State Assignment Using Partition Pairs

- Discuss hypercube method, add slides later on

State Assignment Using Partition Pairs

- This method allows for finding high quality solutions but is slow and complicated
- Only computer approach is practical
- **Definition of Partition.**
  - Set of blocks B_i is a partition of set S if the union of all these blocks forms set S and any two of them are disjoint
  - B1 u B2 u B3 … = S
  - B1 /\ B2={}, B2 /\ B3 … = {}, etc
  - Example 1: {12,45,36}, {{1,2},{4,5},{3,6}}
  - Example 2: {123,345} not a partition but a **set cover**

State Assignment Using Partition Pairs

- **Definition of X-successor of state S_a**
  - The state to which the machine goes from state S_a using input X
- **Definition of Partition Pair**
  - P1=> P2 is a partition pair if for every two elements S_a and S_b from any block in P1 and every input symbol X_i the X_i successors of states S_a and S_b are in the same block of P2

State Assignment Using Partition Pairs

Methods of calculation of Partition Pairs

- Partition pair P1 => P2 calculated with known partition P1
- Partition Pair P1 => P2 calculated with known partition P2

Multi-line method

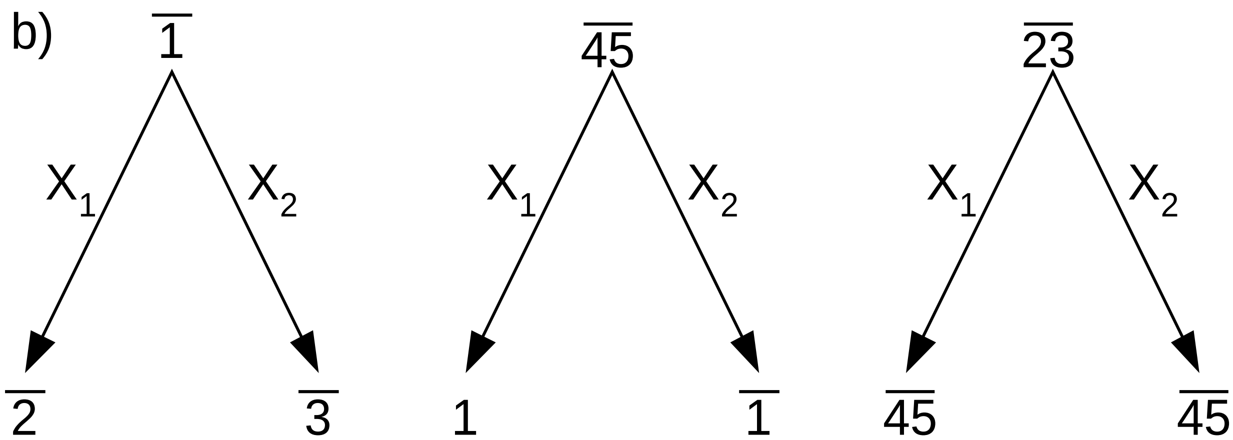
Multi-line method

Multi-line method

Calculation of successor partition from the predecessor partition in the partition pair

{1,23,45} => ???

	X_1	X_2
1	2	3
2	4	5
3	5	4
4	1	1
5	1	--



{1,23,45}=>{1,2,3,45}

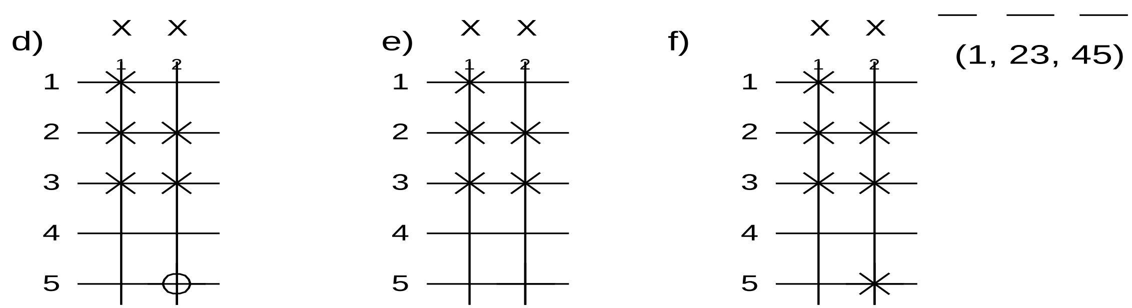
Calculation of successor partition from the successor partition in the partition pair

