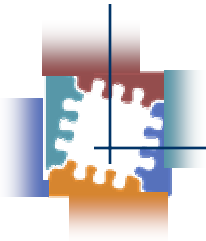


# *GRASP: A Search Algorithm for Propositional Satisfiability*

EE878C Homework #2

2002/11/1

KAIST, EECS ICS Lab

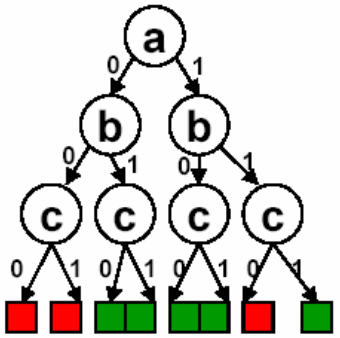
Lee, Dongsoo

# Sat in a Nutshell

- Given a Boolean formula, find a variable assignment such that the formula evaluates to 1, or prove that no such assignment exists.

$$F = (a + b)(a' + b' + c)$$

- For n variable,s there are $2^n$ possible truth assignments to be checked.




- NP-Complete problem.

# Problem Representation

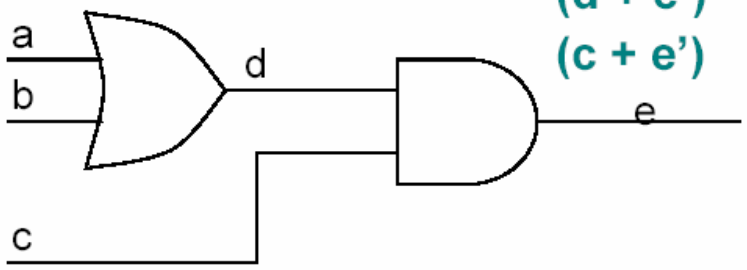
- Conjunctive Normal Form
  - F = (a+b)(a'+b'+c)
  - Simple representation (more efficient data structures)
- Logic circuit representation
  - Circuits have structural and direction information
- Circuit – CNF conversion is straightforward

$d \equiv (a + b)$

$(a + b + d')$
$(a' + d)$
$(b' + d)$

$e \equiv (c \cdot d)$

$(c' + d' + e)$
$(d + e')$
$(c + e')$



# DLL Algorithm

- Davis, Logemann and Loveland
  - M. Davis, G. Logemann and D. Loveland, "A Machine Program for Theorem-Proving", Communications of ACM, Vol. 5, No. 7, pp. 394-397, 1962
- Basic framework for many modern SAT solvers
- Also known as DPLL for historical reasons

# Basic DLL Procedure - DFS

(a' + b + c)
(a + c + d)
(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)

# Basic DLL Procedure - DFS

(a' + b + c)
(a + c + d)
(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)

# Basic DLL Procedure - DFS

(a' + b + c)
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(a + c + d')
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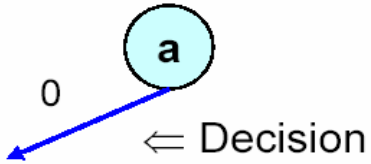
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)



# Basic DLL Procedure - DFS

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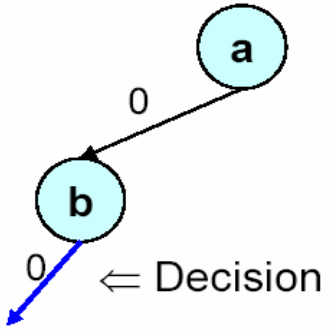
(a + c + d)
(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)



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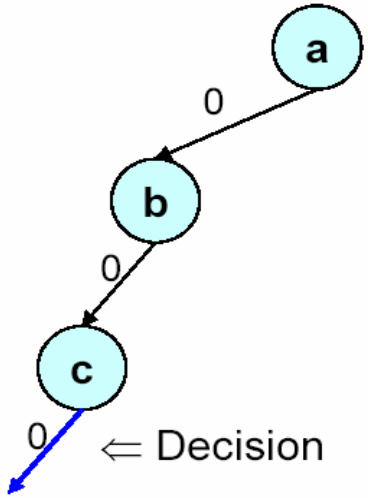

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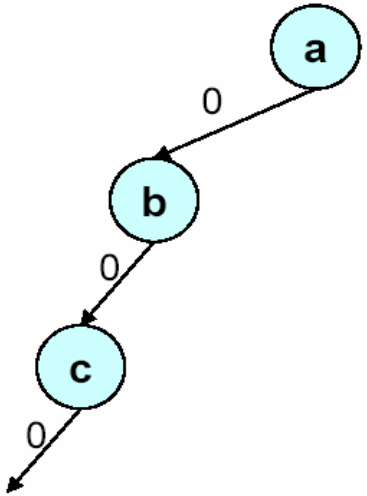
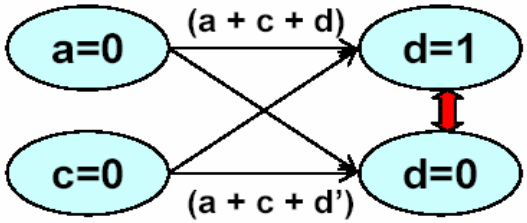



Implication Graph




Conflict!

# Basic DLL Procedure - DFS

(a' + b + c)
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(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
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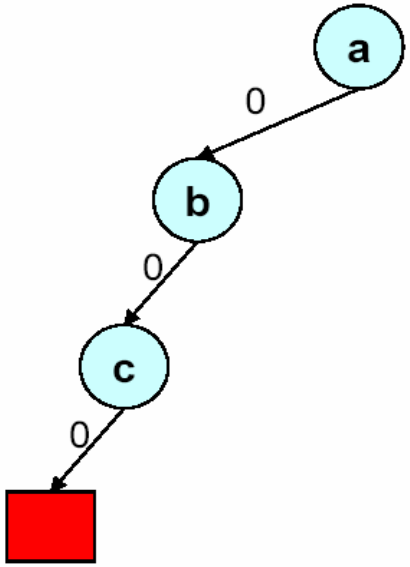
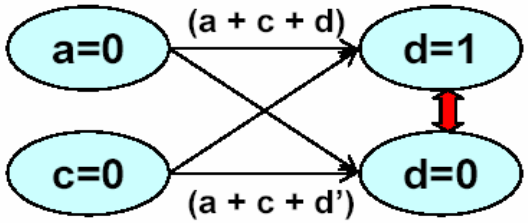



Implication Graph




Conflict!

