



Introduction to Quantum Logic

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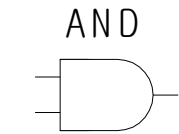


Reversible Logic

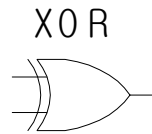
- ❖ *Lose information is equal meaning to lose energy in physical view.*
 - The loss of information is associated with laws of physics requiring that one bit of information lost dissipates $kT \ln 2$ of Energy.
 - Whenever we use a logically **irreversible gate** we **dissipate energy** into the environment.
 - **Reversible computation can reduce heat dissipation** by losing no information through the gate. So, It allows higher densities, higher speed.

Reversible Gate

❖ Irreversible gate

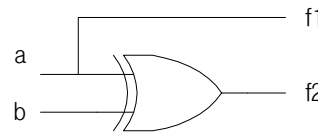


a	b	F
0	0	0
0	1	0
1	0	0
1	1	1

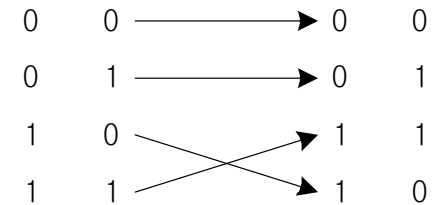


a	b	F
0	0	0
0	1	1
1	0	1
1	1	0

❖ reversible gate



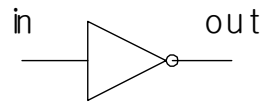
a	b	f1	f2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



- Input and Output is one-to-one mapping
 - ◆ Output reconstruct input.
 - ◆ No information loss through the gate
- The gate of this type is called Feymann gate (so famous physician).
 - ◆ This gate is elementary gate of quantum logic

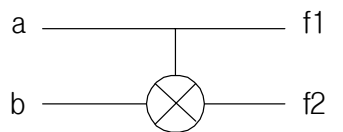
More Reversible Gate

- ❖ Inverter – Only reversible gate in classic logic

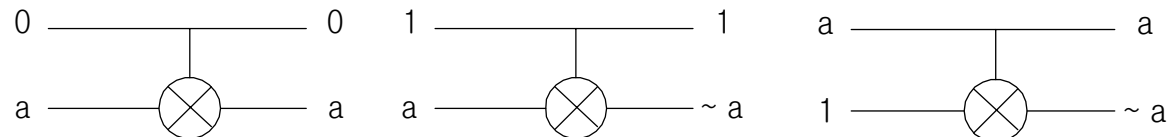
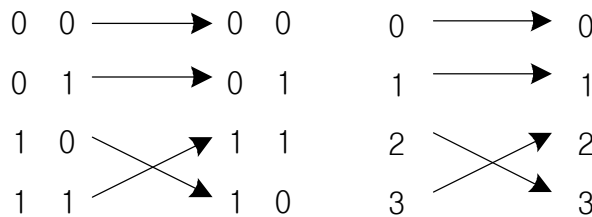


in	out
0	1
1	0

- ❖ Feymann gate – controlled inverter

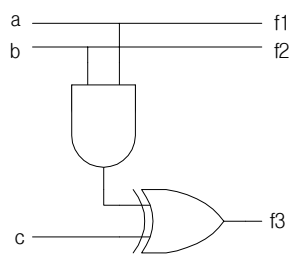


a	b	f1	f2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



More Reversible Gate

❖ Toffoli gate – controlled controlled inverter



$$f1 = a$$

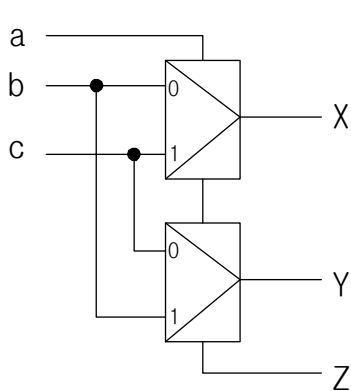
$$f2 = b$$

$$f3 = (a \cdot b) \oplus c$$

a	b	c	f1	f2	f3		
0	0	0	0	0	0	0 0 0	0 → 0
0	0	1	0	0	1	0 0 1	1 → 1
0	1	0	0	1	0	0 1 0	2 → 2
0	1	1	0	1	1	0 1 1	3 → 3
1	0	0	1	0	0	1 0 0	4 → 4
1	0	1	1	0	1	1 0 1	5 → 5
1	1	0	1	1	1	1 1 0	6 → 6
1	1	1	1	1	0	1 1 1	7 → 7

(6 7)
Reversible
&
Universal

❖ Fredkin gate – controlled swap



$$X = \bar{a}b + ac$$

$$Y = ab + \bar{a}c$$

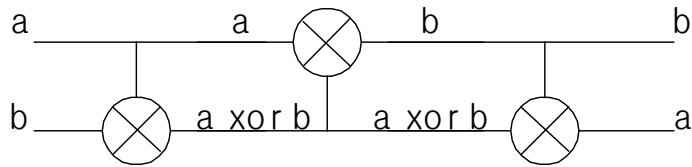
$$Z = a$$

a	b	c	X	Y	Z		
0	0	0	0	0	0	0 0 0	0 → 0
0	0	1	0	1	0	0 0 1	1 → 1
0	1	0	1	0	0	0 1 0	2 → 2
0	1	1	1	1	0	0 1 1	3 → 3
1	0	0	0	0	1	1 0 0	4 → 4
1	0	1	1	0	1	1 0 1	5 → 5
1	1	0	0	1	1	1 1 0	6 → 6
1	1	1	1	1	1	1 1 1	7 → 7