0 1

{WHAT??} => {13,245}

	X_1	X_2
1	2	3
2	4	5
3	5	4
4	1	1
5	1	--

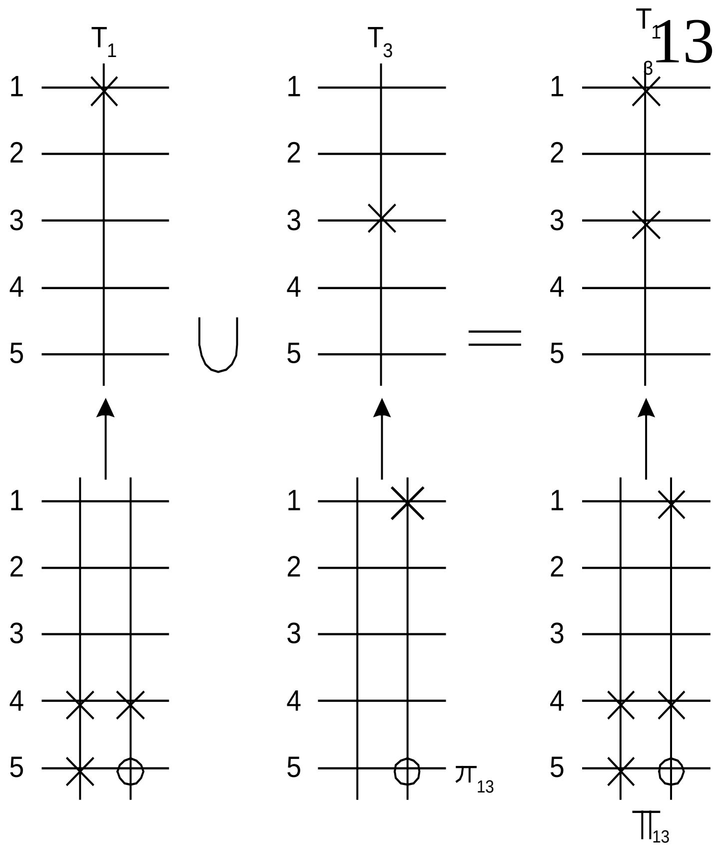
	X_1	X_2
1	1	0
2	1	1
3	1	1
4	0	0
5	0	--



{1,23,45} => {13,245}

Machine M1

Operations on Partitions represented as Multi-lines



{{1}, {2,3,4,5}} +
{{3},{1,2,4,5}}={{1,3},{2,4,5}}

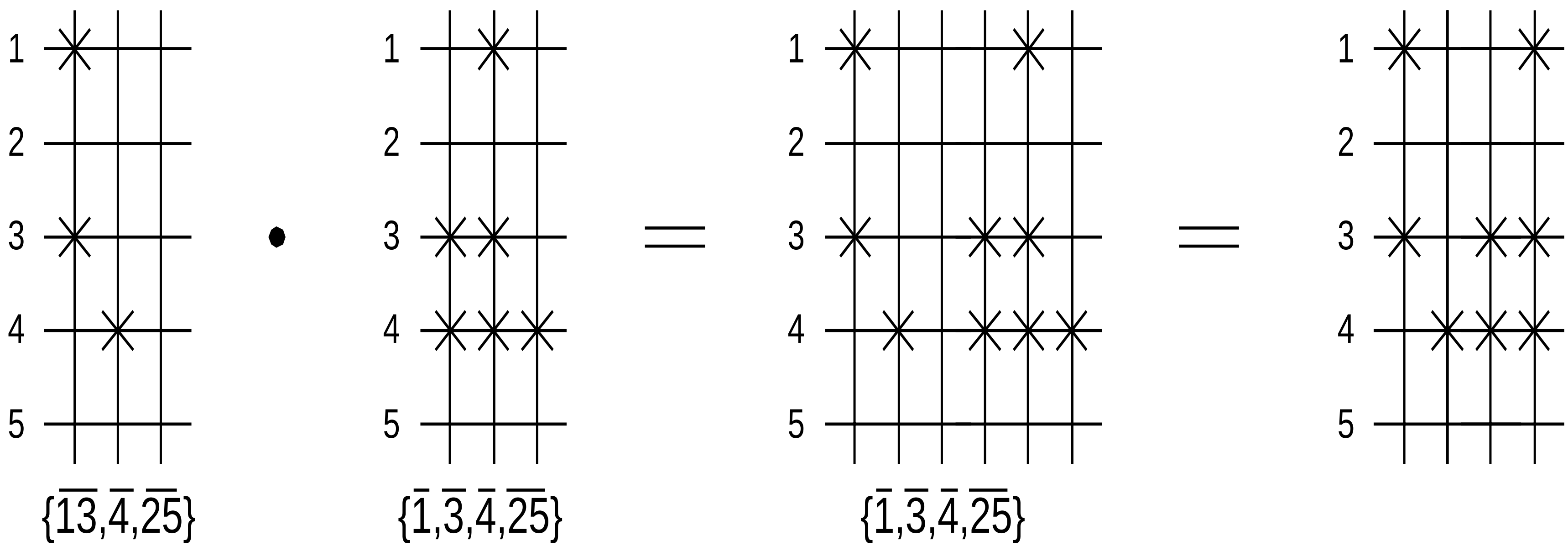
{1, 2345} + {3,1245}={13,245}

Union of images of predecessors

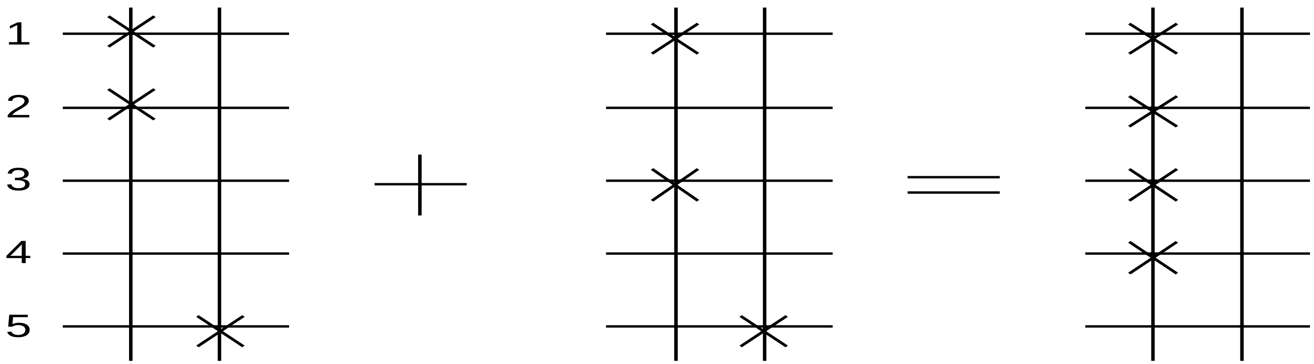
Operations on Partitions represented as Multi-lines

Intersection (called also a product) of partitions

b)

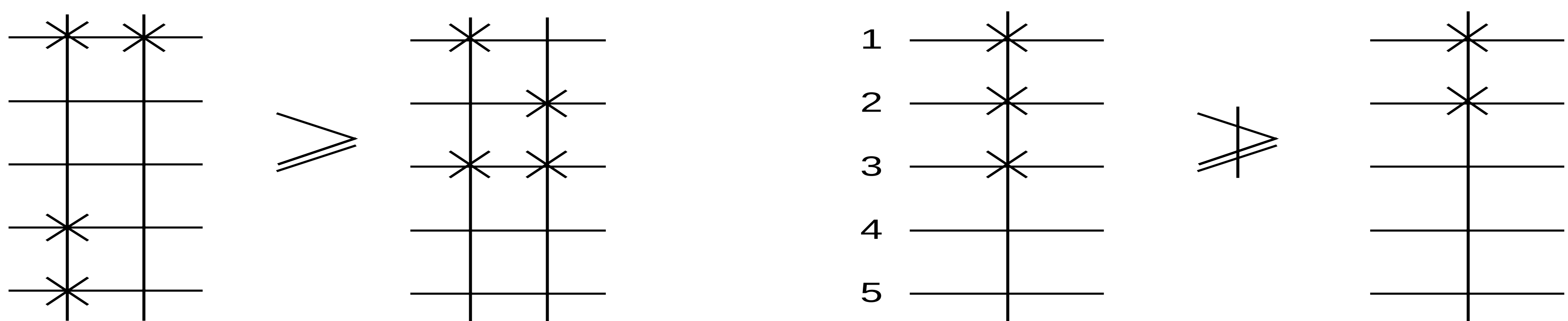


Operations on Partitions represented as Multi-lines



Combine Overlapping Blocks

$\{\overline{12},\overline{34},\overline{5}\} + \{\overline{13},\overline{24},\overline{5}\} = \{\overline{1234},\overline{5}\}$



$\{\overline{1},\overline{23},\overline{45}\} \geqslant \{\overline{1},\overline{2},\overline{3},\overline{45}\}$ $\{\overline{123},\overline{45}\} \ngeqslant \{1\overline{2},\overline{345}\}$

- These methods are used to find a good state assignment.
- This means, the assignment that minimizes the total number of variables as arguments of excitation (and output) functions.
- The is a **correspondence** between the structure of the set of all partition pairs for all two-block (proper) partitions of a machine and the realization (decomposition) structure of this machine
- Simple pairs lead to simple **submachines**