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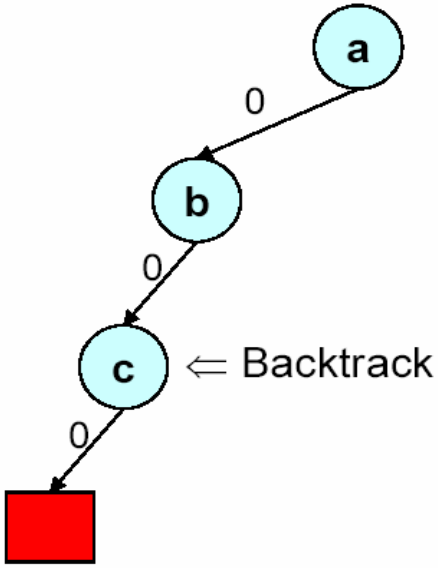

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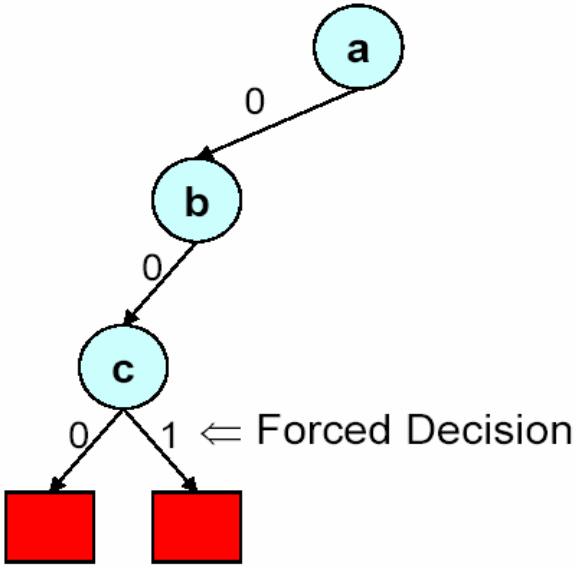


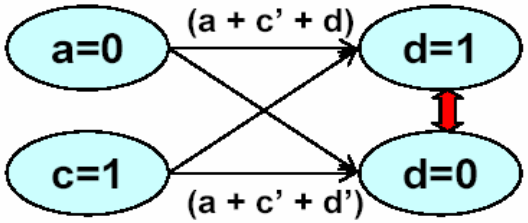


Conflict!

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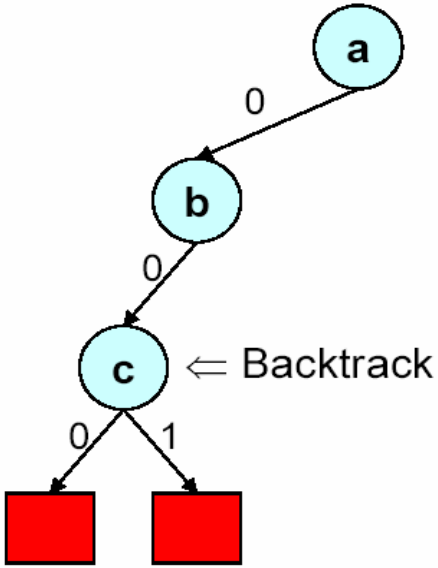

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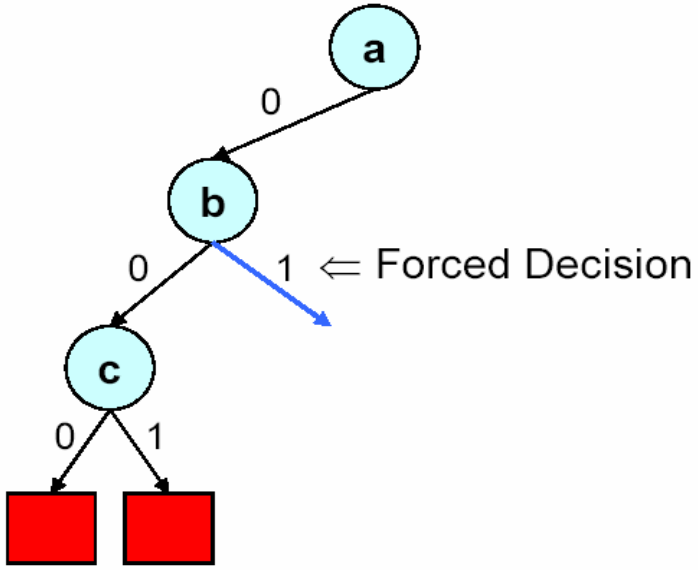

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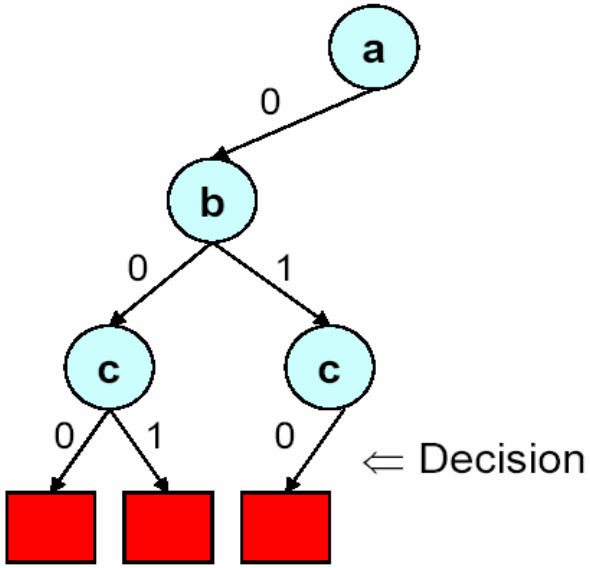


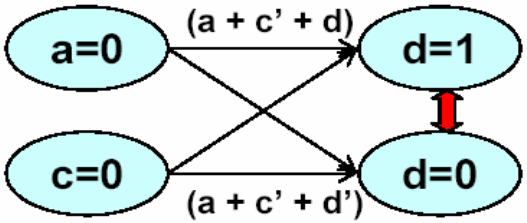


Conflict!

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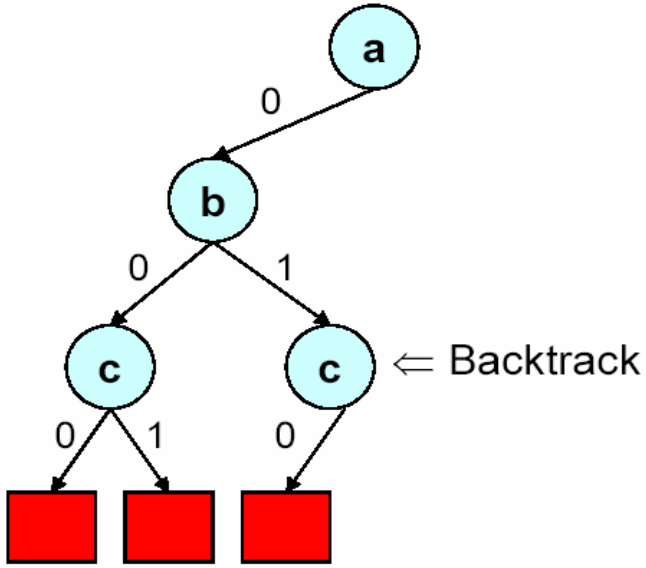

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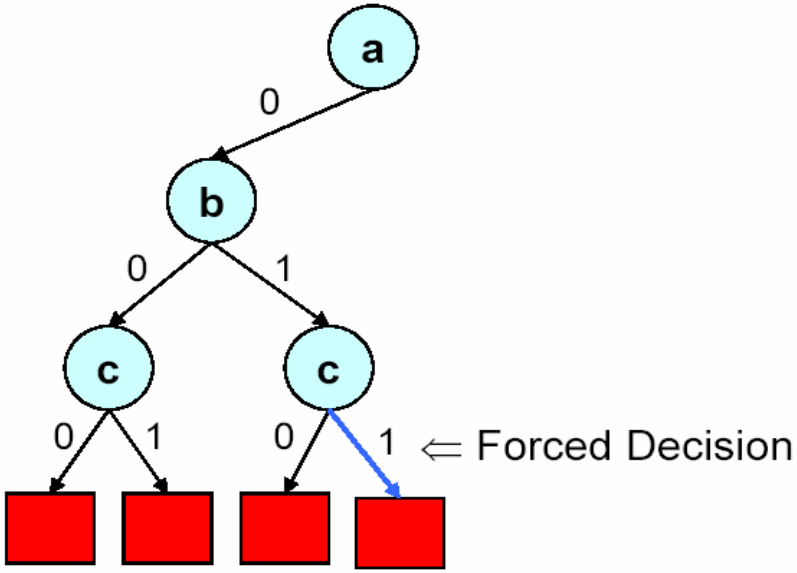


