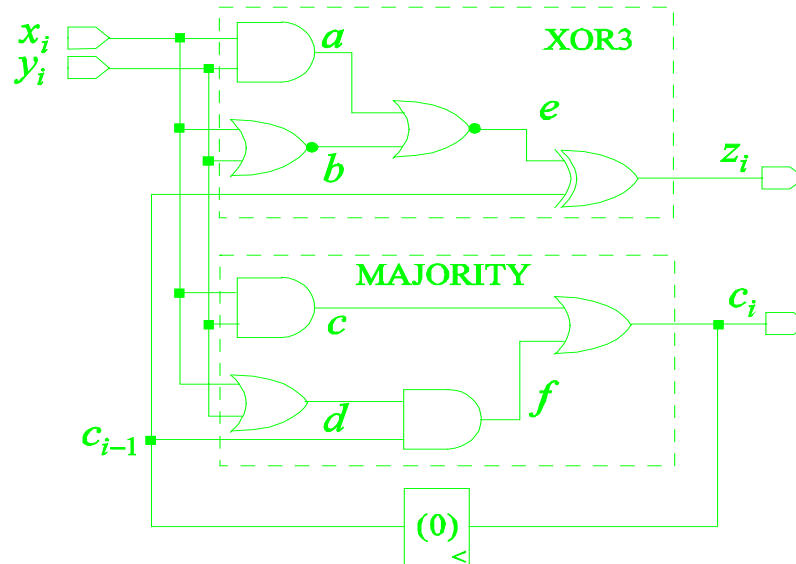
$$z_i = e c_{i-1}' + e' c_{i-1}$$

$$c = x_i y_i$$

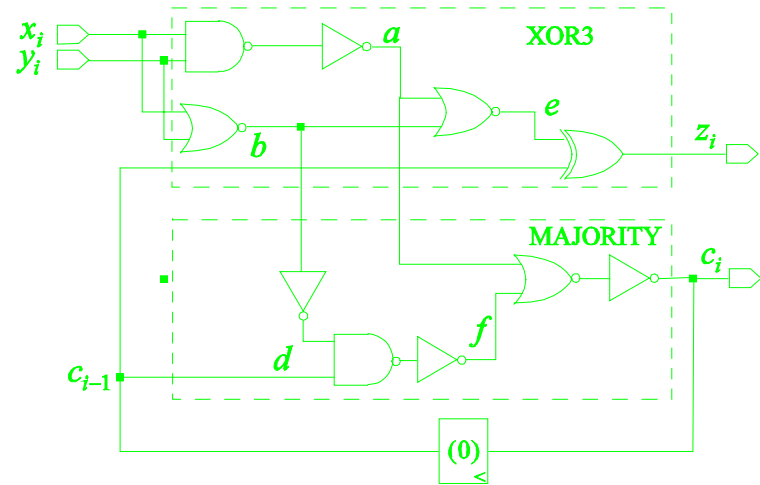
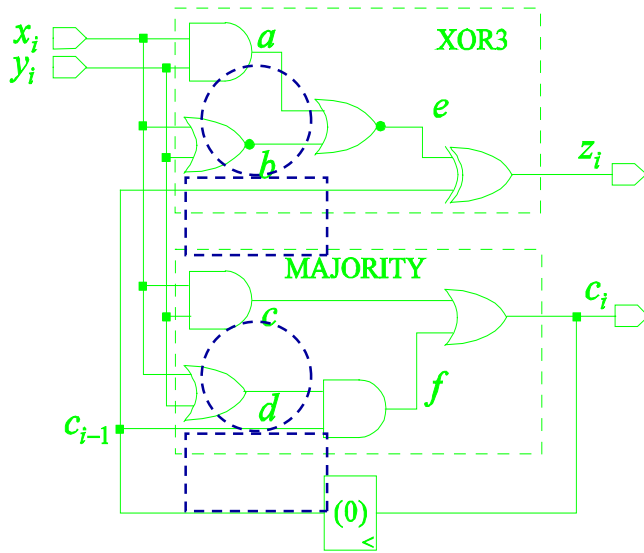
$$d = x_i + y_i + c_{i-1}$$