<u>Theorem 5.3.</u> If there is transition Π_I --> τ_I and Π_i >= $\tau_{s1.....}$ τ_{sn} then D1 is a logic function of only input signals and flip-flops Q_{s1}… Q_{sn}

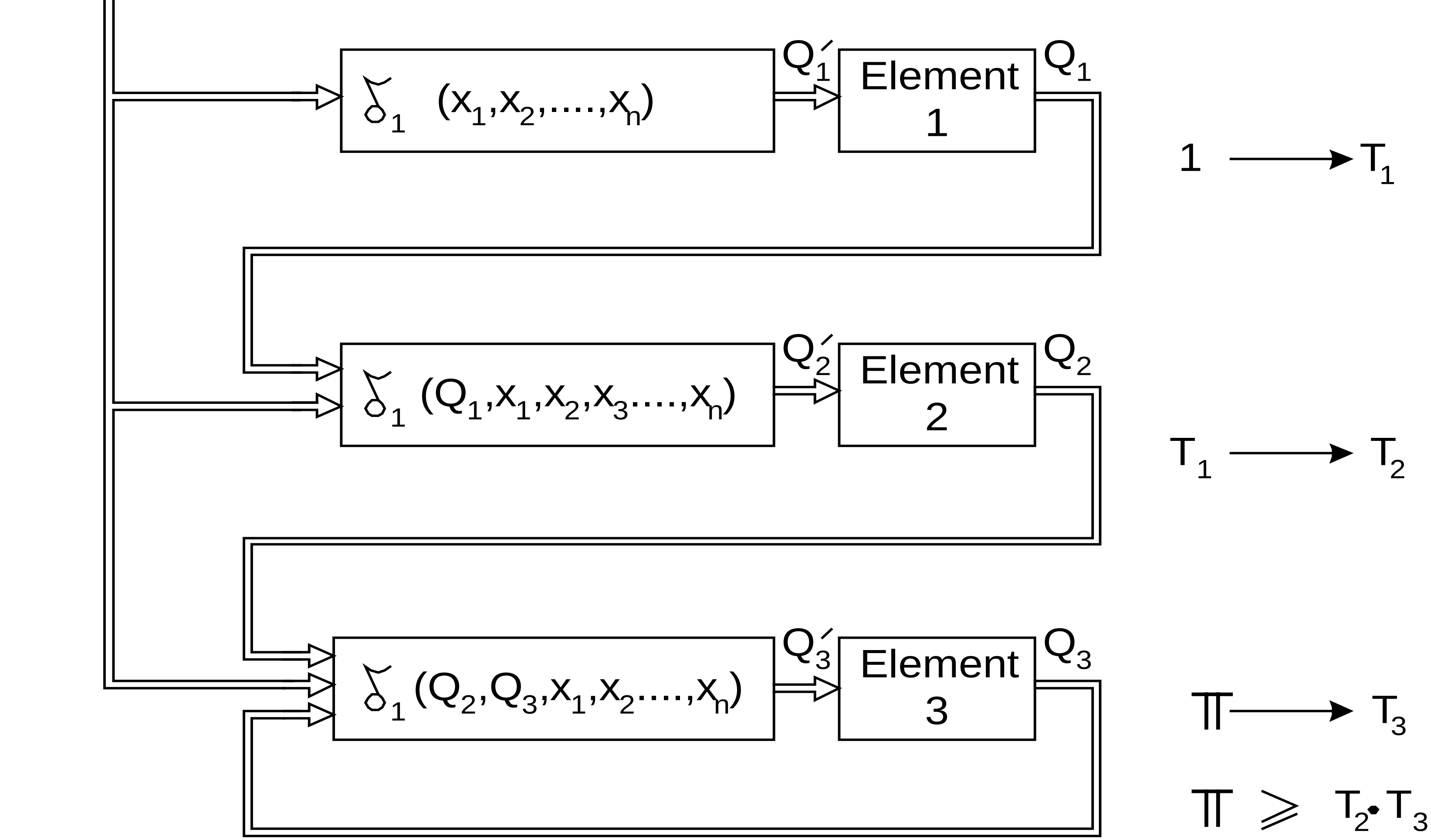


Fig.5.37. Structure of automaton illustrating application of Theorem 5.3

Let us assume D type Flip - Flops

a)

A \ X	X_1	X_2	$y_1 y_2$
1	1	2	0 0
2	1	3	0 0
3	4	3	- 0
4	5	3	1 1
5	1	3	0 1

b)

A \ X	X_1	X_2
1	1 / 1	2 / 1
2	1 / 0	3 / 0
3	4 / 1	3 / 0
4	5 / 0	3 / 0
5	1 / 1	3 / 1

c)

$Q_1Q_2Q_3$ \ X	X_1	X_2
0 0 0	000 / 1	010 / 1
0 1 0	000 / 0	110 / 0
1 1 0	101 / 1	110 / 0
1 0 1	001 / 0	110 / 0
0 0 1	000 / 1	110 / 1

d)

A \ X	X_1	X_2
1	1 / 1	2 / 1
2	1 / 0	3 / 0
3	4 / 1	3 / -
4	5 / -	3 / 0
5	1 / -	3 / 1

$\Pi x_1 (y_1) = \{\overline{125}, \overline{4}, (3)\}$

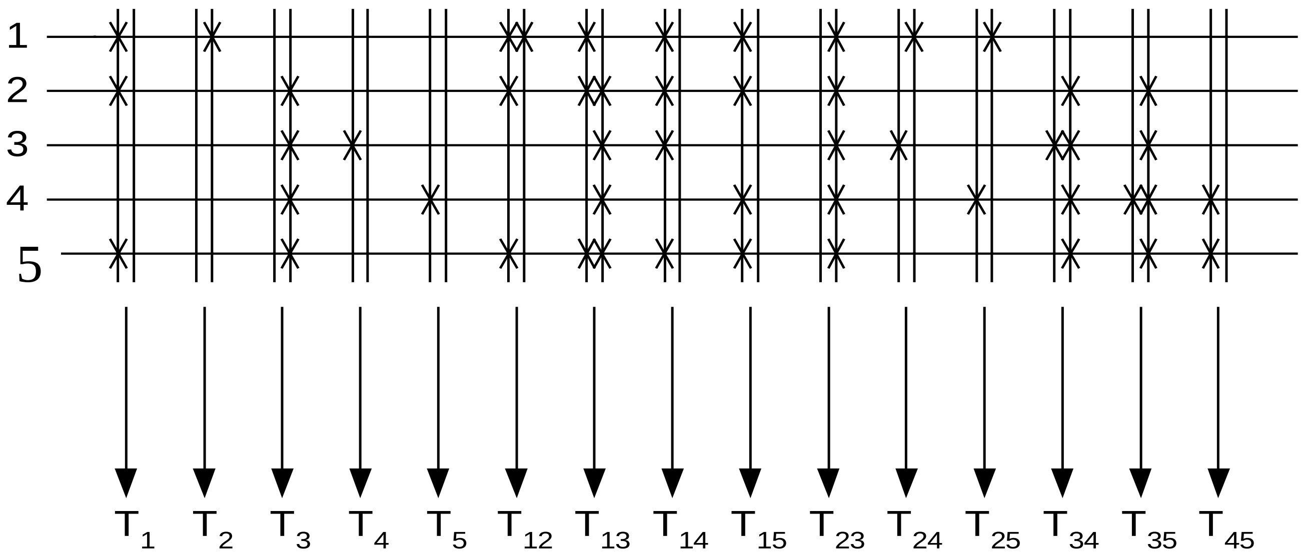
For machine M2 partitions (1235,4) $=T_4$ and (125, 34)$=T_{34}$ are good for y_1