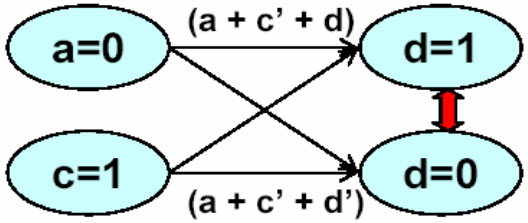


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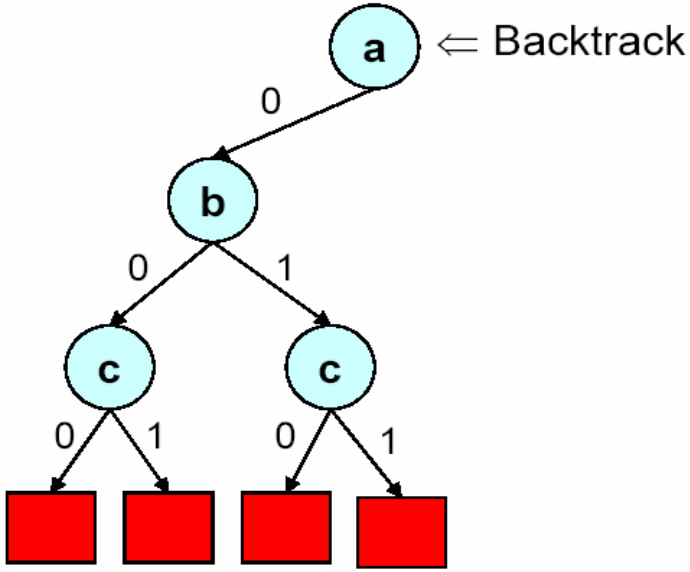

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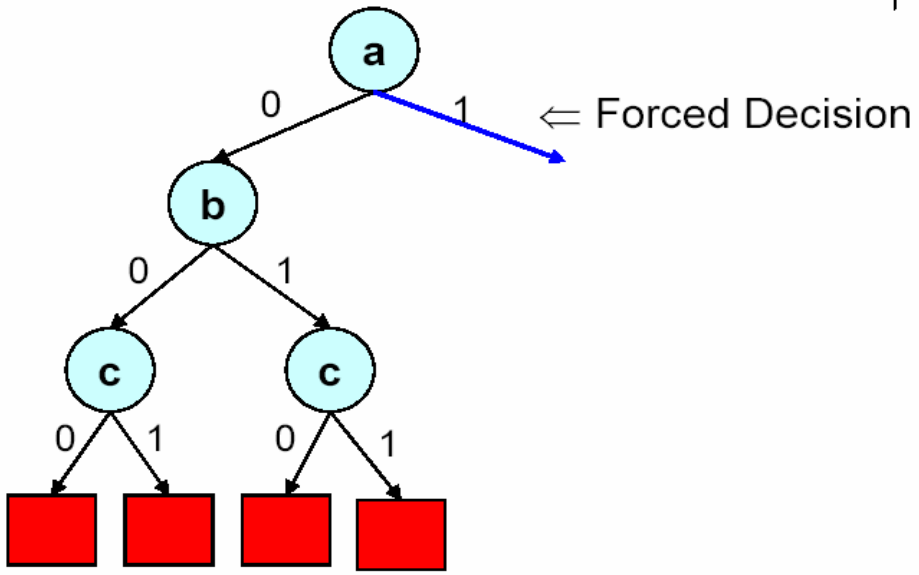

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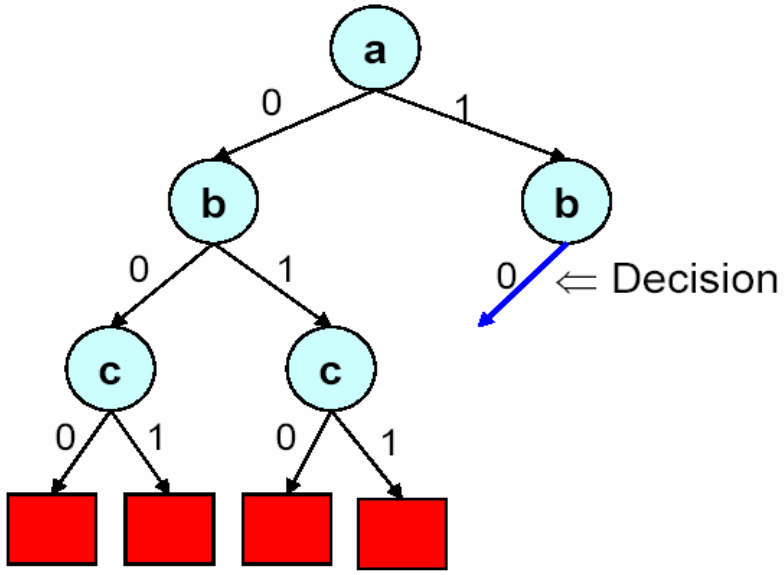

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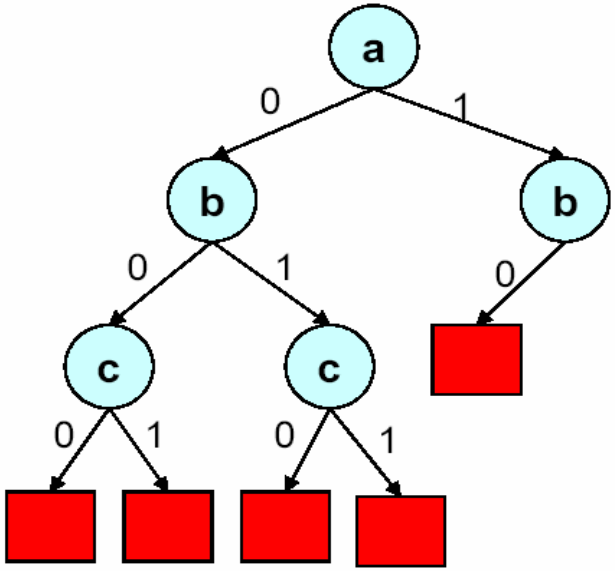