$$f = d c_{i-1}$$

$$c_i = c + f$$



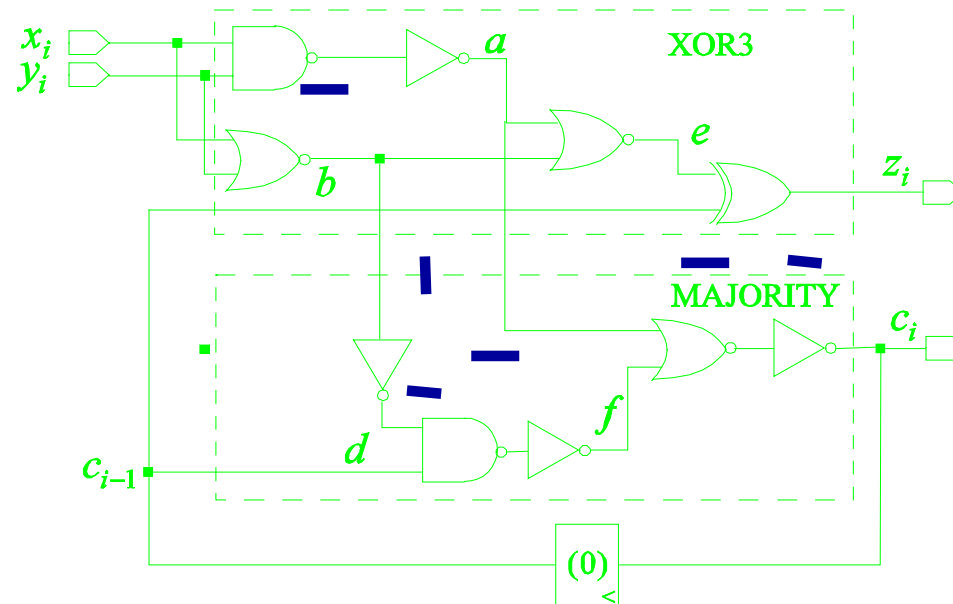
- In this view the gates of the full adder circuit are just logic equations



- Common subfunctions shared
- Functions “Technology Mapped” to negative gates

- **Faults Models**
  - Stuck-at faults
  - Delay faults
- **Test vectors**
  - Fault simulation
  - Automatic test pattern generation
- **Diagnosis**
- **Testable Design**





- First step is to identify the **Critical Path**
- Simplest delay model: “number of logic levels”

- **Static Delay Models:**
  - Levels of Logic
  - Delay function of size, load
  - Worst, best case models
- **Dynamic Delay Models**
  - Simplified device models
  - Full Spice analysis

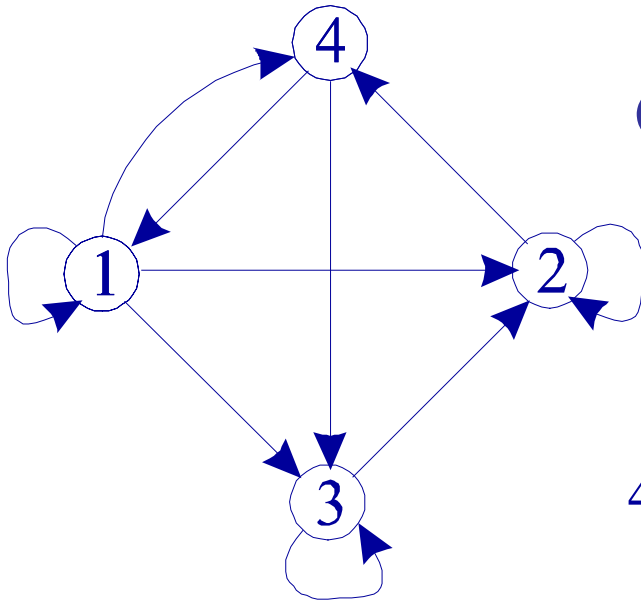


- In the 70s IBM found that 75% of all its cpu cycles went to critical path algorithms
- Now the bottleneck is Formal Verification
- Scalable algorithms required, for which cpu time and space increase:
  - linearly in problem size  $n$  ( $O(n)$ )
  - log-linearly ( $O(n \log(n))$ )
  - even quadratic ( $O(n^2)$ ) too expensive



- **Graph**
- **Edge**
- **Vertex**
- **Undirected graph**
- **Digraph**
  - All edges are directed
- **Mixed graph**
- **DAG (directed acyclic graph)**