For machine M2 partition (123,45)$=T_{45}$ is good for y_2

Machine M2

Calculation of all partition pairs for Machine M2

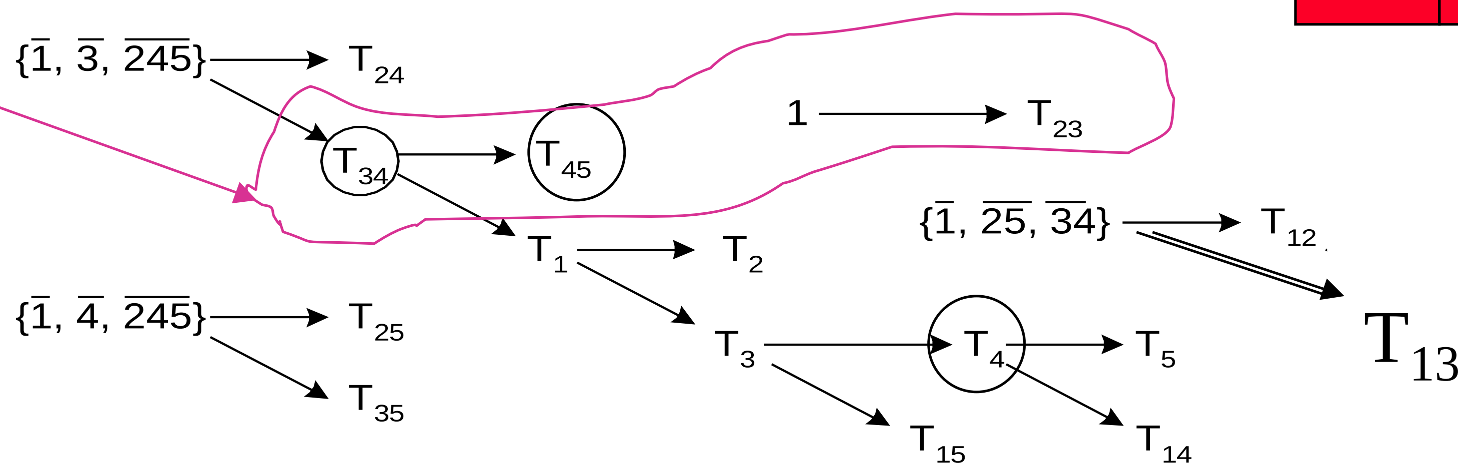
a)



	X_1	X_2
1	1	2
2	1	3
3	4	3
4	5	3
5	1	3

Selection of partitions

b)



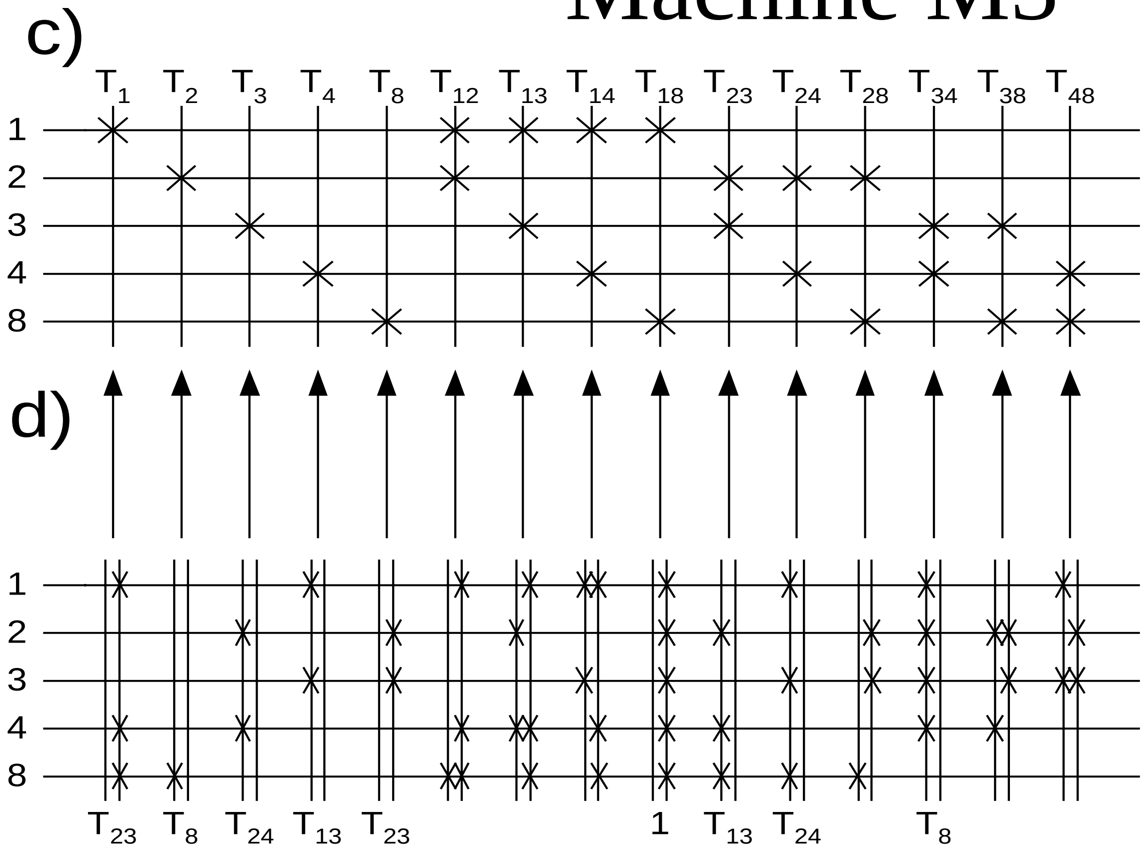
Partitions good for output are circled

Selected Partitions

- T_{23} is always good since it has a predecessor of 1
- Out of many pairs of proper partitions from the graph we select partitions T_{34} and T_{45} because they are both good for outputs
- So now we know from the main theorem that the (logic) excitation function of the Flip-flop encoded with partition T_{23} will depend only on input signals and not on outputs of other flip-flops
- We know also from the main theorem that the excitation function of flip-flop encoded with T_{45} will depend only on input signals and flip-flop encoded with partition T_{34}
- The question remains how good is partition T_{34}. It is good for output but how complex is its excitation function? This function depends either on two or three flip-flops. Not one flip-flop, because it would be seen in the graph. Definitely it depends on at most three, because the product of partitions T_{23} T_{34} T_{45} is a zero partition.
- In class we have done calculations following main theorem to evaluate complexity and the result was that it depends on three.
- Please be ready to understand these evaluation calculations and be able to use them for new examples.