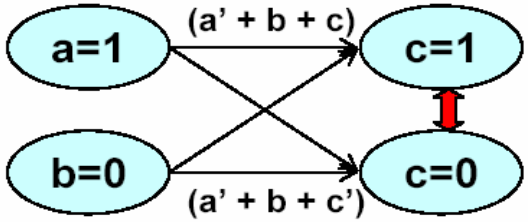


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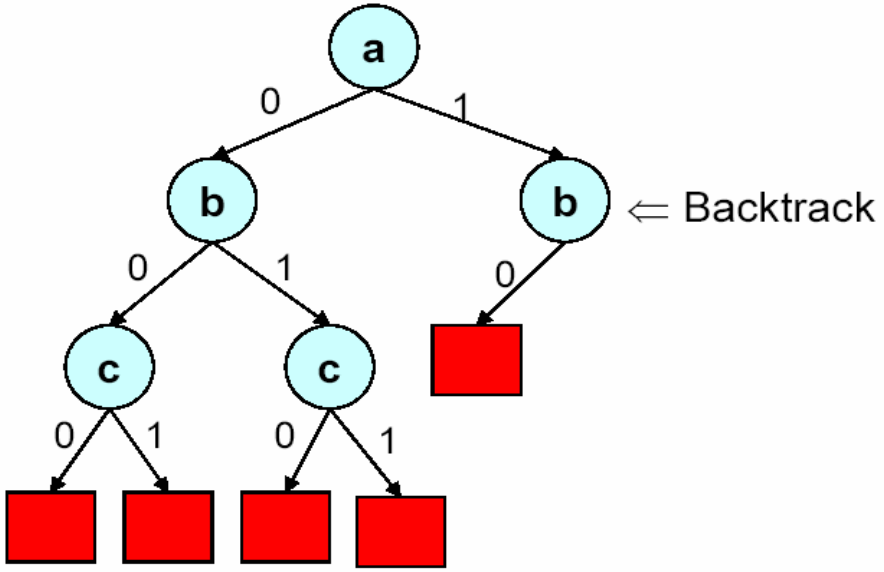

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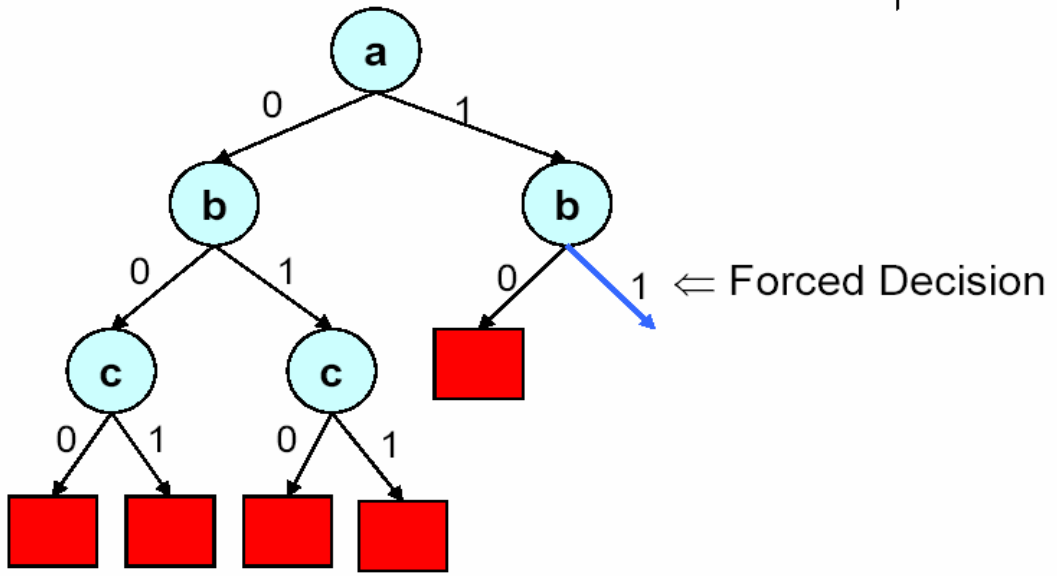


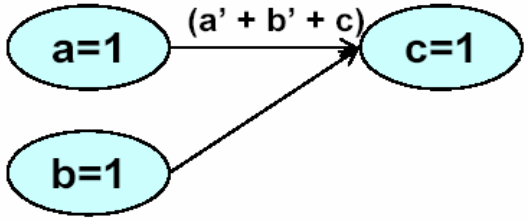

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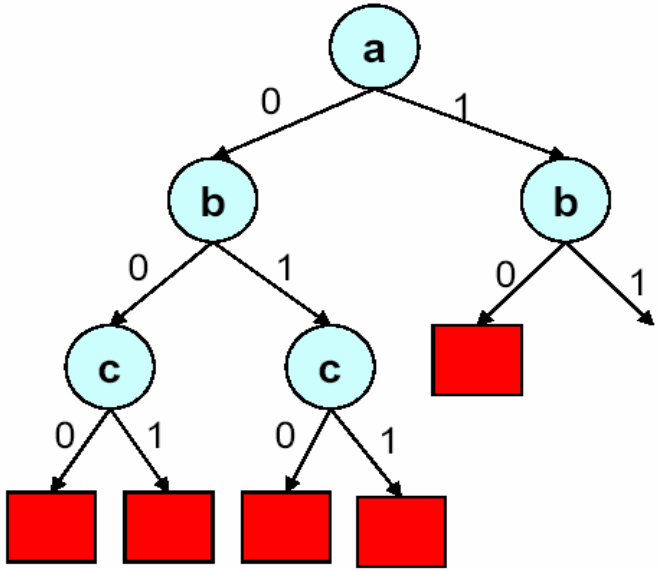


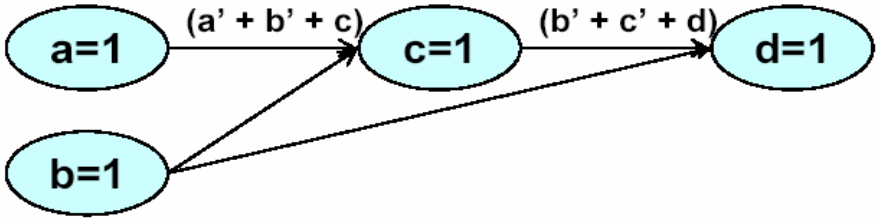

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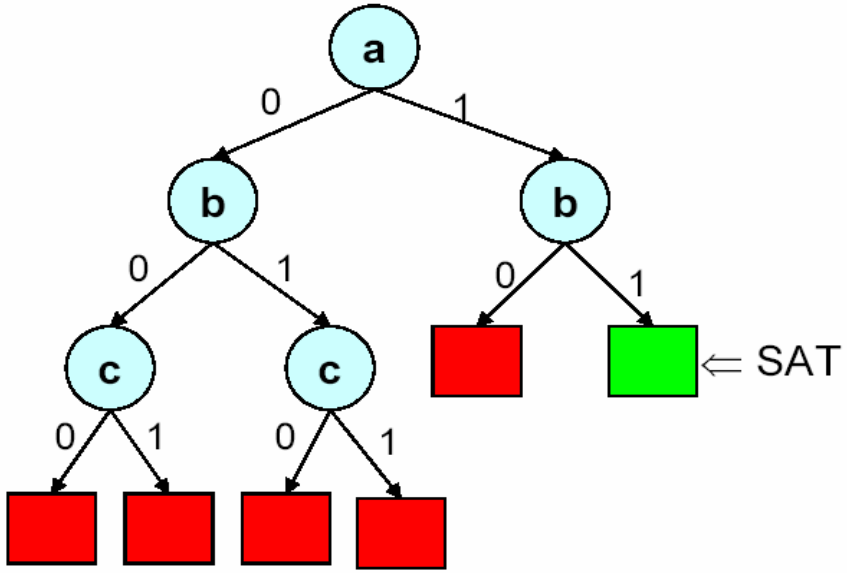
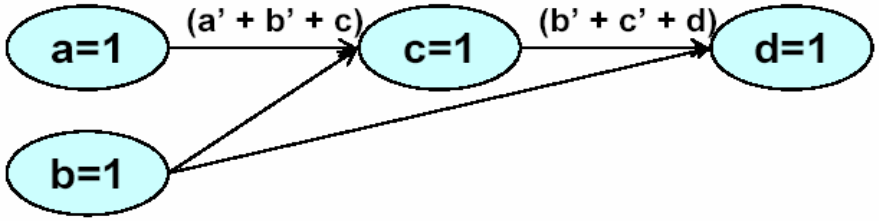
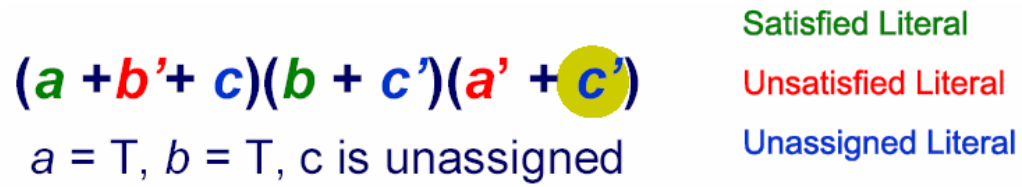