**Graph: ordered set of two sets**

$$G = (V, E)$$

**: a set of vertices or nodes**

**: a set of edges or arcs**

**$4_{\rightarrow}$ : the successors (fanouts) of node 4**

$$4_{\rightarrow} = \{1, 3\}$$

**$\rightarrow 4$ : the predecessors (fanins) of node 4**

$$\rightarrow 4 = \{1, 2\}$$



- **Transitive closure**
- **fanout**
- **fanin**
- **Source**
  - **v has no predecessors**
- **Sink**
  - **v has no successors**



- Intersecting 2 sets of sets

```
Procedure SET_CARTESIAN_PRODUCT(G,H) {  
    m = |G|; n= |H|;  
    P = NULL  
    for (i = 1, 2, ..., m) {  
        for (j = 1, 2, ..., n) {  
            P = P ∪ (Gi ∩ Hj)  
        }  
    }  
    return(P)  
}
```

Ops	Times/call	
	Best	Worst
c1	1	1
c2	1	1
c3m	1	1
c4n	m	m
c5	mn	mnq
c6	1	1



- This problem is modeled as that of finding the **longest path in a DAG (Directed Acyclic Graph)**
- Thus we digress for a while, and introduce the notions of sets and graphs
- Then we discuss a scalable (linear) complexity algorithm for finding the longest path in a DAG



## Longest Paths

*Graph Algorithms*

Line	Procedure LONGEST_PATH( $V, E, L, I, spec$ ) {	Ops	Times/call	
			Best	Worst
0	$n =  V ; m =  E ; g =  I $	$c_0$	1	1
	for( $v \in V$ ) {			
1	$\lambda(v) = 0$	$c_1$	$n$	$n$
2	$D_v =  \rightarrow v $	$c_2$	$n$	$n$
	}			
3	$Q = I$	$c_3g$	1	1
4	while( $Q \neq \emptyset$ ) {	$c_4$	$n$	$n$
5	$v = \text{DEQUEUE}(Q)$	$c_5$	$n$	$n$
6	foreach( $a \in v \rightarrow$ ) {	$c_6$	$m$	$m$
7	$\lambda_a = \max(\lambda_a, (\lambda_v + L_{v,a}))$	$c_7$	$m$	$m$
8	$D_a = D_a - 1$	$c_8$	$m$	$m$
9	if( $D_a = 0$ ) QUEUE( $Q, a$ ) }	$c_9$	$m$	$m$
10	}	$c_{10}$	$n$	$n$
11	$\lambda^* = \max_{v \in V} \{\lambda_v\}$	$c_{11}n$	1	1
12	$v^* = \text{SELECT1}(V, \lambda^*)$	$c_{12}n$	1	1
13	$\pi = \text{BACK\_TRACE}(V, E, L, \lambda, v^*, (spec - \lambda^*))$	$c_{13}$	1	1
14	return( $\pi, \lambda$ ) }	$c_{14}$	1	1

- The slack of an edge  $(a,v)$  is the slack of  $v$  plus the difference between the length of the longest path to  $v$  and the longest path to  $v$  through  $(a,v)$
- In formula,  $\text{slack}_{a,v} = \text{slack}_v + (\lambda_v - (\lambda_a + L_{a,v}))$
- Here  $\lambda_v$  is the length of the longest path to  $v$ , and  $(\lambda_a + L_{a,v})$  is the length of the longest path to  $v$  that passes through the edge  $(a,v)$
- The slack of a node  $u$  is defined as the minimum of its fanout edge slacks, so
$$\text{slack}_u = \min \text{slack}_v$$



- A function  $F(n)$  is in the set  $O(g(n))$  if and only if there exist positive constants  $c_o$  and  $n_o$  such that
$$F(n) \leq c_o g(n) \quad \text{for all } n \geq n_o$$
- This means that  $F(n)$  is asymptotically bounded from above by a linear function of  $g(n)$
- A function  $F(n)$  is in the set  $\Omega(g(n))$  if and only if there exist positive constants  $c_\Omega$  and  $n_\Omega$  such that
$$F(n) \geq c_\Omega g(n) \quad \text{for all } n \geq n_\Omega$$
- This means that  $F(n)$  is asymptotically bounded from below by a linear function of  $g(n)$