Machine M3

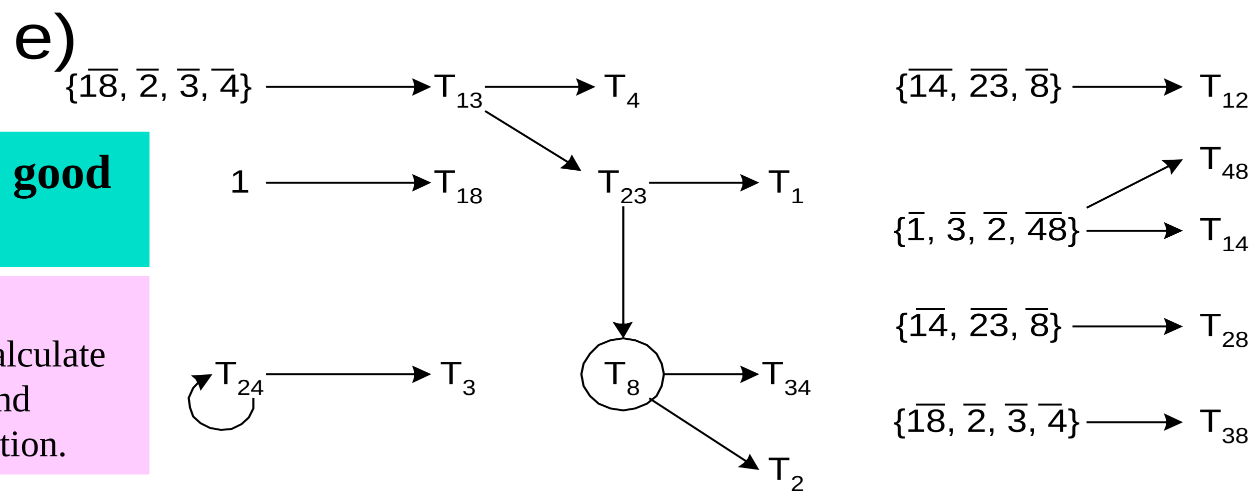
Calculation of partition pair graph from multi-line for machine



	X_1	X_2
1	**4**	**1**
2	**3**	**8**
3	**4**	**8**
4	**3**	**1**
8	**2**	**1**

T_8 is a good output partition

Select T_{18}, T_{24} and T_8

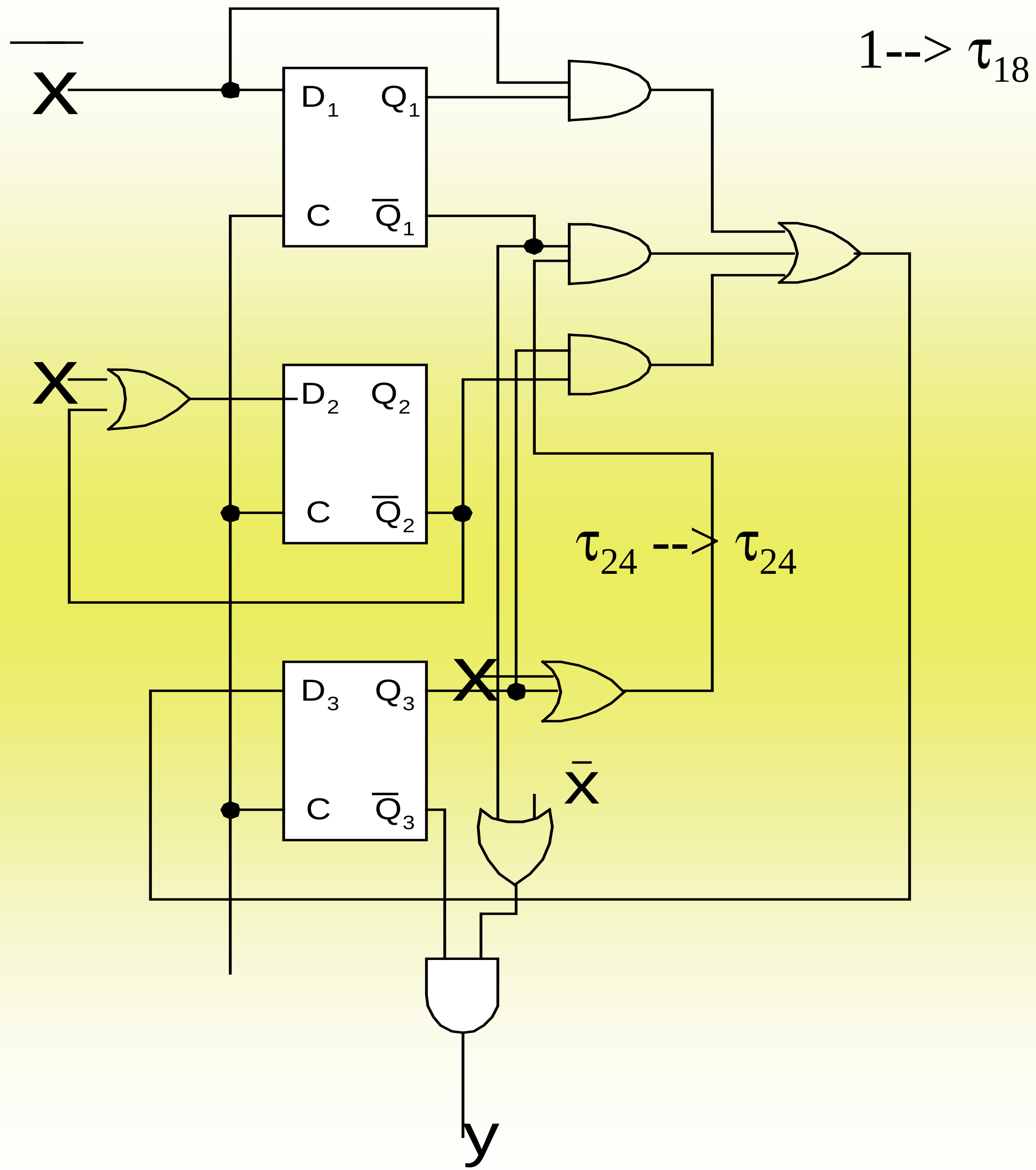


GRAPH OF PARTITION PAIRS

Explain why this is a good choice

Evaluate complexities of all excitation functions. Next calculate the functions from Kmaps and compare. Give final explanation.

