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Lattice Diagrams using Reed-Muller Logic

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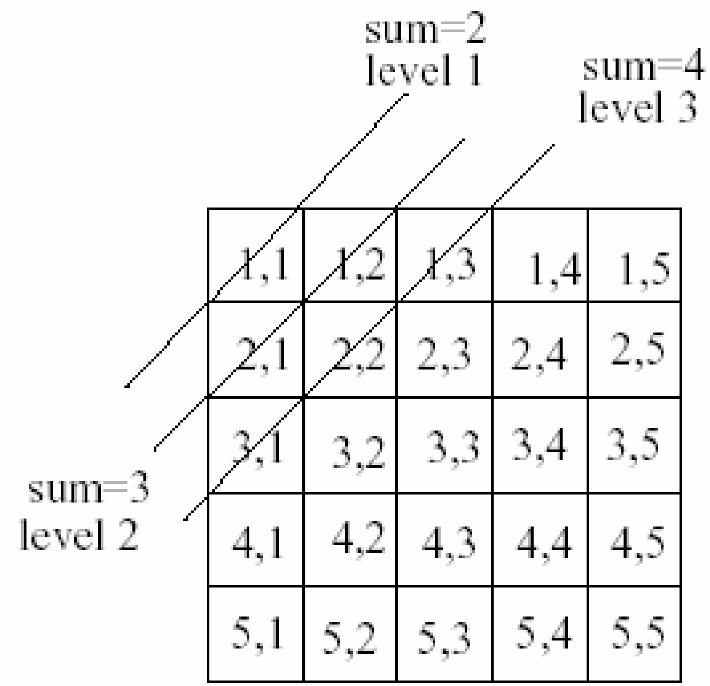
Introduction

- To realize arbitrary Boolean functions in a regular and planar layout
  - Universal Akers Array (UAA)

- This paper presents an extension of UAAs, called "Lattice Diagrams" and the efficient method of mapping arbitrary multi-output functions.

Background

- Lattice Diagram
  - Data structures that describe both regular geometry of connections and a logic of a circuit.



-Array to explain Lattice concepts
-Each block means a function.

- Expansion type
  - Shannon : $F = a \cdot F_a @ a' \cdot F_{a'}$
  - Positive Davio (pD)

    : $F = a(F_a @ F_{a'}) @ 1 \cdot F_{a'}$
  - Negative Davio (nD)

    : $F = 1 \cdot F_a @ a'(F_a @ F_{a'})$

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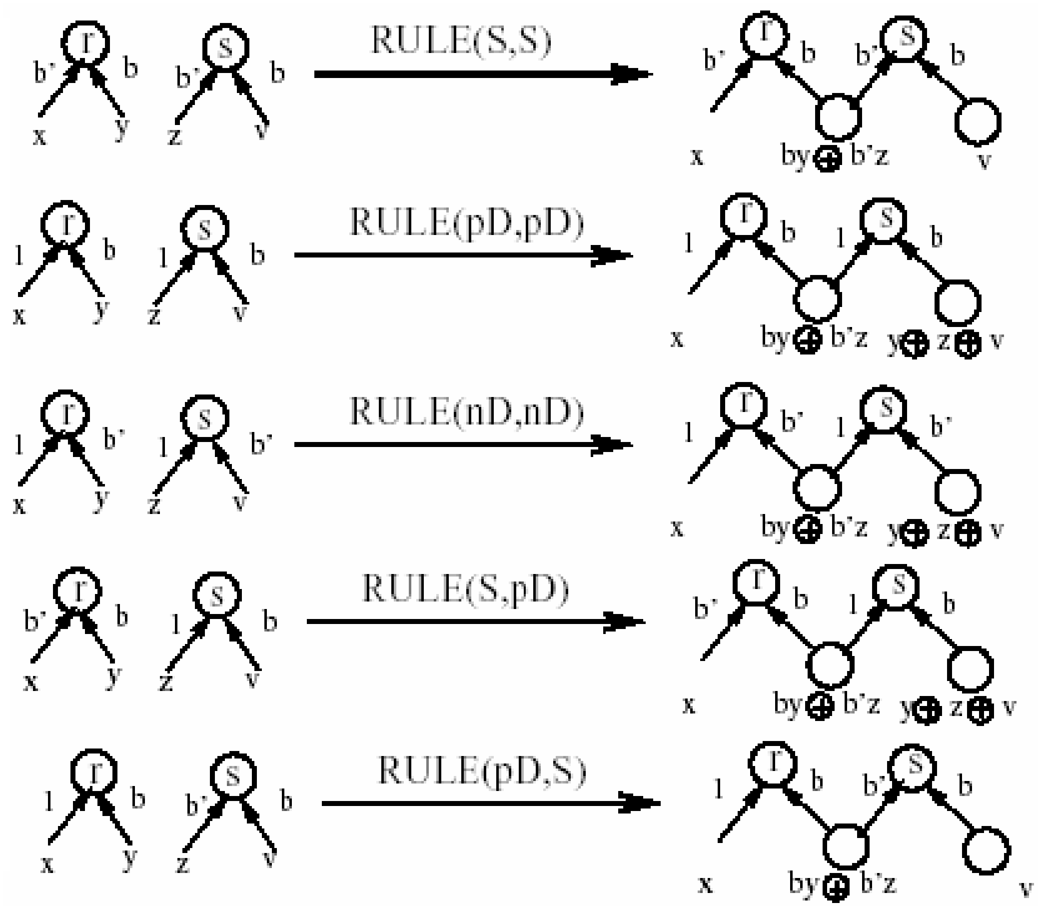
Classification of Lattice Diagram