# *Implications and Boolean Constraint Propagation*

- Implication
  - A variable is forced to be assigned to be True or False based on previous assignments.
- Unit clause rule (rule for elimination of one literal clauses)
  - An unsatisfied clause is a unit clause if it has exactly one unassigned literal.




  - The unassigned literal is implied because of the unit clause.
- Boolean Constraint Propagation (BCP)
  - Iteratively apply the unit clause rule until there is no unit clause available
- Workhorse of DLL based algorithms.

# GRASP

- Marques-Silva and Sakallah [SS96,SS99]
  - J.P.Marques-Silva and K.A.Sakallah,"GRASP – A New Search Algorithm for Satisfiability," Proc. ICCAD 1996. (49 citations)
- Incorporates conflict driven learning and non-chronological backtracking
- Practical SAT instances can be solved in reasonable time
- Bayardo and Schrag's RelSAT also proposed conflict driven learning [BS97]

# Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

# Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
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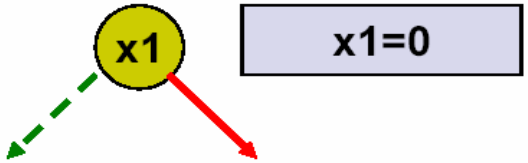


x1=0

# Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
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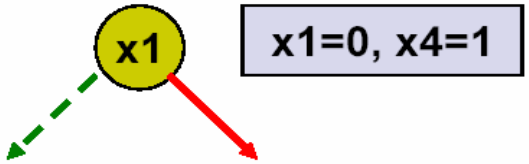


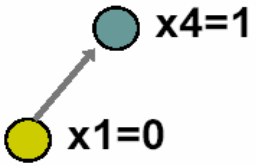

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x1 + x3' + x8'
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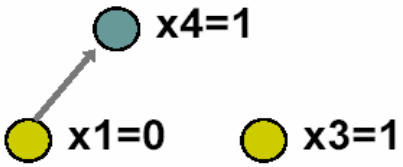
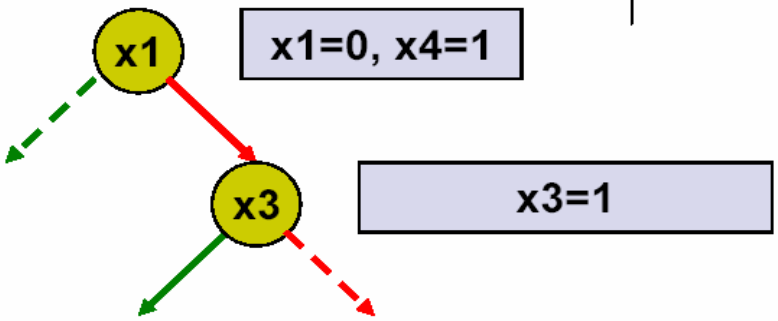
x7 + x10 + x12'



# Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
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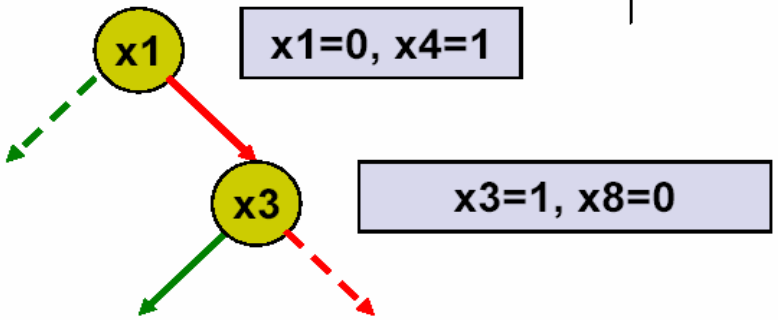
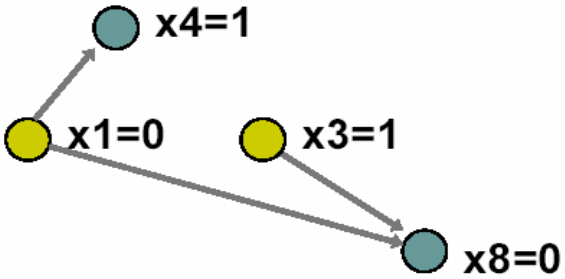
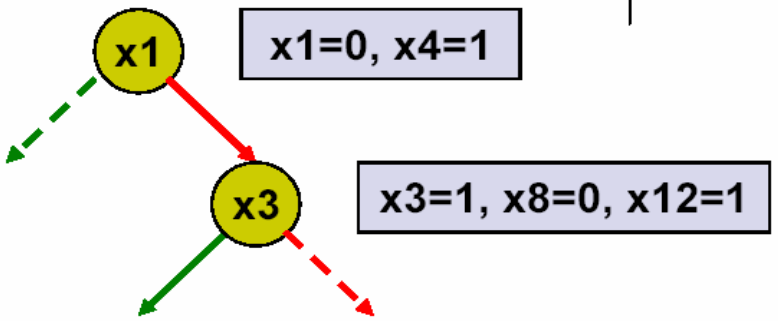
x7 + x10 + x12'

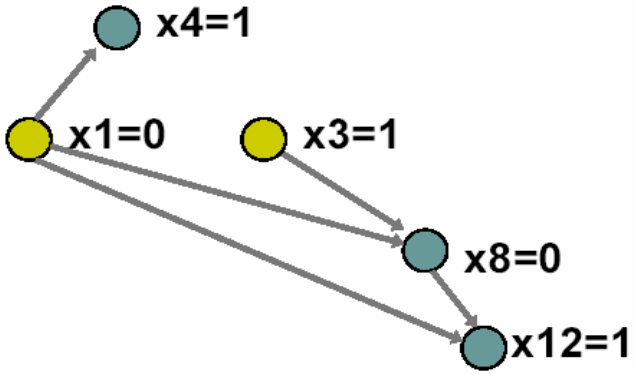


# Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'