Schematic of machine M3 realized using D Flip-Flops



JK flip-flops are very important since they include D and T as special cases - you have to know how to prove it

Relation between excitation functions for D and JK flip-flops

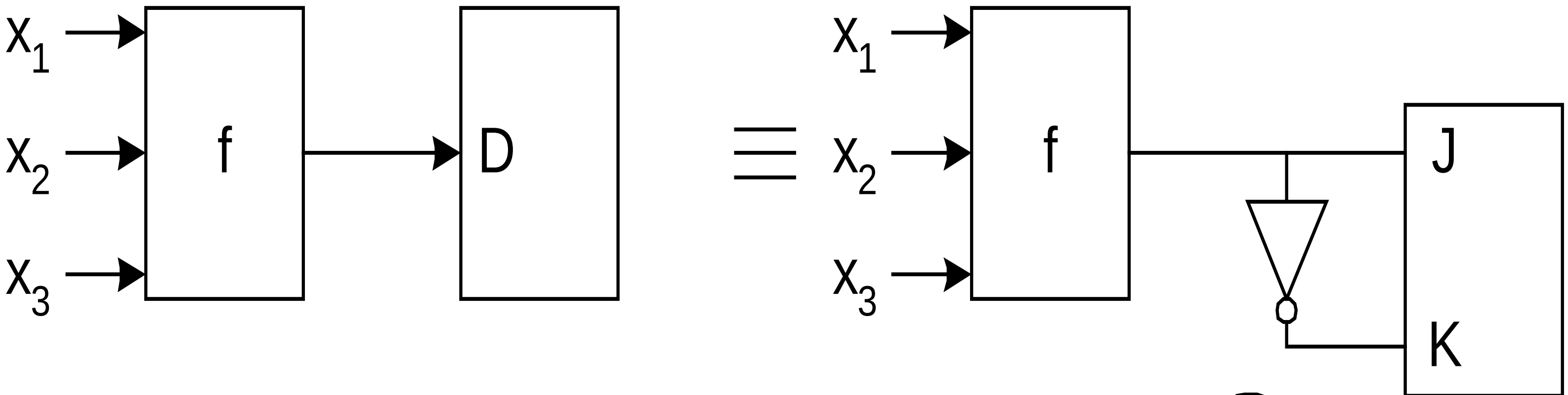


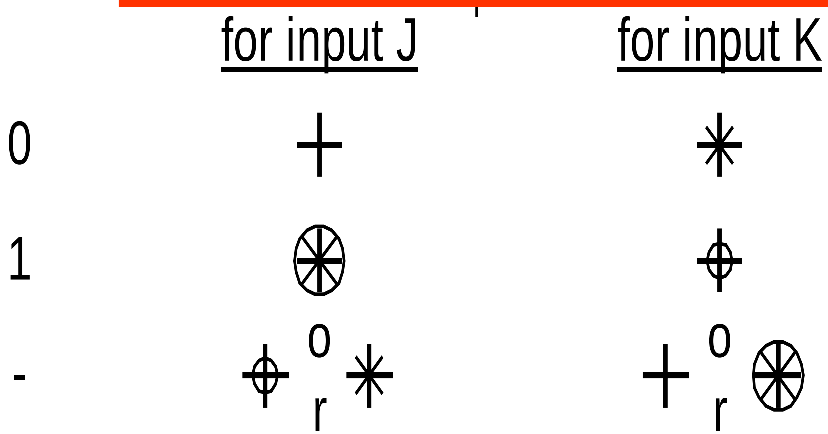
Fig 5.36. Example of excitation function for D and JK flip-flops

Fig.5.36

QUESTION: How to do state assignment for JK flip-flops?

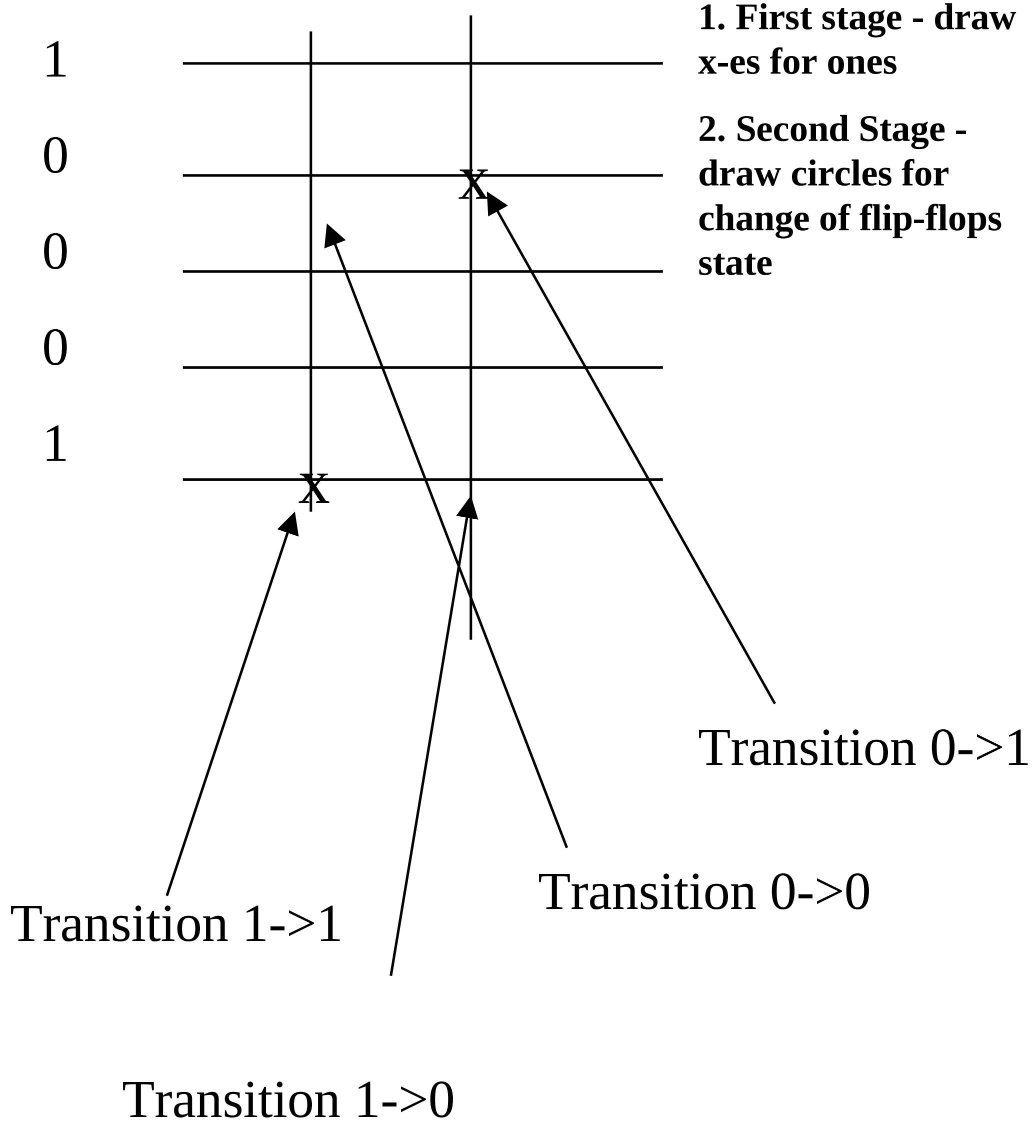
Let us first recall excitation tables for JK Flip-flops