- Order
  - Ordered ~ : one variable on a diagonal (=level)
  - Ordered ~ with Repeated Variables : one variable in a level, but the same variable may appear on various levels.
  - Free ~ : different orders of variables in the paths leading from leafs to the root
    - Folded ~ : Free ~, but the order of variables in levels must be the same, with some variables possibly missing.
- Expansion
  - All expansions are of the same type.
    - Shannon ~ : Shannon
    - Functional ~ : pD.
    - Negative Functional ~ : nD.
  - All expansions in every level are of the same type.
    - Reed-Muller ~ : pD, nD.
    - Kronecker ~ : Shannon, pD, nD.
  - All expansions in every level are either of some expansion types.
    - Pseudo Reed-Muller ~ : pD, nD
    - Pseudo S/pD ~ : Shannon, pD
    - Pseudo S/nD ~ : Shannon, nD
    - Pseudo Kronecker ~ : Shannon, pD, nD

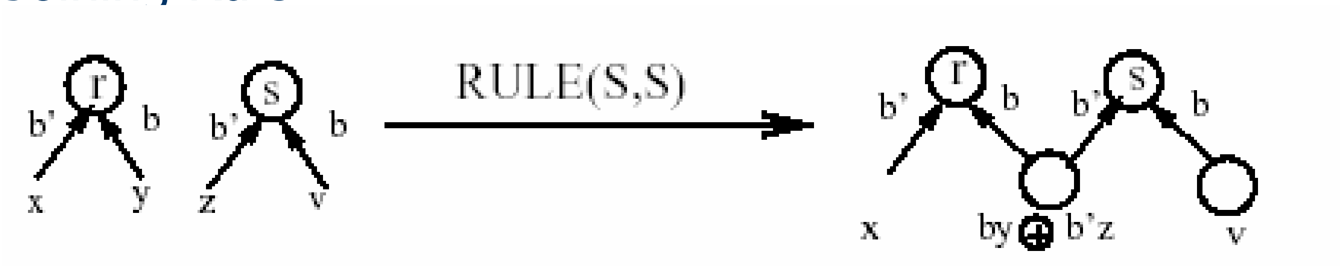
Background

- Joining Rule
  - R= 1·x@b(by@b'z)
    = 1·x@by
  - S= 1·(by@b'z)@
    b·(y@z@v)
    = (by@by)@
    z(b'@b)@
    bv
    = 0@(z·1)@bv
    = 1·z@bv