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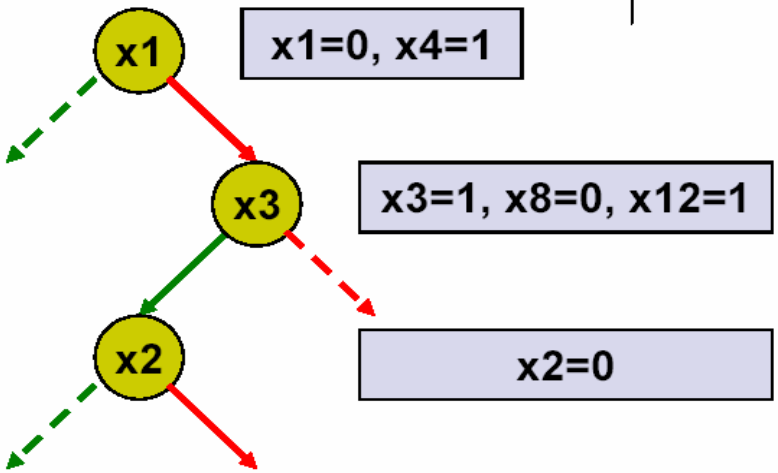


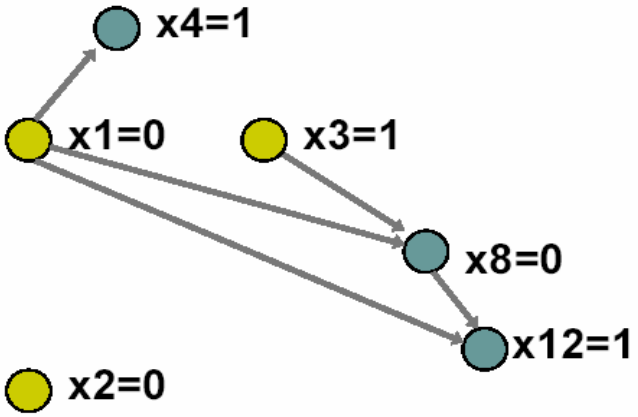

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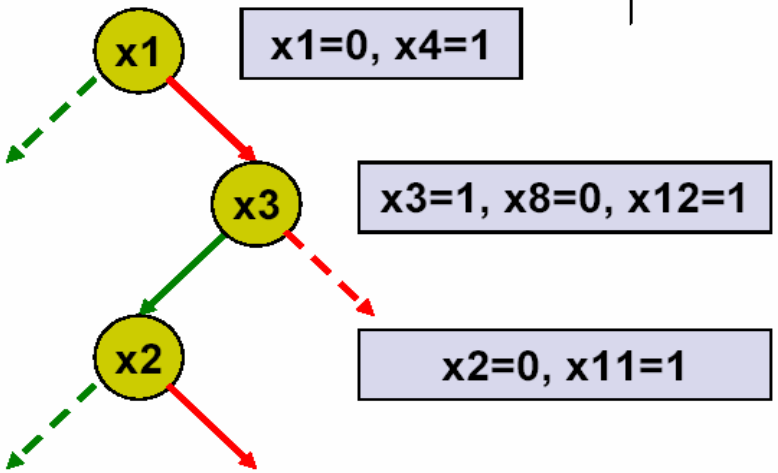


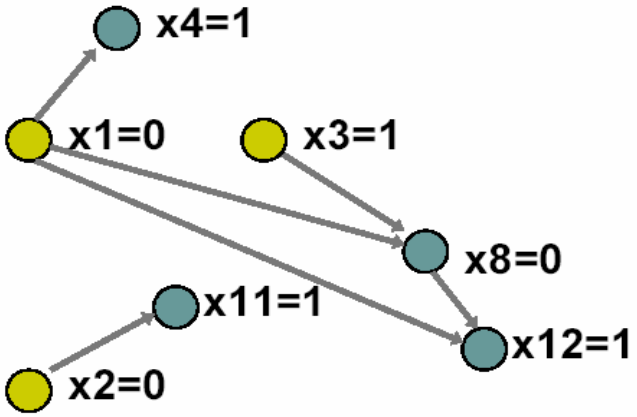

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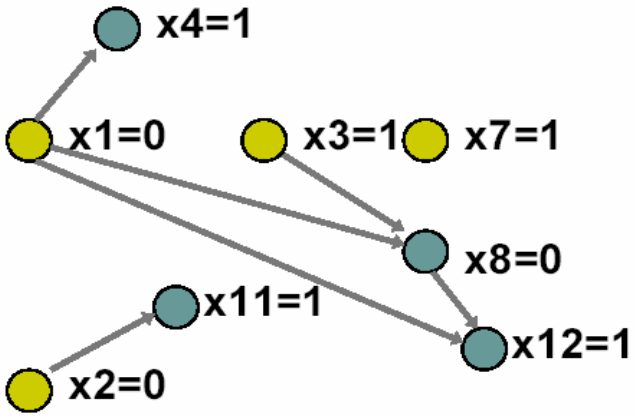


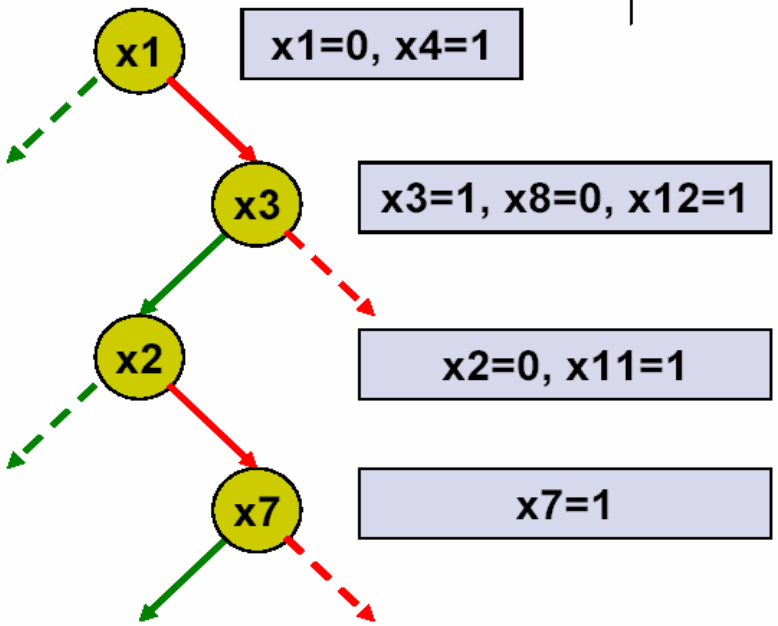

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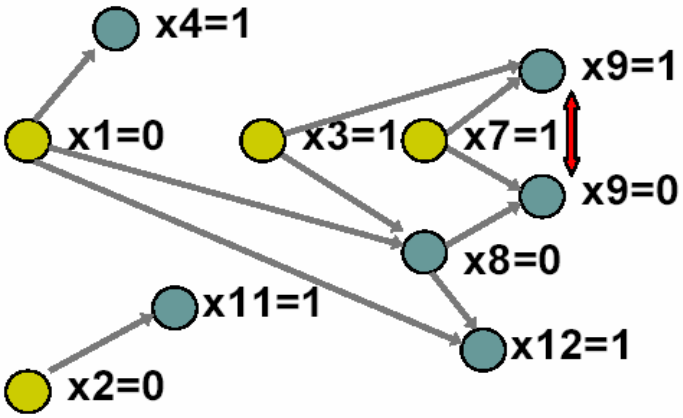