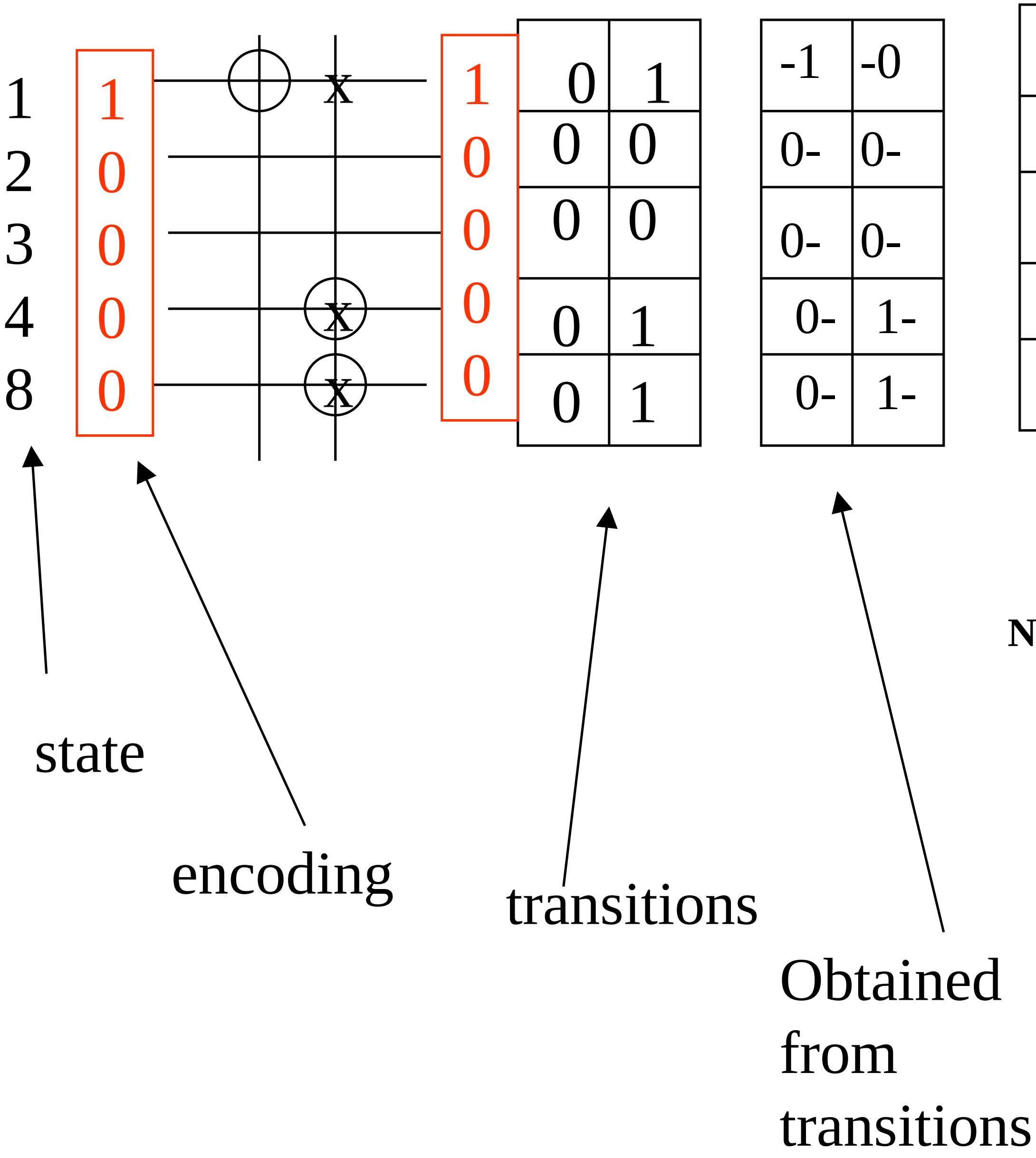
Q	Q^+	J	K
0	0	0	-
0	1	1	-
1	0	-	1
1	1	-	0



1. First stage - draw x-es for ones

2. Second Stage - draw circles for change of flip-flops state





-	-
0	0
0	0
0	1
0	1

J

1	0
-	-
-	-
-	-
-	-

K

Now, thanks to don't cares from J we can write :

$(123,48){-}{-}{>}T_1$

$(23,148){-}{-}{>}T_1$

From K we can write :

1 $-->T_1$

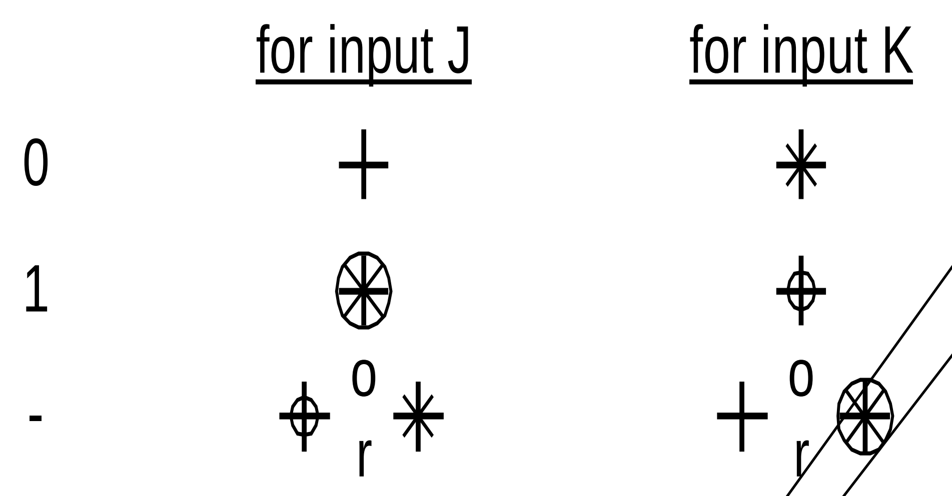
For this task we will adapt the Multi-line method

Rules for State Assignment of JK Flip-Flops

	<u>for input J</u>	<u>for input K</u>
0	+	✳
1	⊛	⊕
-	⊕ or ✳	+ or ⊛

These are the mechanical rules for you to follow, but where they come from?