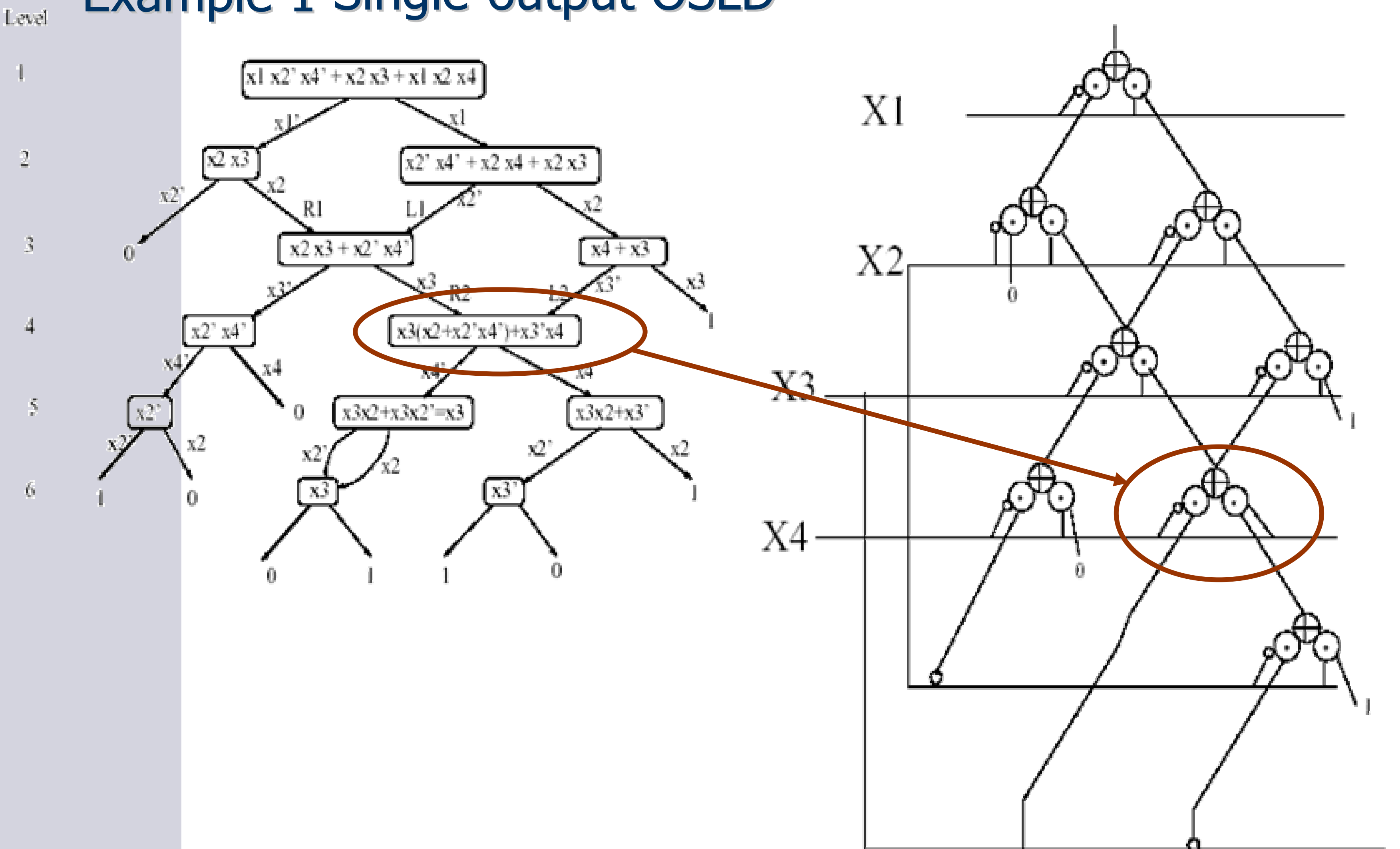
Method of Creating Lattice Diagram

- Ordered Shannon Lattice Diagram (OSLD)
  - It is expanded level-by-level, starting from the root level, and from left to right in every level.
  - In contrast to standard BDDs, the joining operation combines also non-isomorphic nodes of trees.
    - Shannon Expansion: $F = a \cdot F_a \oplus a' \cdot F_{a'}$
    - Joining Rule