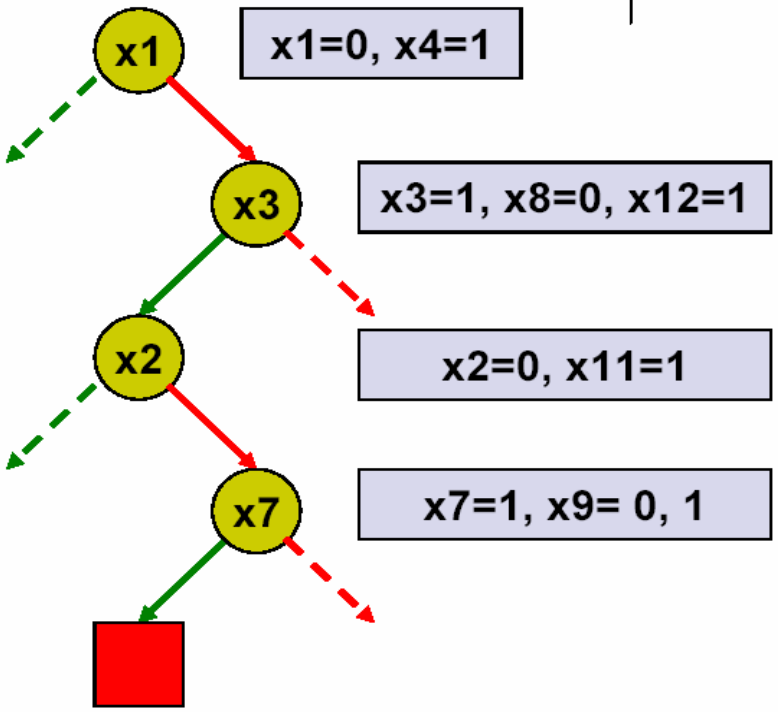

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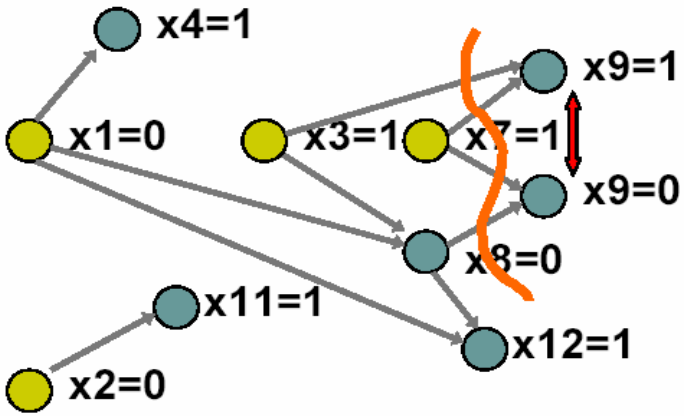


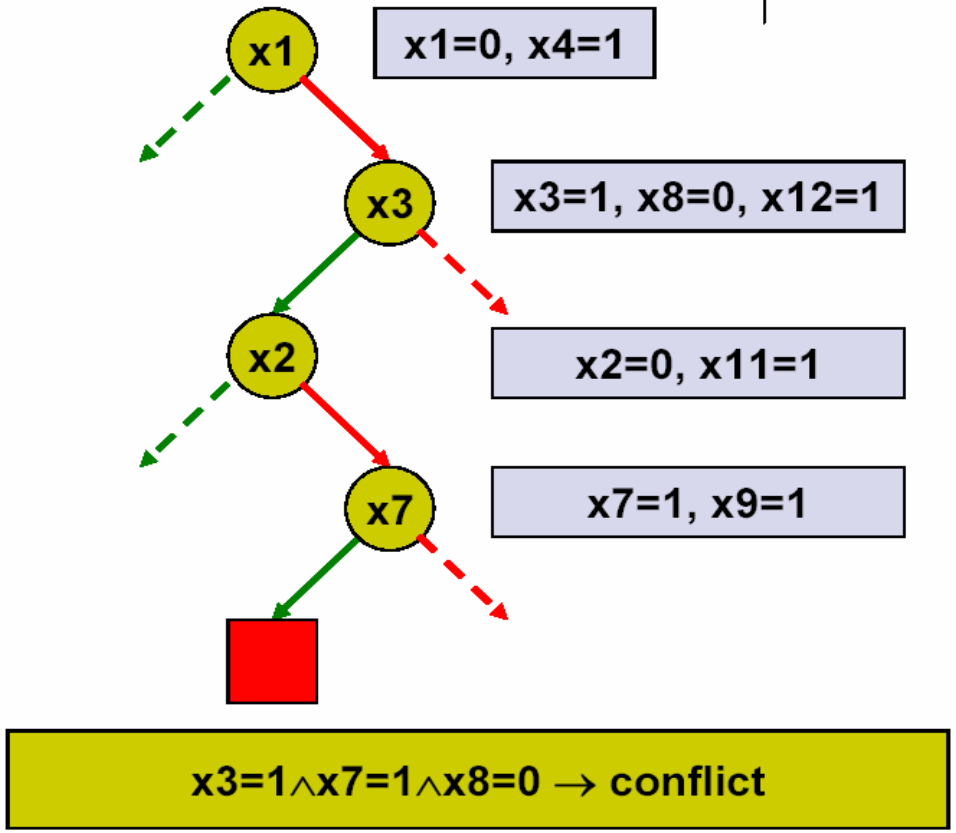

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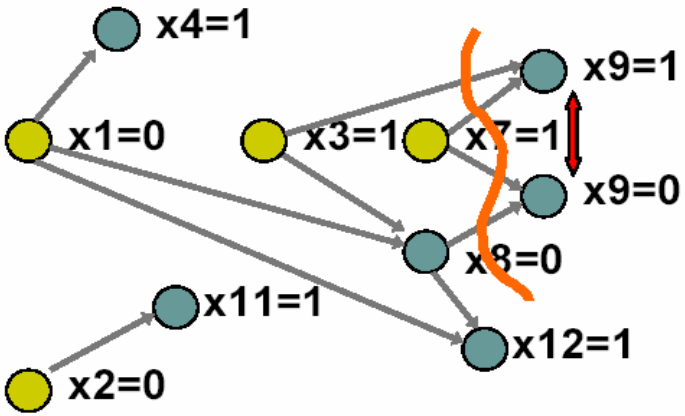