Calculation of partition pairs assuming JK flip-flops for machine M3



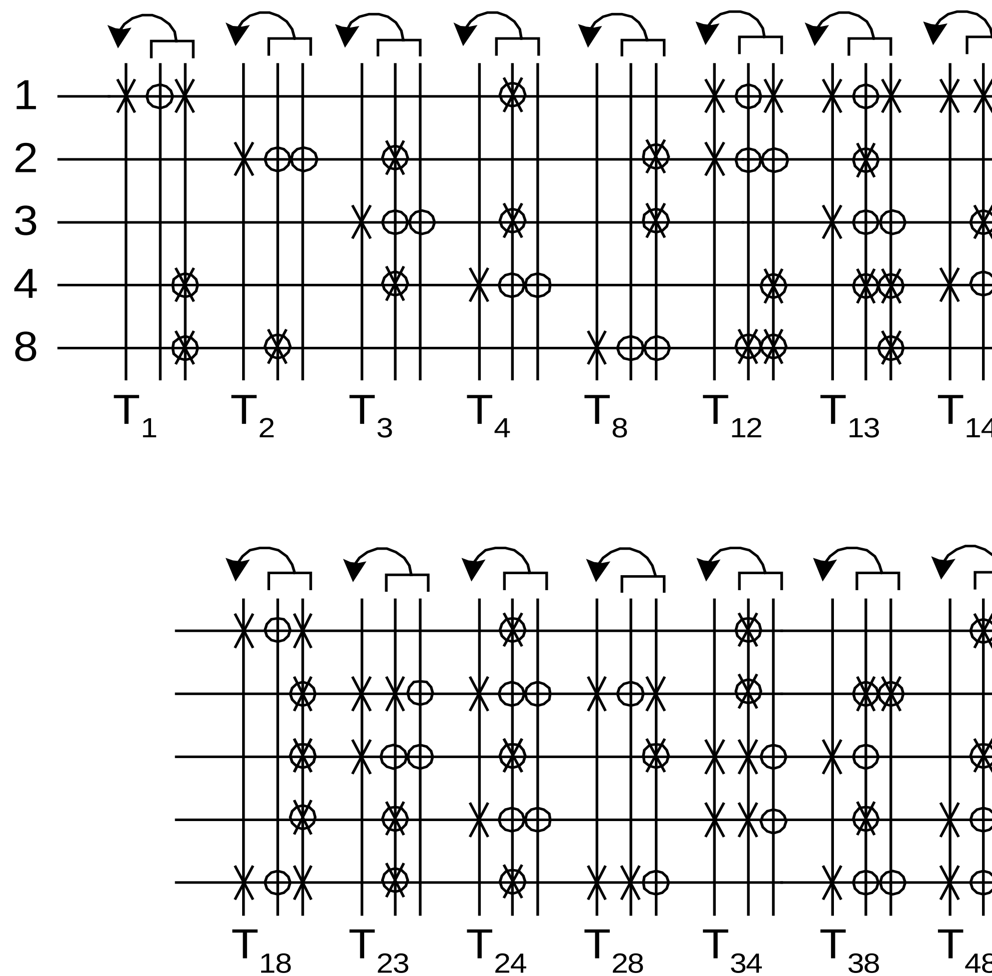
Transition 0->0 has excitations 0- for JK.

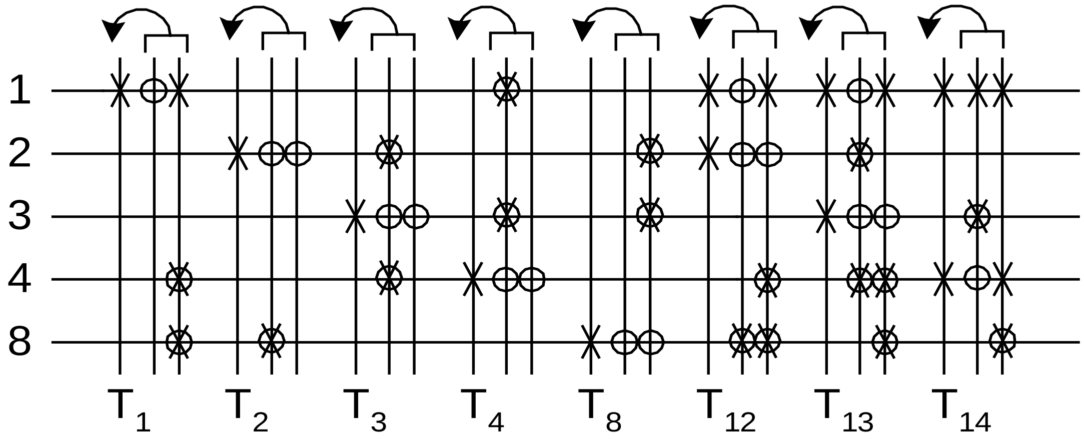
Transition 1->1 has excitations -0 for JK.

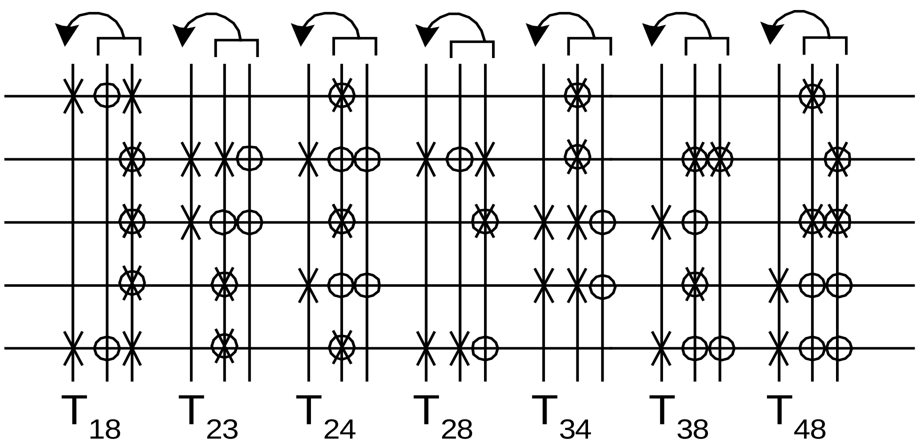
Current encoding

	X_1	X_2			
1	4	1	1	0	1
2	3	8	0	0	0
3	4	8	0	0	0
4	3	1	0	0	1
8	2	1	0	0	1

Machine M3







The subsequent stages are the following.

1. From multiline draw the graph of transitions for both J and K inputs.

2. Mark partitions good for output

3. Find partition pairs that simplify the total cost, exactly the same as before.

There fore the multi-line method can be extended for any type of flip-flops and for incompletely specified machines.

Fig.5.43. Schematic of FSM from Example 5.7 realized with JK Flip-flops