Method of Creating Lattice Diagram

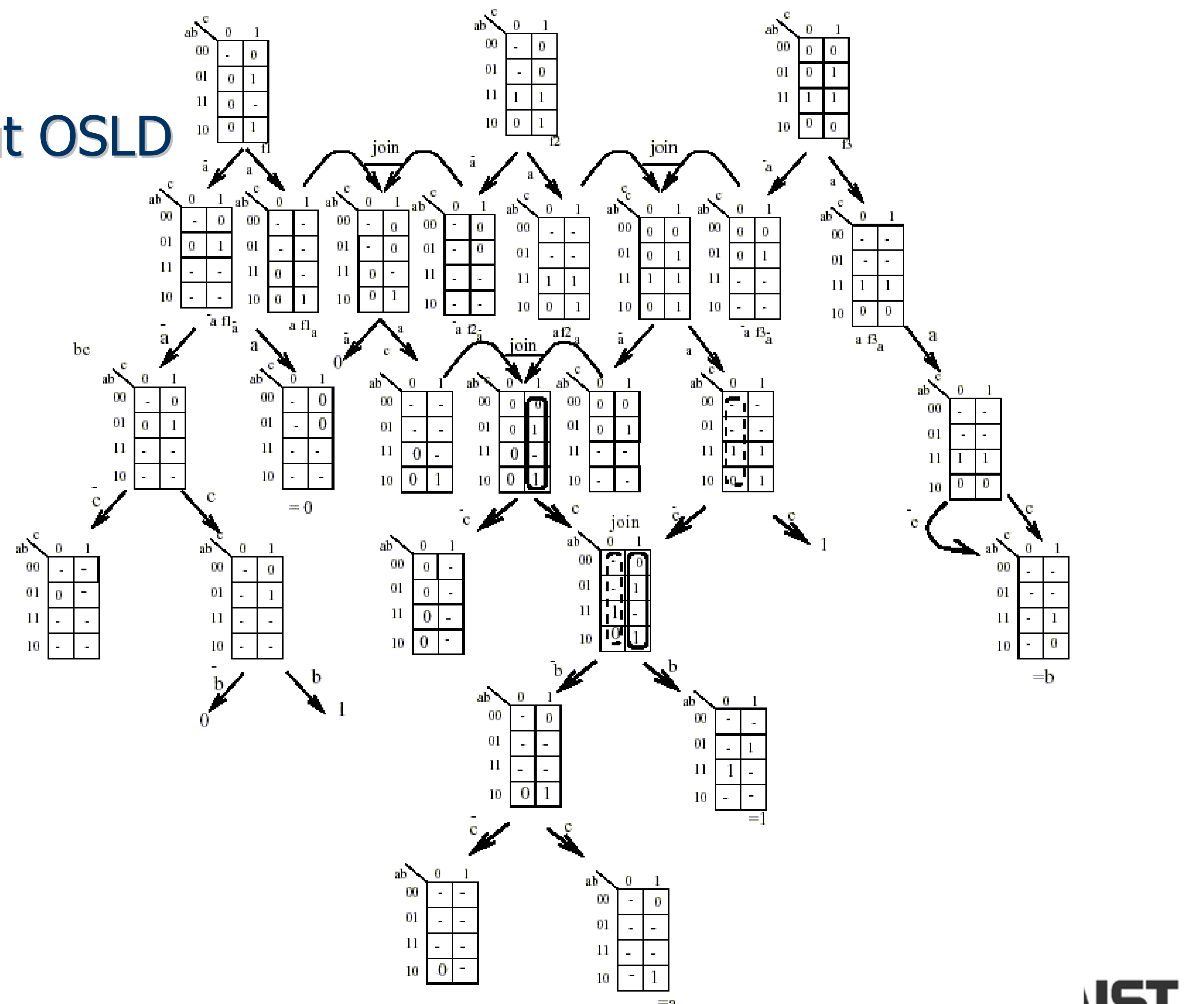
Example 1 Single-output OSLD



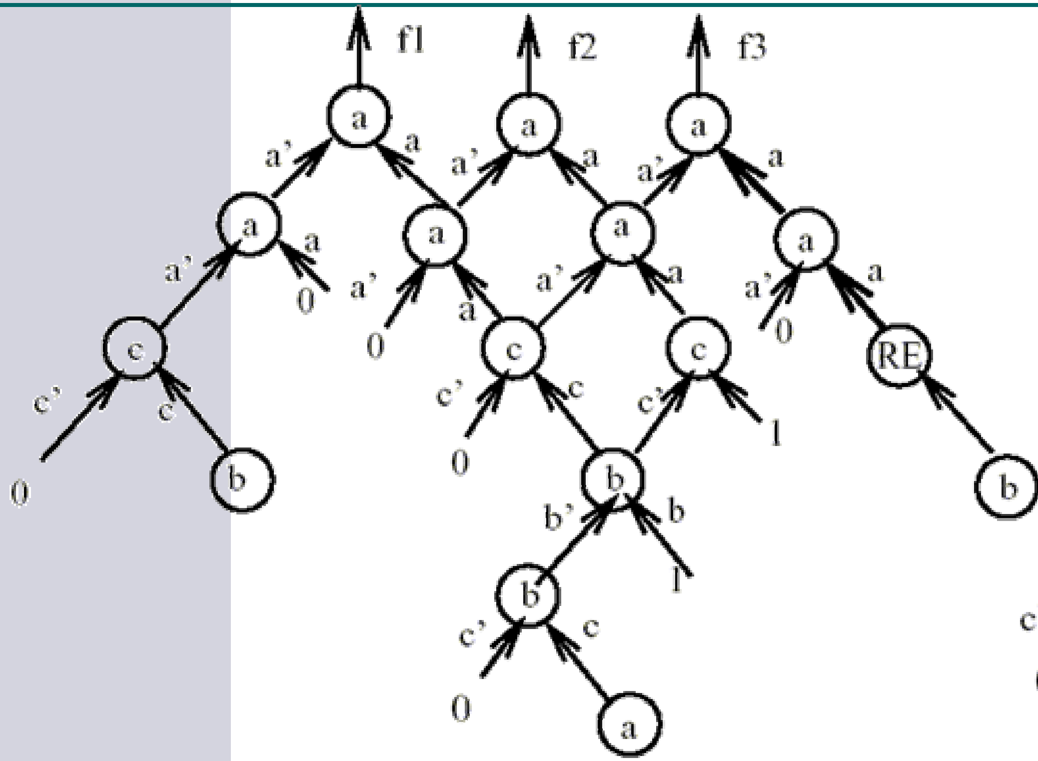
Method of Creating Lattice Diagram

Example 2

Multi-output OSLD

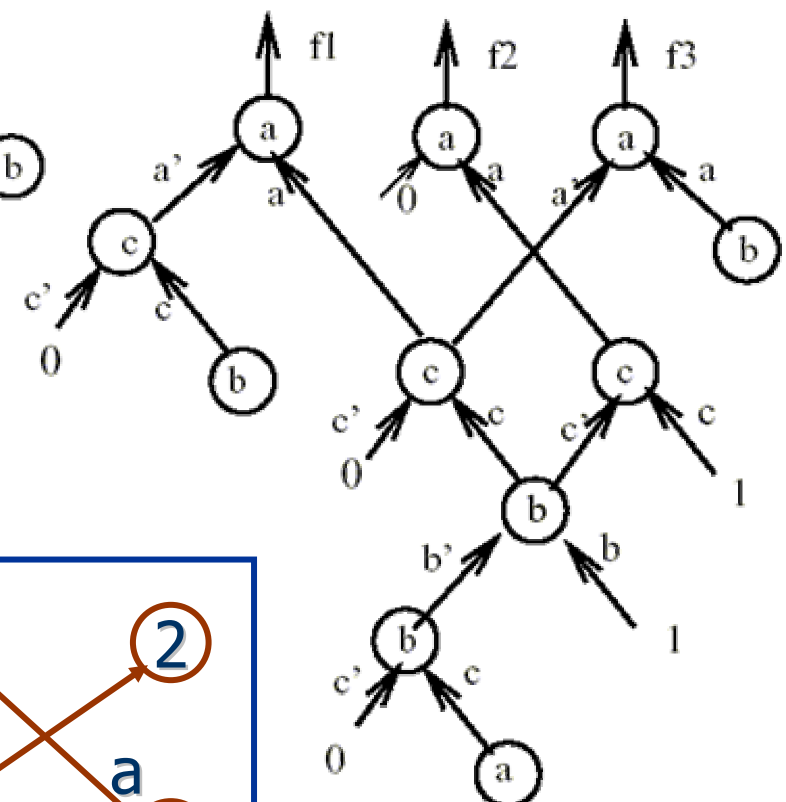


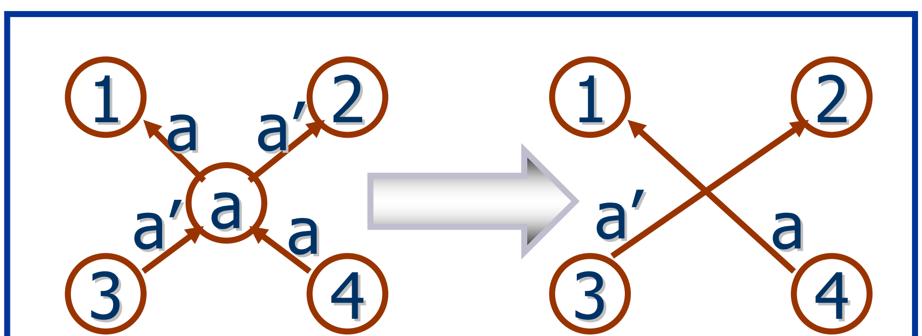
Method of Creating Lattice Diagram



Multi-output OSLD

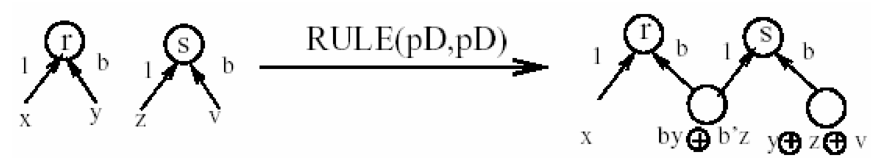
Folded SLD





Method of Creating Lattice Diagram

- Functional Lattice Diagram
  - It is like for OSLD.
  - Positive Davio expansions are used instead of Shannon and the (pD,pD) joining rules instead of the (S,S) joining rules.
    - Positive Davio (pD) : $F = 1 \cdot F_{a'} @ a(F_{a'} @ F_a)$