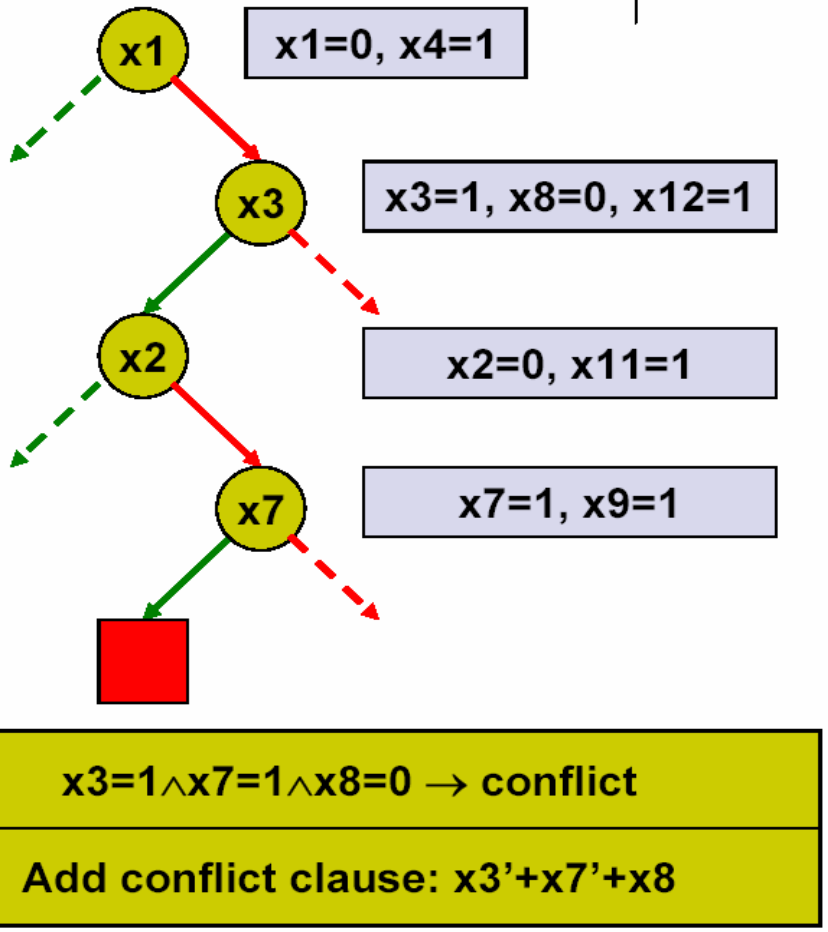

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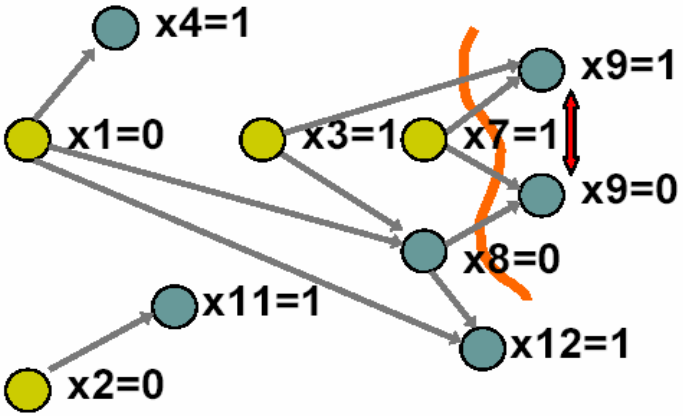
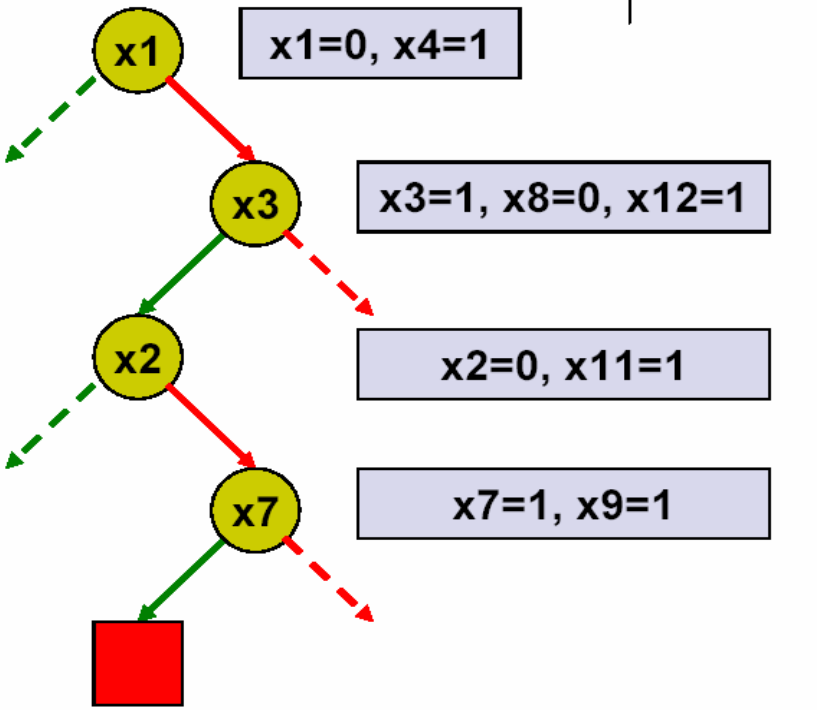
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

> x3'+x7'+x8




x3=1∧x7=1∧x8=0 → conflict

Add conflict clause: x3'+x7'+x8

# Conflict Driven Learning and Non-chronological Backtracking

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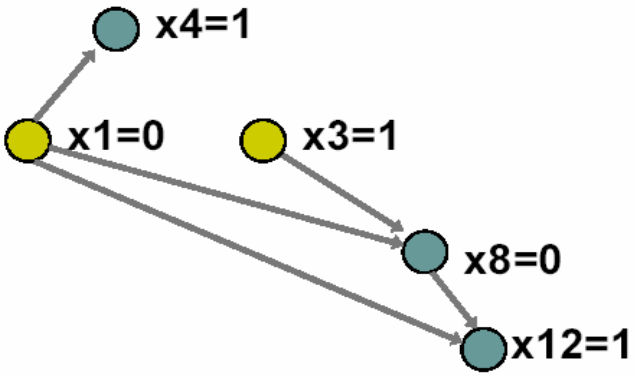
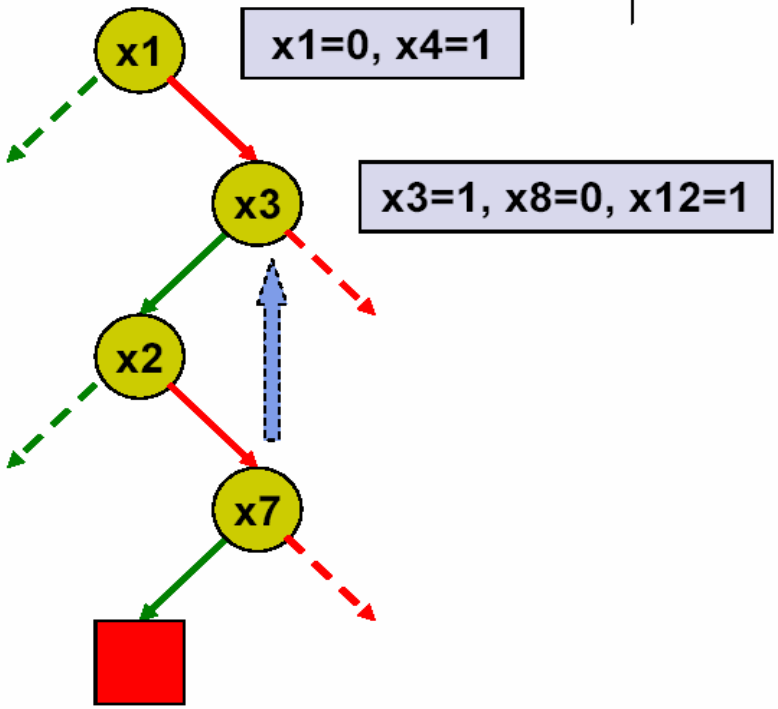
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'
x3' + x8 + x7'




Backtrack to the decision level of x3=1
x7 = 0

# *Conflict Clause*

- Significantly prune the search speace
  - Learned clause is useful forever
- Useful in generating future conflict clauses