State Assignment Using Partition Pairs
Discuss hypercube method, add slides later on
State Assignment Using **Partition Pairs**

- This method allows for finding high quality solutions but is slow and complicated.
- Only computer approach is practical.
- **Definition of Partition.**
  - Set of blocks $B_i$ is a partition of set $S$ if the union of all these blocks forms set $S$ and any two of them are disjoint.
  - $B_1 \cup B_2 \cup B_3 \ldots = S$
  - $B_1 \cap B_2 = \{\}$, $B_2 \cap B_3 \ldots = \{\}$, etc
  - **Example 1:** $\{12,45,36\}$, $\{\{1,2\},\{4,5\},\{3,6\}\}$
  - **Example 2:** $\{123,345\}$ not a partition but a **set cover**
State Assignment Using Partition Pairs

- **Definition of X-successor of state** $S_a$
  - The **state** to which the machine goes from state $S_a$ using input $X$

- **Definition of Partition Pair**
  - $P_1 \rightarrow P_2$ is a partition pair if for every two elements $S_a$ and $S_b$ from any block in $P_1$ and every input symbol $X_i$ the $X_i$ successors of states $S_a$ and $S_b$ are in the same block of $P_2$
State Assignment Using **Partition Pairs**

Methods of calculation of **Partition Pairs**

- Partition pair $P_1 \Rightarrow P_2$ calculated with known partition $P_1$
- Partition Pair $P_1 \Rightarrow P_2$ calculated with known partition $P_2$
Calculation of successor partition from the predecessor partition in the partition pair

\[ \{1, 23, 45\} \Rightarrow \{1, 2, 3, 45\} \]
Calculation of successor partition from the successor partition in the partition pair

<table>
<thead>
<tr>
<th></th>
<th>X_1</th>
<th>X_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
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<tr>
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<table>
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<tr>
<th></th>
<th>X_1</th>
<th>X_2</th>
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<td>5</td>
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</tbody>
</table>

\{1,23,45\} \Rightarrow \{13,245\}

Machine M1
Operations on Partitions represented as Multi-lines

\[
\begin{align*}
\{ \{1\}, \{2,3,4,5\} \} + \\
\{ \{3\}, \{1,2,4,5\} \} &= \{ \{1,3\}, \{2,4,5\} \}
\end{align*}
\]

\[
\{1, 2345\} + \{3,1245\} = \{13,245\}
\]

Union of images of predecessors
Operations on Partitions represented as Multi-lines

Intersection (called also a product) of partitions

b)

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
\{13,4,25\} & \{1,3,4,25\} & \{1,3,4,25\} & \{1,3,4,25\} \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
\{13,4,25\} & \{1,3,4,25\} & \{1,3,4,25\} & \{1,3,4,25\} \\
\end{array}
\]
Operations on Partitions represented as Multi-lines

\[
\{12,34,5\} + \{13,24,5\} = \{1234,5\}
\]

\[
\{1,23,45\} \geq \{1,2,3,45\}
\]

\[
\{123,45\} \geq \{12,34,5\}
\]
• These methods are used to find a good state assignment.

• This means, the assignment that minimizes the total number of variables as arguments of excitation (and output) functions.

• The is a correspondence between the structure of the set of all partition pairs for all two-block (proper) partitions of a machine and the realization (decomposition) structure of this machine.

• Simple pairs lead to simple submachines.
Theorem 5.3. If there is transition $\prod_1 \rightarrow \tau_1$ and $\prod_i \geq \tau_{s1} \ldots \tau_{sn}$ then $D1$ is a logic function of only input signals and flip-flops $Q_{s1} \ldots Q_{sn}$.

Fig. 5.37. Structure of automaton illustrating application of Theorem 5.3.
Let us assume D type Flip - Flops

For machine M2 partitions (1235,4) = T₄ and (125, 34)=T₃₄ are good for y₁

For machine M2 partition (123,45)=T₄₅ is good for y₂
Calculation of all partition pairs for Machine M2

Selection of partitions

Partitions good for output are circled
Selected Partitions

- $T_{23}$ is always good since it has a predecessor of 1
- Out of many pairs of proper partitions from the graph we select partitions $T_{34}$ and $T_{45}$ because they are both good for outputs
- So now we know from the main theorem that the (logic) excitation function of the Flip-flop encoded with partition $T_{23}$ will depend only on input signals and not on outputs of other flip-flops
- We know also from the main theorem that the excitation function of flip-flop encoded with $T_{45}$ will depend only on input signals and flip-flop encoded with partition $T_{34}$
- The question remains how good is partition $T_{34}$. It is good for output but how complex is its excitation function? This function depends either on two or three flip-flops. Not one flip-flop, because it would be seen in the graph. Definitely it depends on at most three, because the product of partitions $T_{23} \cdot T_{34} \cdot T_{45}$ is a zero partition.
- In class we have done calculations following main theorem to evaluate complexity and the result was that it depends on three.
- Please be ready to understand these evaluation calculations and be able to use them for new examples.
Calculation of partition pair graph from multi-line for machine

T_8 is a good output partition

Select T_{18}, T_{24} and T_8

Explain why this is a good choice

Evaluate complexities of all excitation functions. Next calculate the functions from Kmaps and compare. Give final explanation.
Schematic of machine M3 realized using D Flip-Flops
JK flip-flops are very important since they include D and T as special cases - you have to know how to prove it.

Relation between excitation functions for D and JK flip-flops

![Diagram of JK flip-flop](image)

**Fig 5.36. Example of excitation function for D and JK flip-flops**

**QUESTION:** How to do state assignment for JK flip-flops?

**Fig.5.36**
Let us first recall excitation tables for JK Flip-flops

<table>
<thead>
<tr>
<th>Q</th>
<th>Q⁺</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
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<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
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<td>0</td>
</tr>
</tbody>
</table>

for input J

for input K

1. First stage - draw x-es for ones
2. Second Stage - draw circles for change of flip-flops state

Transition 0->1
Transition 1->1
Transition 0->0
Transition 1->0
Now, thanks to don’t cares from J we can write:

$$(123,48) \rightarrow T_1$$

$$(23,148) \rightarrow T_1$$

From K we can write:

$$1 \rightarrow T_1$$
For this task we will adapt the Multi-line method

Rules for State Assignment of JK Flip-Flops

<table>
<thead>
<tr>
<th></th>
<th>for input J</th>
<th>for input K</th>
</tr>
</thead>
<tbody>
<tr>
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<td>+</td>
<td>*</td>
</tr>
<tr>
<td>1</td>
<td>⊕</td>
<td>⊕</td>
</tr>
<tr>
<td></td>
<td>+ 0 ⊗</td>
<td>+ 0 ⊗</td>
</tr>
</tbody>
</table>

These are the mechanical rules for you to follow, but where they come from?
### Calculation of partition pairs assuming JK flip-flops for machine M3

<table>
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<th>for input J</th>
<th>for input K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>8</td>
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**Current encoding**

- Transition 0->0 has excitations 0- for JK.
- Transition 1->1 has excitations -0 for JK.

**Machine M3**
The subsequent stages are the following.

1. From multiline draw the graph of transitions for both J and K inputs.

2. Mark partitions good for output

3. Find partition pairs that simplify the total cost, exactly the same as before.

Therefore the multi-line method can be extended for any type of flip-flops and for incompletely specified machines.
Fig. 5.43. Schematic of FSM from Example 5.7 realized with JK Flip-flops