    - Joining Rule (pD, pD) :



1@ad@bd@abd → 1: 1 ; d: (1)@(1@a@b@ab) = a@b@ab

a@b@d@bd → 1: a@b ; d: (a@b)@(a@1) = b@1

d(a@b@ab) @ d'(a@b) =
a(d@d')@b(d@d')@abd = a@b@abd

a@b@ab @ a@b @ b@1
=ab@b@1

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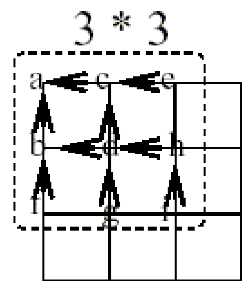
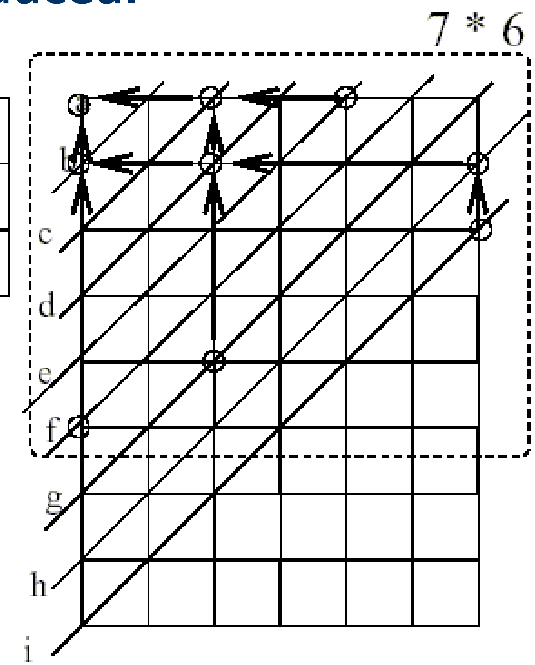
Method of Creating Lattice Diagram

- Ordered Kronecker Lattice Diagrams (OKLD)
  - It uses the joining rules, (S,S), (pD,pD), (nD, nD) because all expansions in every level are of the same type.

- Folded Kronecker Lattice Diagram
  - The rectangular envelope area has been reduced.





Method of Creating Lattice Diagram

- Pseudo-Kronecker Lattice Diagram
  - Pseudo S/pD Kronecker Lattice Diagram can be solved. (only mixture of S and pD nodes in a level)
  - But, Joining rules cannot be created for combinations of expansion nodes (pD,nD) and (nD,S)
  - Open problem whether creating of Pseudo-Kronecker Lattice Diagrams for (S,pD,nD) can be solved analogously to the previous method.

Conclusion

- Introduce method of creating various type of lattice diagram by combining together non-isomorphic nodes at the same level.
- There is no constraint on repeating variables consecutively.
- Use any subset of S, pD, and nD expansions.
- Support arbitrary multi-output functions.