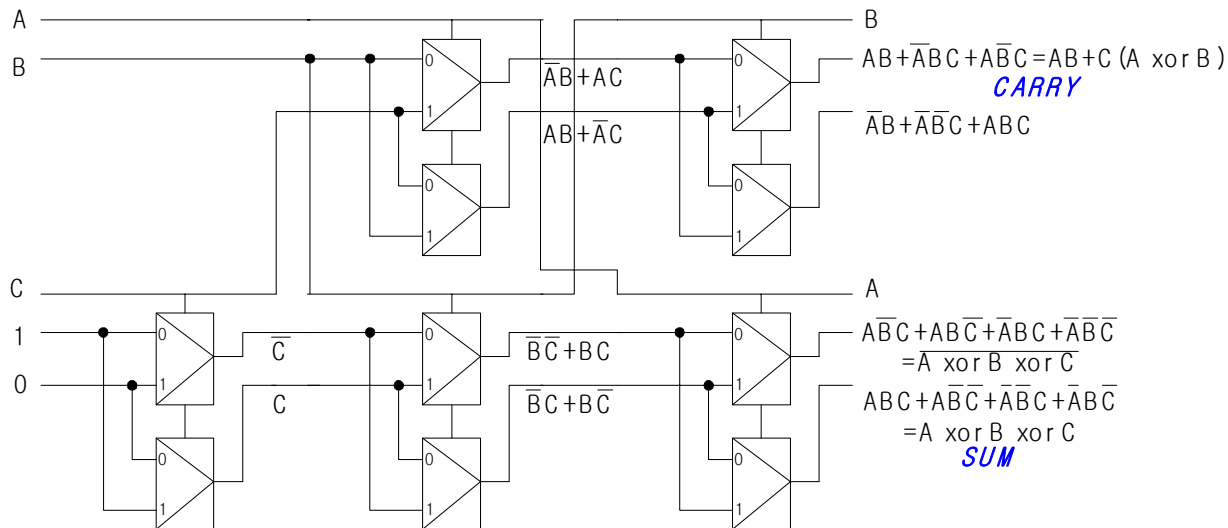
(1 4 2)
(3 6)

Circuit with reversible gate

❖ Swapper with Feymann gate



❖ Full Addder with 5 Fredkin gate





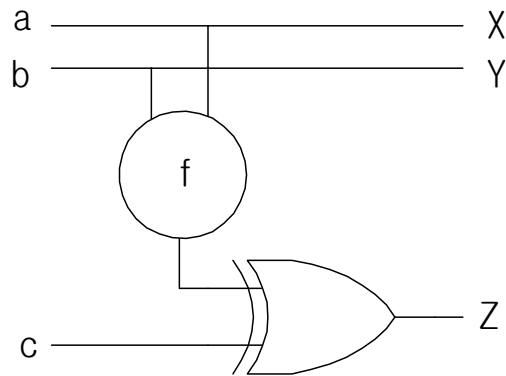
General reversible gate

❖ Four reversible gate

- Inverter 1-by-1 gate
- Feymann 2-by-2 gate
- Toffoli 3-by-3 gate
- Fredkin 3-by-3 gate
- More?? 4-by-4 gate? ... N-by-N gate is possible

General reversible gate

❖ G gate (General gate of inv, Feymann, Toffoli)

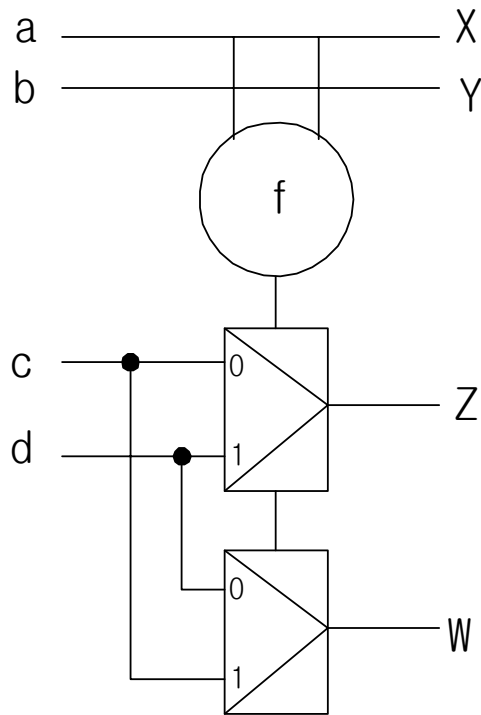


- $f =$ arbitrary function of a & b
- G gate is reversible

$C = 0$	$f(a,b)$	c	$f(a,b)$	Z
		0	0	0
		0	1	1
		1	0	1
		1	1	0

General reversible gate

- ❖ H gate (General gate of Fredkin gate)
 - H gate is reversible gate



$f(a,b)$	c	d	Z	W
0	0	0	0	0
	0	1	0	1
	1	0	1	0
	1	1	1	1
1	0	0	0	0
	0	1	1	0
	1	0	0	1
	1	1	1	1

$$X = a$$

$$Y = b$$

$$Z = \overline{f(a,b)c} + f(a,b)d$$

$$W = f(a,b)c + \overline{f(a,b)d}$$